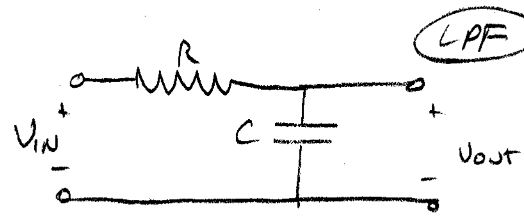
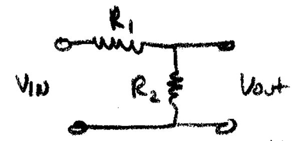
FRIDAY | OCT 1

9.2 SINUSOIDAL STEADY STATE AMP RESPONSE

SYSTEM FCN OR XFER FCN

LPF





$$V_o = \frac{R_2}{R_1+R_2} V_{IN}$$

VOLTAGE DIVISION

$$\text{XFER FCN} = \frac{V_{OUT}}{V_{IN}} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1+j\omega RC}$$ LPF

$\omega \ll \frac{1}{RC}$ $\omega RC \ll 1$ $\left|\frac{V_{out}}{V_{IN}}\right| = 1$

CAP SHUNT TO GND.

12.2 SINUSOIDAL FREQUENCY ANALYSIS

A POWER SYSTEM NETWORK OPERATES AT ONLY ONE FREQUENCY - IN GENERAL WE WANT TO ANALIZE NETWORK BEHAVIOR AS A FUNCTION OF FREQUENCY.

NETWORK FUNCTION $H(j\omega) = M(\omega)e^{j\phi(\omega)}$

$M(\omega) = |H(j\omega)|$ magnitude and phase characteristics

$\phi(\omega) = \measuredangle H(j\omega)$

FREQUENCY RESPONSE USING BODE PLOT - SEMILOG OR LOG-LOG

MAGNITUDE IS PLOTTED IN dB VS. $\log_{10}(\omega)$

dB - originally used to describe power ratio

$$\boxed{\#\text{ in dB} = 10\log\frac{P_2}{P_1}}$$

$P_2 = \frac{V_2^2}{R}$ $P_1 = \frac{V_1^2}{R}$

$P_2 = I_2^2R$ $P_1 = I_1^2R$

$$= 10\log\frac{V_2^2/R}{V_1^2/R} = 20\log\frac{V_2}{V_1} = 20\log_{10}\frac{I_2}{I_1}$$

$$H(j\omega) = \frac{K_o(j\omega)^{\pm N}(1+j\omega\tau_1)\left[1+2\zeta_3(j\omega\tau_3)+(j\omega\tau_3)^2\right]\cdots}{(1+j\omega\tau_a)\left[1+2\zeta_b(j\omega\tau_b)+(j\omega\tau_b)^2\right]\cdots}$$

1. K_o is Frequency independent factor $K_o > 0$
2. Poles or zeros at origin of the form $j\omega$ $(j\omega)^{+N}$ for zeros $(j\omega)^{-N}$ for poles
3. Ploes or zeros of the form $(1+j\omega\tau)$
4. QUADRATIC POLES OR ZEROS OF THE FORM $1+2\zeta(j\omega\tau)+(j\omega\tau)^2$

$$20\log_{10}|H(j\omega)| = 20\log_{10}K_o \pm 20N\log_{10}|j\omega| + 20\log_{10}|1+j\omega\tau_1|$$
$$+ 20\log_{10}|1+2\zeta_3(j\omega\tau_3)+(j\omega\tau_3)^2| + \ldots$$
$$- 20\log_{10}|1+j\omega\tau_a| - 20\log_{10}|1+2\zeta_b(j\omega\tau_b)+(j\omega\tau_b)^2|\ldots$$

log of product = sum of logs

$\log ab = \log a + \log b$

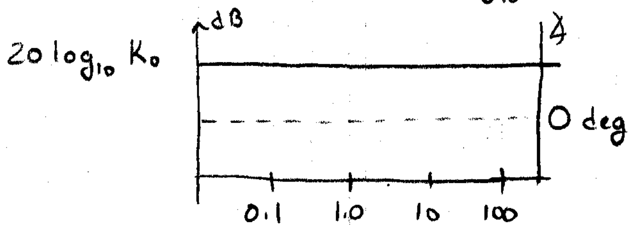
$\log a^n = n\log a$

$$\measuredangle H(j\omega) = 0 \pm N(90^\circ) + \tan^{-1}\omega T_1 + \tan^{-1}\left(\frac{2\zeta_3\omega T_3}{1-\omega^2 T_3^2}\right) + \cdots$$

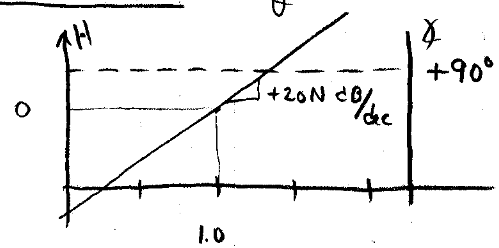
$$- \tan^{-1}\omega T_a - \tan^{-1}\left(\frac{2\zeta_b\omega T_b}{1+\omega^2 T_b^2}\right) \cdots$$

PLOT EACH FACTOR INDEPENDENTLY ON A COMMON GRAPH THEN SUM THEM ALGEBRAICALLY.

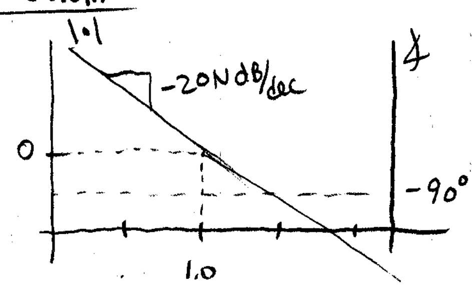
CONSTANT TERM: $20\log_{10} K_o$



ZERO AT ORIGIN $(j\omega)^N$



POLE AT ORIGIN



SIMPLE POLE OR ZERO $(1 + j\omega\tau)$ LINEAR APPROXIMATIONS.

