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Constructions of Strict Lyapunov Functions: An Overview

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Abstract—Mathematical control theory provides the theoretical foundations that undergird many modern technologies, including aeronautics, biotechnology, communications networks, manufacturing, and models of climate change. During the past fifteen years, there have been numerous exciting developments at the interface of control engineering and mathematical control theory. Many of these advances were based on new Lyapunov methods for analyzing and stabilizing nonlinear systems. Constructing strict Lyapunov functions is a central and challenging problem. On the other hand, non-strict Lyapunov functions are often constructed easily, using passivity, backstepping, or forwarding, or by taking the Hamiltonian for Euler-Lagrange systems. Roughly speaking, non-strict Lyapunov functions are characterized by having negative semi-definite time derivatives along all trajectories of the system, while strict Lyapunov functions have negative definite derivatives along the trajectories. Even when we know a system to be globally asymptotically stable, it is often still important to have an explicit global strict Lyapunov function, e.g., to design feedbacks that give input-to-state stability to actuator errors.

One important research topic involves finding necessary and sufficient conditions for different kinds of stability, in terms of the existence of Lyapunov functions, such as Lyapunov characterizations for hybrid systems, or for systems with measurement uncertainty and outputs. Some of the most significant recent work in this direction has been carried out by Andrew Teel and his co-workers, who employ systems on hybrid time domains that encompass continuous time and discrete time systems as special cases. Converse Lyapunov function theory implies the existence of strict Lyapunov functions for large classes of globally asymptotically stable nonlinear systems. However, the Lyapunov functions given by converse theory are often abstract or non-explicit, and so may not always lend themselves to feedback design. Explicit strict Lyapunov functions are also important for quantifying the effects of uncertainty, because, e.g., they can be used to build the comparison functions in the input-to-state stability estimate. In fact, once we construct a suitable global strict Lyapunov function, several significant stabilization and robustness problems can be solved almost immediately, using standard arguments.

In some situations, non-strict Lyapunov functions are enough, because they can be used in conjunction with LaSalle Invariance or Barbalat's Lemma to show global asymptotic

stability. In other cases, it suffices to analyze the system around a reference trajectory, or near an equilibrium point, so linearizations having simple local quadratic Lyapunov functions apply. However, it is now well appreciated that linearizations and non-strict Lyapunov functions are insufficient to analyze general time-varying nonlinear systems. Non-strict Lyapunov functions are not well suited for robustness analysis, because their negative semi-definite time derivatives along the trajectories could become positive under small uncertainties of the system. Uncertainties usually arise in applications, because of unknown model parameters, or noise entering controllers. For this reason, input-to-state stability and other robustness proofs often rely on finding global strict Lyapunov functions. Also, there are important classes of nonlinear systems (such as chemostat models) that often evolve far from their equilibria. This has motivated a significant body of research on ways to explicitly construct strict Lyapunov functions.

One approach to designing explicit strict Lyapunov functions, which has received a lot of attention in the past few years, is the so-called strictification approach. This entails transforming given non-strict Lyapunov functions into explicit global strict Lyapunov functions, under appropriate nondegeneracy conditions on the non-strict Lyapunov function. The approach has been successfully employed in many contexts, including adaptive control, Hamiltonian systems satisfying the conditions from the Jurdjevic-Quinn theorem, and time-varying hybrid dynamical systems with mixtures of continuous and discrete time evolutions. Strictification reduces difficult strict Lyapunov function construction problems to oftentimes much simpler non-strict Lyapunov function construction problems. This talk will present an overview of the strictification approach for systems satisfying the conditions from Matrosov's Theorem, which express the nondegeneracy of the nonstrict Lyapunov function in terms of auxiliary scalar functions. The simplicity of our strict Lyapunov function constructions makes them suitable for quantifying the effects of uncertainty, and for feedback design. We illustrate our work using two important biotechnological examples, the first involving anaerobic digesters and the second involving Lotka-Volterra systems.

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