(A) FOR $\omega\tau << 1$ $\quad (1 + j\omega\tau) \approx 1 \quad 20 \log (1 + j\omega\tau) = 0\,dB$

(B) $\omega\tau >> 1$ $\quad (1 + j\omega\tau) \approx j\omega\tau \quad 20 \log (1 + j\omega\tau) \cong 20 \log_{10} \omega\tau$

(A) Response is FLAT, 0dB
(B) SAME AS POLE OR ZERO AT ORIGIN

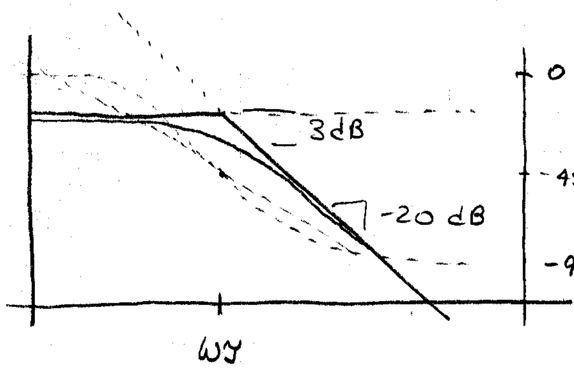
THE INTERSECTION OF THESE 2 ASYMPTOTES IS CALLED A <u>BREAK FREQUENCY</u>, $\omega = \frac{1}{\tau}$

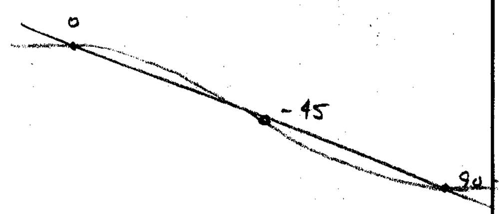
$$\omega = 1/\tau \qquad 20 \log_{10} |1 + j1| = 20 \log (2)^{1/2} = 3\,dB$$

CURVE DEVIATES FROM asymptotes by 3dB at the break freq.

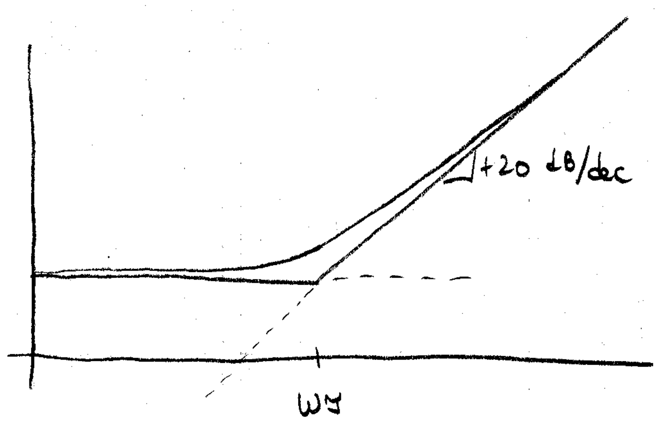
<u>PHASE</u> $\quad \phi = \tan^{-1} \omega\tau \quad$ at $\quad \omega\tau = 0 \quad \phi = 0°$
$\omega\tau >> 0 \quad \phi = 90°$
$\omega\tau = 1 \quad \phi = 45°$

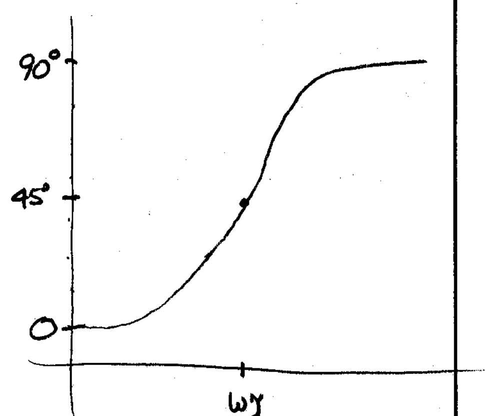
POLE





ZERO





IF MULTIPLE POLES $(1 + j\omega\tau)^N$

SLOPE OF HIGH FREQ ASYMP. IS MULTIPLIED BY N
DEVIATION FROM ACTUAL CURVE AT BREAK FREQUENCY IS $3N\,dB$

PHASE CURVE EXTENDS FROM 0 to $N(90°)$ AND IS $N(45°)$ AT BREAK FREQUENCY.

QUADRATIC POLES OR ZEROS $\quad 1 + 2\zeta(j\omega\tau) + (j\omega\tau)^2$

FUNCTION OF ω
AND ζ damping ratio

$\zeta > 1$ ROOTS ARE REAL AND UNEQUAL

$\zeta = 1$ REAL AND EQUAL

$\zeta < 1$ ROOTS COMPLEX CONJUGATES

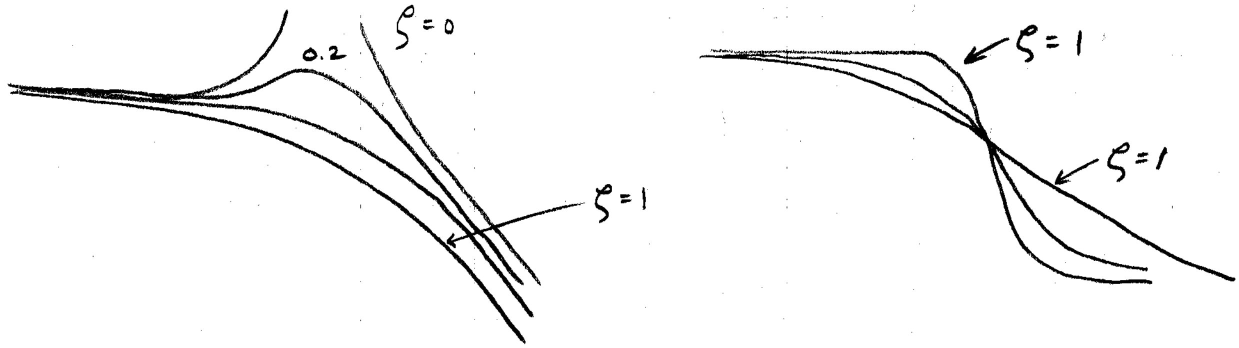
FOR $\omega\tau \ll 1$ $\quad 20\log[\;] = 0$ dB

FOR $\omega\tau \gg 1$ $\quad 20\log_{10}\left|1-(\omega\tau)^2 + 2j\zeta(\omega\tau)\right| \approx 20\log\left|(\omega\tau)^2\right| = 40\log_{10}|\omega\tau|$

SLOPE OF log Mag curve IS $+40$ dB/dec FOR QUAD. ZERO
-40 dB/dec FOR QUAD. POLE

BETWEEN THE EXTREMES, BEHAVIOR IS DEPENDENT ON ζ

FIG 12.13 p678



PHASE FROM 0 to $-180°$ FOR POLES
0 to $+180$ FOR ZERO (INVERTED) $+90 \to 0°$

EX 12.5 $G_v(j\omega) = \dfrac{25 j\omega}{(j\omega+0.5)[(j\omega)^2 + 4j\omega + 100]}$

PUT IN STANDARD FORM

$$\frac{2}{2} = \frac{50 j\omega}{(2j\omega+1)[(j\omega)^2 + 4j\omega + 100]}$$

$$\frac{\frac{1}{100}}{\frac{1}{100}} = \frac{0.5 j\omega}{(2j\omega+1)\left(\left(\frac{j\omega}{10}\right)^2 + \frac{j\omega}{25} + 1\right)}$$

CONSTANT: 0.5 $20\log_{10}(0.5) = -6$ dB $0°$ dB

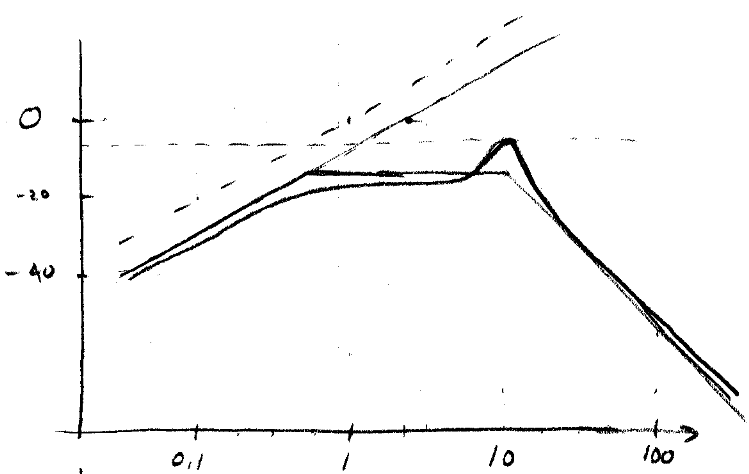
ZERO AT ORIGIN: +20 dB/dec +90° CONSTANT

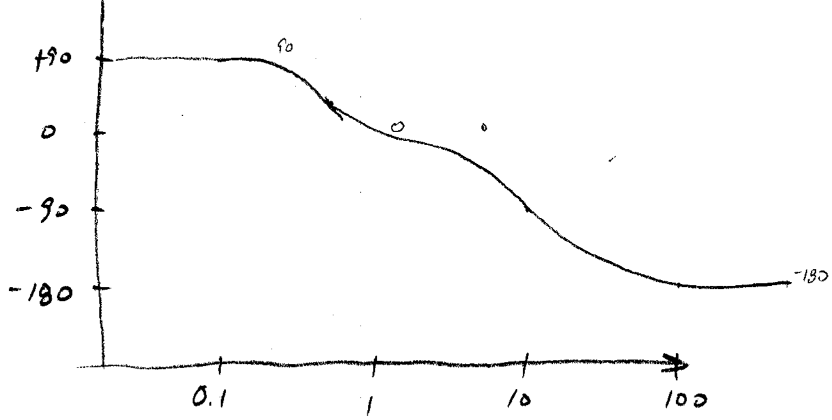
POLE AT $\omega = \frac{1}{2}$: $0 \rightarrow -90°$ -45 @ $\omega = \frac{1}{2}$

QUADRATIC POLE: $\left(\frac{j\omega}{10}\right)^2$ $\tau = \frac{1}{10}$

$\frac{j\omega}{25}$ $2\zeta\tau = \frac{1}{25}$ $\zeta = \frac{1}{25}\left(\frac{10}{1}\right)\left(\frac{1}{2}\right) = \frac{10}{50} = 0.2$

pole at $\omega = 10$, $\zeta = 0.2$ -40 $0 \rightarrow -180°$





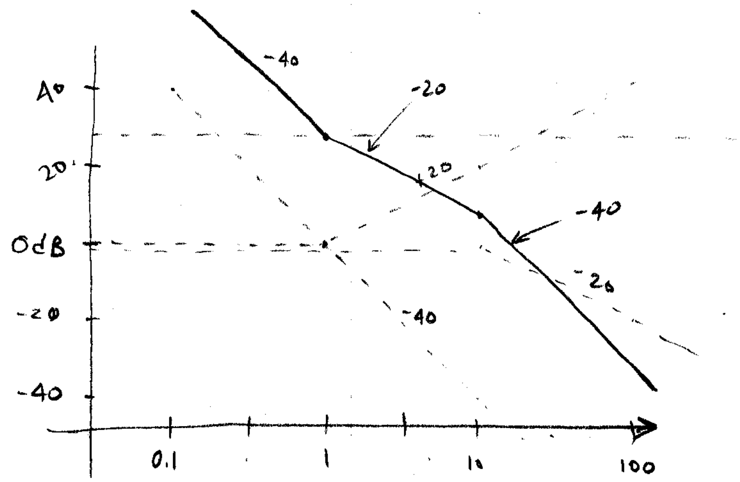
EX 12.4 $G(j\omega) = \dfrac{25\,(j\omega+1)}{(j\omega)^2\,(0.1j\omega+1)}$

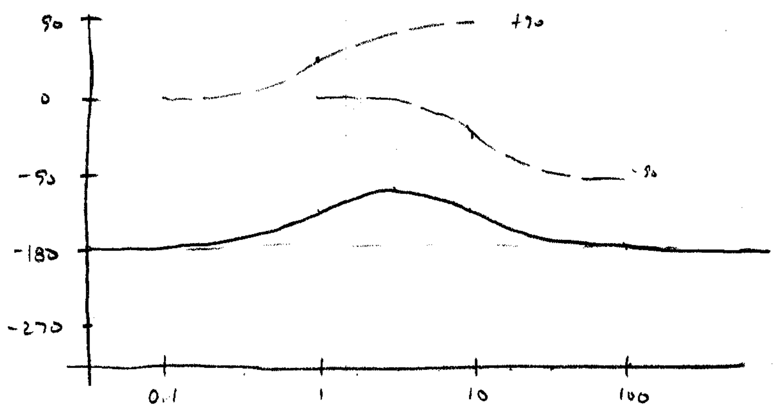
CONSTANT: 25 $\quad 20\log_{10} 25 = 28\text{dB} \quad 0°$

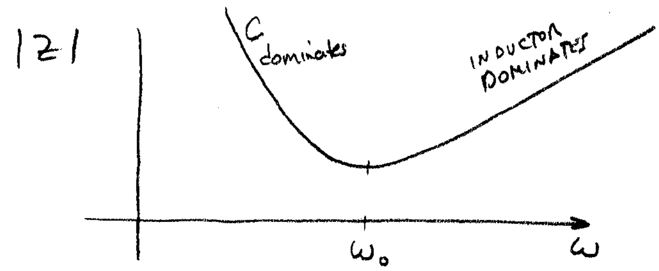
ORIGIN: DOUBLE POLE $\quad -40\,\text{dB/dec} \quad -180°$ for all ω

ZERO AT $\omega=1$ $\quad 0 \to 90$

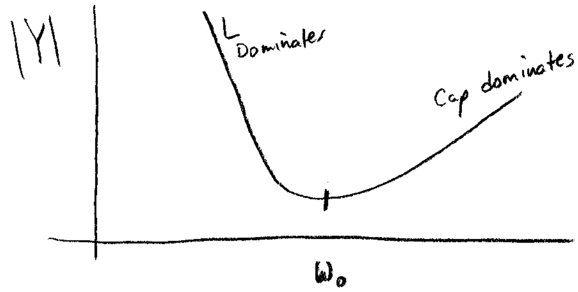
Pole at $\omega = 10$ $\quad 0 \to -90$







series eg goes to open as L goes to open.

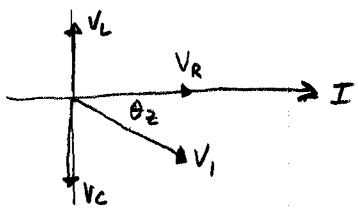


Parallel eg goes to short as $\frac{|}{|}$ goes to short

SERIES RLC - CURRENT IS COMMON TO ALL ELEMENTS
PARALLEL RLC - VOLTAGE " " " "

SERIES CASE WHEN $\omega < \omega_0$ C DOMINATES $V_C > V_L$ θ_Z IS NEG

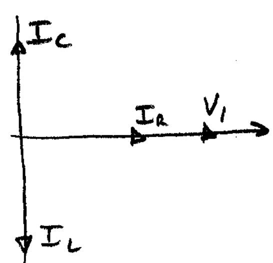
V_1 LAGS I



$\omega = \omega_0$ $V_L = V_C$ $\theta_Z = 0$ V_1 IN PHASE w/ I

$\omega > \omega_0$ $V_L > V_C$ $\theta_Z > 0$ V_1 leads I

Parallel case



$\omega < \omega_0$ L DOMINATES $I_L > I_C$ V_1 leads I

$\omega = \omega_0$ $I_L = I_C$

$\omega > \omega_0$ C DOMINATES $I_C > I_L$ I leads V

QUALITY FACTOR $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

Q IS AN IMPORTANT FACTOR

EX 12.3

$$G_v(j\omega) = \frac{10(0.1j\omega + 1)}{(j\omega+1)(0.02j\omega+1)}$$

CONSTANTS

$K_0 = 10$ $\quad 20\log_{10} K_0 = 20 \text{ dB}$

Zeros

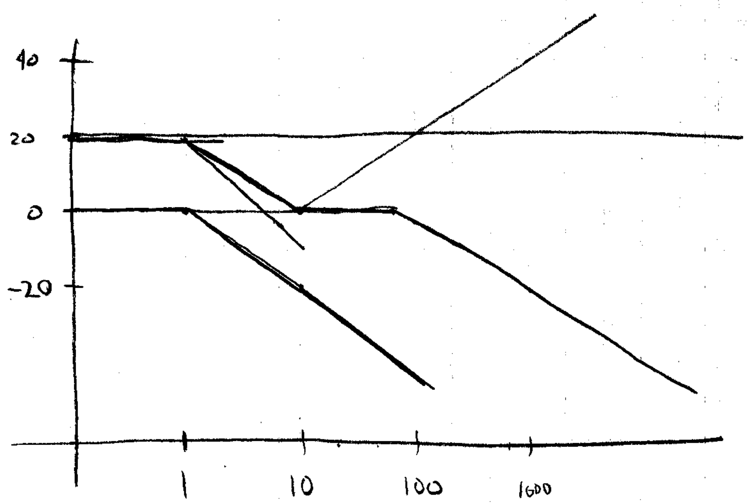
$(0.1j\omega+1)$ $\quad 20\log_{10}|0.1j\omega+1|$ $\quad$ 0 dB FOR $0.1\omega \ll 1$

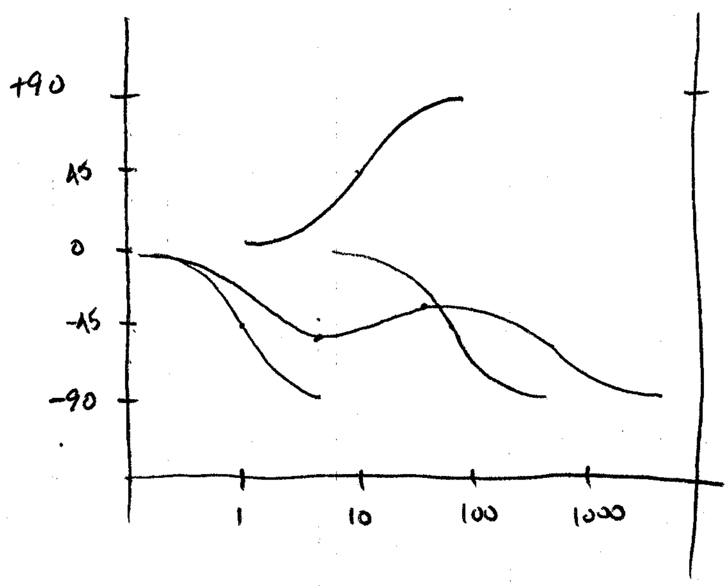
+20 dB/dec FOR $0.1\omega \gg 1$

BREAK AT $\omega = 10$ rad/s

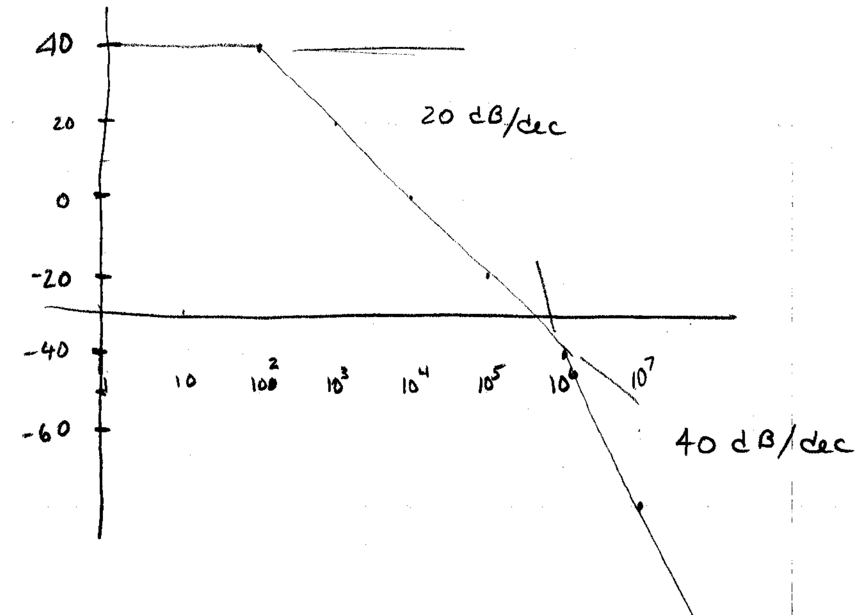
POLES

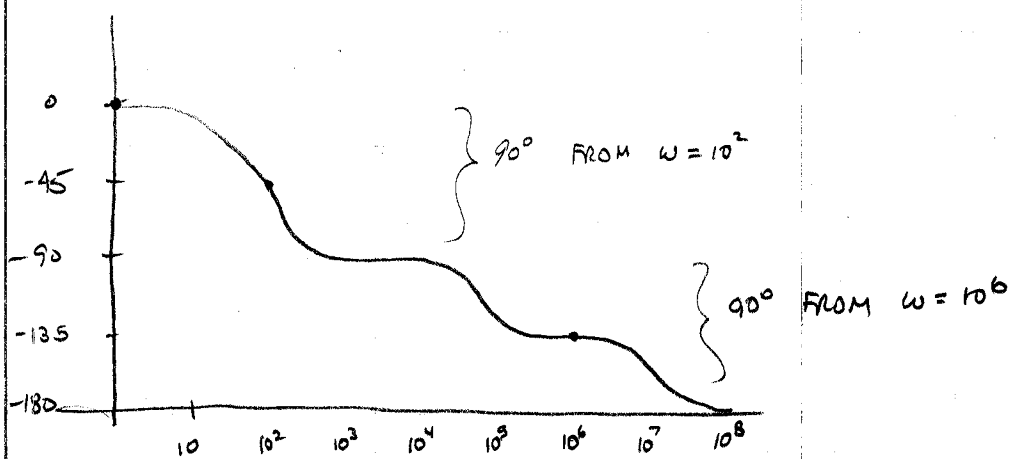
	Break
$(j\omega+1)$	$\omega = 1$
$(0.02j\omega+1)$	$\omega = 50$ rad/s





$$H(j\omega) = \frac{100}{\left(1 + j\omega/10^2\right)\left(1 + j\omega/10^6\right)}$$

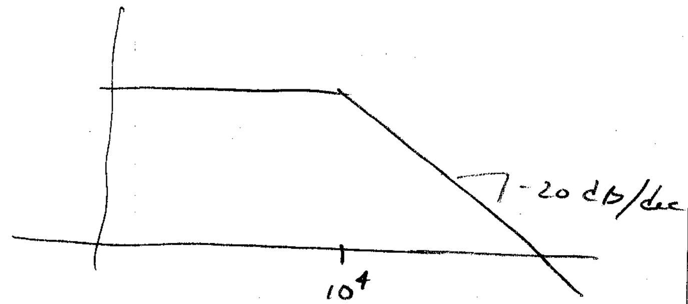




SUPERPOSITION OF POLES

Ex 1

$$H(j\omega) = \frac{1}{1 + j\omega/10^4}$$



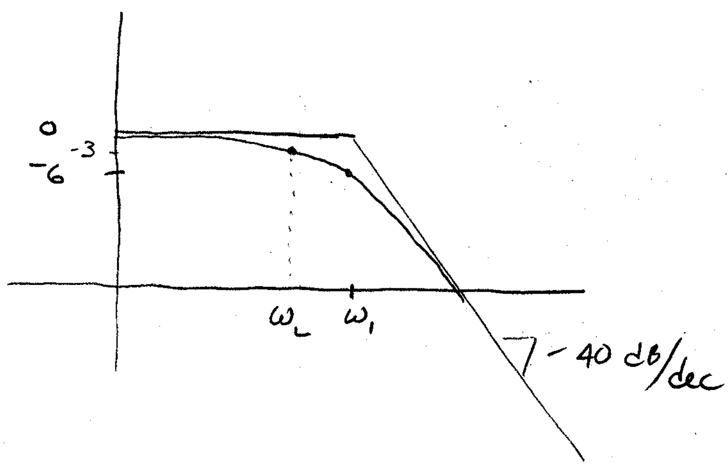
Ex 2

quadratic

$$H(j\omega) = \frac{1}{(1 + j\omega/10^4)(1 + j\omega/10^4)} = \frac{1}{(1 + j\omega/10^4)^2} = \frac{1}{1 + \frac{2j\omega}{10^4} - \frac{\omega^2}{10^8}}$$

$$\left|H(j\omega)\right|_{\omega = 10^4} = \left|\frac{1}{(1+j)(1+j)}\right| = \left|\frac{1}{1 + 2j - 1}\right| = \left|\frac{1}{2j}\right| = \frac{1}{2}$$

$\frac{1}{2}$ or $20\log_{10} .5 = -6$ dB

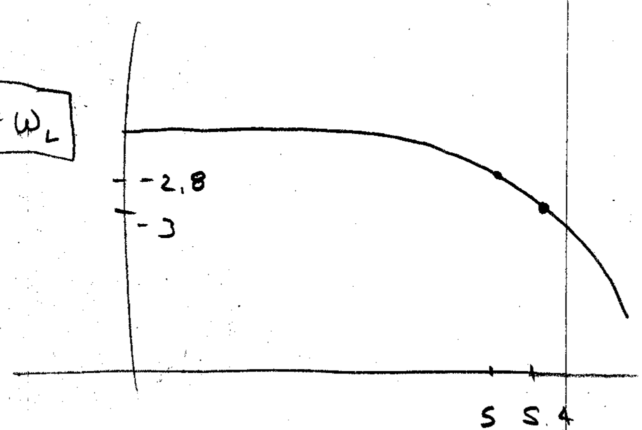


$$\left(1^2 + \frac{\omega^2}{10^8}\right) = \frac{1}{\sqrt{2}} = .707$$

$$\frac{\omega^2}{10^8} = .293$$

$$\omega^2 = .293 \times 10^8 = \boxed{5.4 \times 10^3 = \omega_L}$$

$$\frac{1}{\omega_L} \approx \frac{1}{10^4} + \frac{1}{10^4} = \frac{1}{5 \times 10^3}$$



CONT'D

INTRODUCTION TO THE SUPERPOSITION OF POLES

$$H(j\omega) = \frac{1}{\left(1 + \frac{j\omega}{10^4}\right)\left(1 + \frac{j\omega}{2\times10^4}\right)}$$

$\omega_1 = 10^4$

$\omega_2 = 2\times10^4$

FIND ω_H : $\left|H(j\omega)\right| = \frac{1}{\sqrt{2}}$

$$\frac{1}{\sqrt{2}} = \frac{1}{1 + \frac{j\omega}{2\times10^4} + \frac{2j\omega}{2\,10^4} + \frac{-\omega^2}{2\times10^8}}$$

$+$ $\quad -.5$

$1.5 + 1.5j = 2.12$

$$1 + \frac{3j\omega}{2\times10^4} = \sqrt{2}$$

$\frac{1}{2.12}$

$\boxed{@\ 10^4 \quad -6.5\text{ dB}}$

$$\frac{3\omega}{2\times10^4} = 1$$

$$\omega_H = \frac{2\times10^4}{3} = \boxed{0.67\times10^4}$$

	-1.5 dB	-2.07 dB	-2.8 dB
@	.55	.67	.8
	.151	-.224	-.320
	.275	j.335	j.4
	j.55	j.670	j.8
	j.825 + .85 (.544)	1j + .78	

$\boxed{\approx 0.8\times10^4}$

$$\frac{1}{\omega_H} \cong \frac{1}{\omega_1} + \frac{1}{\omega_2}$$

$$= \frac{1}{10^4} + \frac{1}{2\times10^4}$$

$$\frac{1}{\omega_H} = \frac{2+1}{2\times10^4}$$

$$\omega_H = \frac{2\,10^4}{3} = 0.67\times10^4$$

Estimation is conservative so is acceptable to use.

ALL POLES ω_1 through ω_n are higher than ω_H, so evaluated at ω_H, $(j\omega)^2\left(\frac{1}{\omega_1\omega_2}+\frac{1}{\omega_1\omega_3}+\cdots+\right)$ and higher order terms ~ 0 can be ignored.

$$H_H(j\omega) \cong \frac{A_0}{1+j\omega\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}+\cdots\frac{1}{\omega_n}\right)} = \frac{A}{1+j\frac{\omega}{\omega_Z}}$$

PARALLEL COMBINATION OF POLES
$\Rightarrow$ SUPERPOSITION OF POLES.

$$\left(\frac{1}{10^4}+\frac{1}{2\times10^4}\right) = .67\times10^4 = \omega_H$$

LO-FREQ LIMIT

$$H(j\omega) = A_0 H_L = \frac{A_0 (j\omega/\omega_a)(j\omega/\omega_b)(j\omega/\omega_n)}{(1+j\omega/\omega_a)(1+j\omega/\omega_b)\cdots}$$

$$H(j\omega) = \frac{A_0}{\left(\frac{\omega_a}{j\omega}+1\right)\left(\frac{\omega_b}{j\omega}+1\right)}\cdots = \frac{A_0}{1+\frac{1}{j\omega}(\omega_a+\omega_b+\cdots\omega_n)+\frac{1}{(j\omega)^2}(\omega_a\omega_b\cdots}$$

$$+\frac{1}{(j\omega)^3}(\quad)$$

$\omega_a, \omega_b \ldots \omega_n$ are lower in frequency than the actual ω_L so evaluating $H_L(j\omega)\Big|_{\omega=\omega_L}$, we can ignore terms of $\frac{1}{\omega^2}$ or higher. $\frac{1}{(j\omega_L)^2}\left(\omega_a\omega_b+\cdots\omega_a\omega_c\right.$

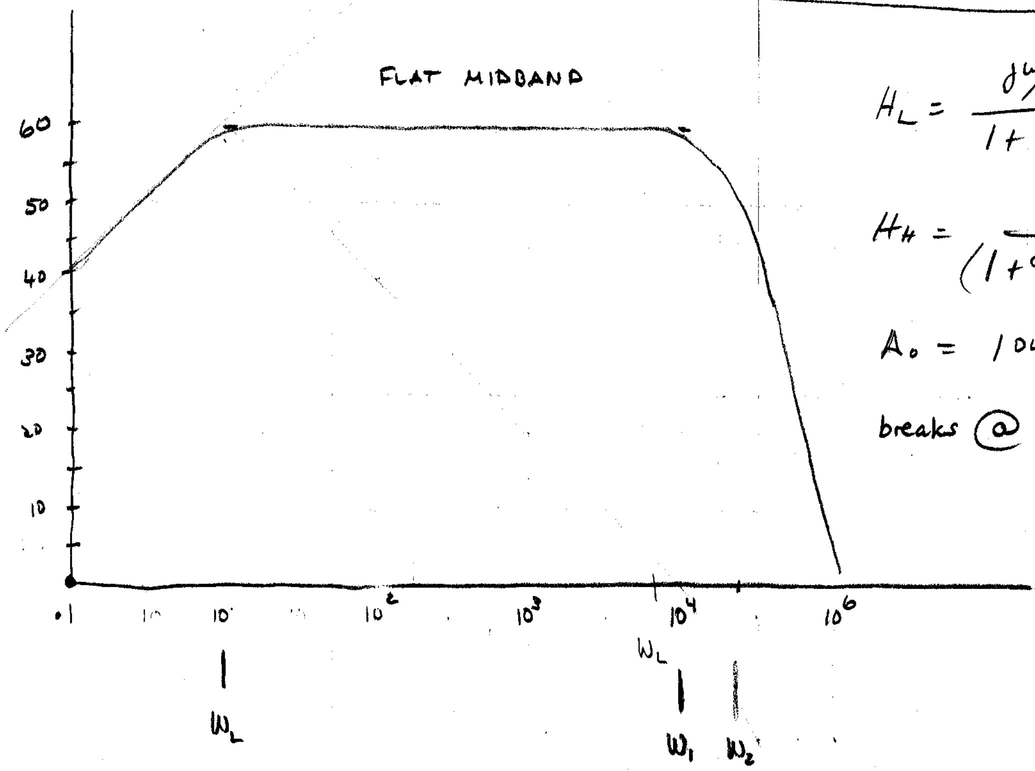
$$A_0 H_L(j\omega) = \frac{A_0}{1+\frac{1}{j\omega}(\omega_a+\omega_b+\cdots+\omega_m)} \cdot \frac{j\omega/(\omega_a+\cdots}{j\omega/(\omega_a+\cdots}$$

$$= A_0\frac{j\omega/(\omega_a+\omega_b+\cdots)}{1+j\omega/(\omega_a+\omega_b)} = \frac{A_0\, j\omega/\omega_L}{1+j\omega/\omega_L}$$

$$\omega_L = \omega_a+\omega_b+\cdots$$

"series" superposition of lo-FREQ poles

EX $H(j\omega) = 1000 \dfrac{j\omega/10}{(1+j\omega/10)(1+j\omega/10^4)[1+j\omega/(2\times10^4)]}$



$H_L = \dfrac{j\omega/10}{1+j\omega/10}$

$H_H = \dfrac{1}{(1+j\omega/10^4)(1+\frac{j\omega}{2\times10^4})}$

$A_o = 1000$

breaks @ $\omega_a = 10$ (zero)
$\omega_1 = 10^4$ pole
ω_2 2×10^4 pole

LO 3dB FREQ

$$|H|_{\omega_H} \equiv \frac{1000}{\sqrt{2}} = \frac{100\ \omega_H}{[1+(\omega_H/10^2)]^{1/2}([1+(\omega_H/10^4)^2])^{1/2}((1+\omega_H/2\times10^4)^2)^{1/2}}$$

$$\omega_H = 0.84\times10^4$$

SUPER POSITION OF POLES

$$H(j\omega) = A_o\, H_L \cdot H_H$$

$$= A_o\left[\left(\frac{j\omega/\omega_a}{1+j\omega/\omega_a}\right)\left(\frac{j\omega/\omega_b}{1+j\omega/\omega_b}\right)\cdots\right]\left[\left(\frac{1}{1+j\omega/\omega_1}\right)\left(\frac{1}{1+j\omega/\omega_2}\right)\cdots\right]$$

NEAR ω_H: $H(j\omega) \cong A_o H_H = \dfrac{A}{\left(1+\frac{j\omega}{\omega_1}\right)\left(1+\frac{j\omega}{\omega_2}\right)\cdots}$ $\qquad H_L(j\omega) \cong 1$

$$1 + j\omega\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}+\cdots\frac{1}{\omega_n}\right) + j\omega^2\left(\frac{1}{\omega_1\omega_2}+\frac{1}{\omega_1\omega_n}+\frac{1}{\omega_2\omega_n}+\cdots\right.$$

$$\left. + j\omega^3\left(\frac{1}{\omega_j\omega_k\omega_n}\right) + j\omega^n \frac{1}{\omega_1\omega_2\cdots\omega_n}\right.$$

SUMMARY

$$\omega_H = \omega_1 \| \omega_2 \| \cdots \| \omega_n$$

$$\omega_L = \omega_a + \omega_b \cdots \omega_m$$

parallel superposition of HI-FREQ POLES

series combination of LO-FREQ POLES.

EX

9.34

$$H(j\omega) = \frac{100\,(j\omega)^2}{\left(1+j\omega/20\right)\left(1+j\omega/50\right)\left(1+j\omega/75\right)\left(1+j\omega/10^4\right)\left(1+j\omega/2\times10^4\right)}$$

$$H(j\omega) = A_o\, H_L\, H_H = 100\left[\frac{(j\omega)^2}{\left(1+\frac{j\omega}{20}\right)\left(1+\frac{j\omega}{50}\right)\left(1+\frac{j\omega}{75}\right)}\right]\left[\frac{1}{\left(1+\frac{j\omega}{10^4}\right)\left(1+\frac{j\omega}{2\times10^4}\right)}\right]$$

AT $\omega < 10^4$ $\qquad \omega \sim 10^3$

$$H(j\omega) \approx \frac{100\,(j10^3)^3}{\left(\frac{10^2}{2}\right)\left(\frac{10^2}{5}\right)\left(\frac{10^2}{7.5}\right)}\left[\frac{1}{(1+.1j)(1+.05j)}\right]$$

$\cancel{97.5}$ 137.5

AT $\omega \sim 10^2$ $\qquad H(j\omega) \approx \dfrac{100\,(10^2)^3}{\left(\frac{10}{2}\right)\left(\frac{10}{5}\right)\left(\frac{10}{7.5}\right)}$

137.5

Ch 9

FREQUENCY (LIMITATIONS) AND TIME-DEPENDENT RESPONSE (TIME DELAYS)

CAUSED BY ENERGY STORAGE ELEMENTS : CAPACITORS, INDUCTORS.

CAPACITORS

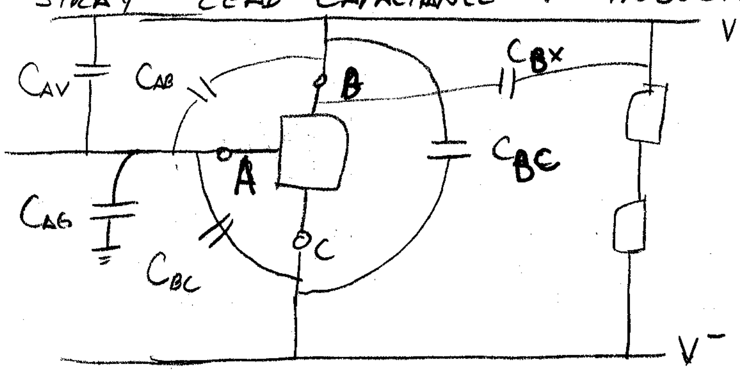
PURPOSELY USED
1. SHAPE FREQUENCY RESPONSE
2. ISOLATE BIAS VOLTAGES

NATURALLY PRESENT IN PHYSICAL STRUCTURE.
STRAY CAPACITANCE AND INDUCTANCE OF WIRES, INTERCONNECTIONS

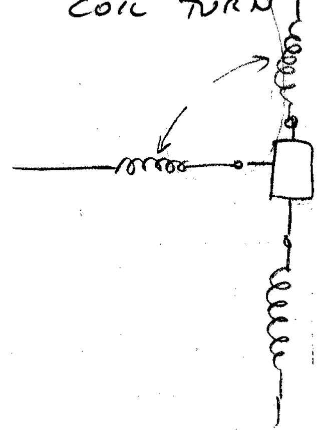
INDUCTANCE

MAINLY PARASITIC IMPORTANT WELL ABOVE BREAKPOINT FREQ'S.

STRAY LEAD CAPACITANCE + INDUCTANCE



CAUSED BY CLOSED LOOP ACTING AS A SINGLE COIL TURN



FOR pn Junction

DEPLETION CAPACITANCE C_j REVERSED BIAS C_{CB}

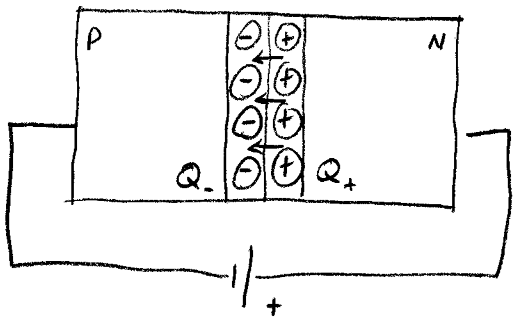
DIFFUSION CAPACITANCE C_d FORWARD BIAS C_{BE}

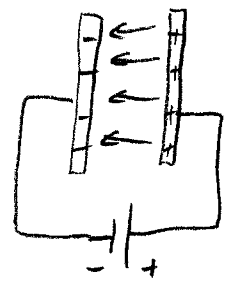
IN BJT: BE FORWARD BIASED C_π BC REVERSED BIAS C_μ

LATERAL BASE resistance r_x MUST BE INCLUDED WHEN CONSIDERING C_π, C_μ

INTERNAL CAPACITANCE IN PN JUNCTION

REVERSED BIAS :





IMMOBILE ION CORES CREATE INTERNAL BUILT-IN-FIELD
SIMILAR TO CONDUCTING PLATES CAPACITOR

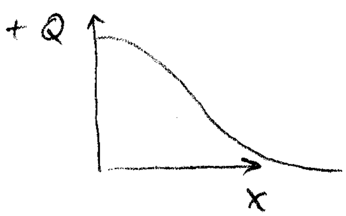
CAPACITANCE OF pn JUNCTION or JUNCTION CAPACITANCE C_j

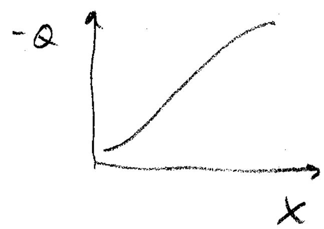
FORWARD BIAS :

current flow through the junction :

Holes injected by v_D from p-side to n-side
e^- " from n-side to p-side

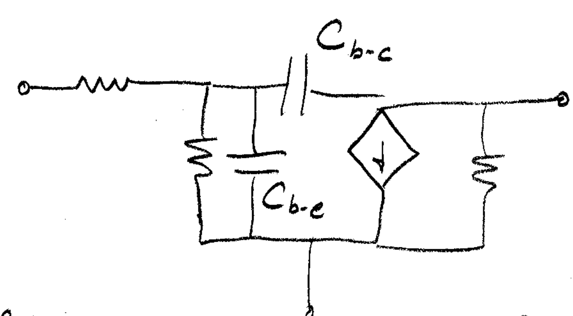
INJECTED CHARGES BUILD UP CARRIER-CONCENTRATION GRADIENTS THAT DECAY FROM THE DEPLETION REGION.





C_d

IN A BJT B-E IS FWD BIAS
B-C IS REV BIAS.



NO CAPACITANCE FROM e-C

9.1.6 JFET CAPACITANCE

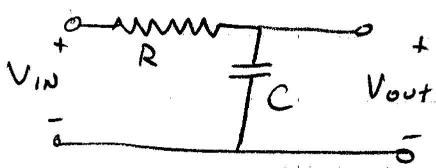
$$C_{gs} = A\left(\frac{q\epsilon}{2}\frac{N_A N_D}{N_A+N_D}\right)^{1/2}\left(\psi_0 - v_{gs}\right)^{-1/2}$$

$$C_{gs} \rightarrow 1-3\,pF$$

9.2 SINUSOIDAL STEADY-STATE AMPLIFIER RESPONSE

SYSTEM FUNCTION or TRANSFER FUNCTION

Bode Plots



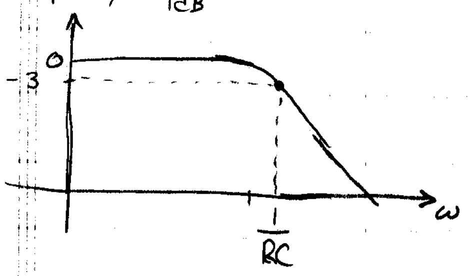
$$\text{XFER FCN} = \frac{V_{OUT}}{V_{IN}} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1+j\omega RC} = \frac{1}{1+j\omega/\omega_1}$$

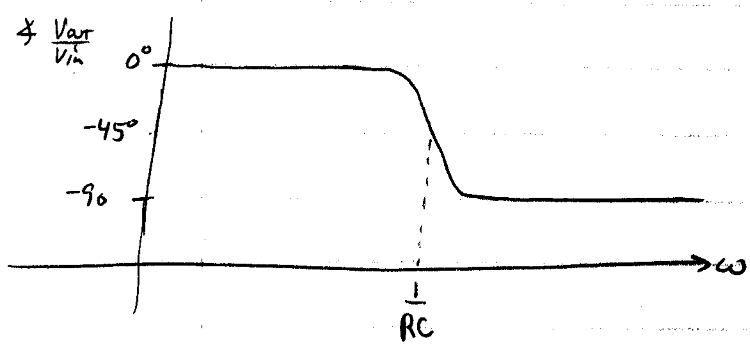
FOR $\omega \ll \frac{1}{RC}$ $\quad \omega RC \ll 1 \quad \left|\frac{V_{out}}{V_{IN}}\right| = 1 \quad \angle = 0°$

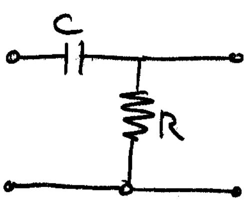
FOR $\omega \gg \frac{1}{RC}$ $\quad \omega RC \gg 1 \quad \left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\omega RC} \quad \angle\frac{1}{j\omega RC} = -90°$

FOR $\omega = \frac{1}{RC}$ $\quad \omega RC = 1 \quad \left|\frac{V_{OUT}}{V_{in}}\right| = \left|\frac{1}{1+j}\right| = \frac{1}{\sqrt{2}} = .707$

$$\angle\frac{V_{OUT}}{V_{in}} = \angle\frac{1}{1+j} = -45°$$



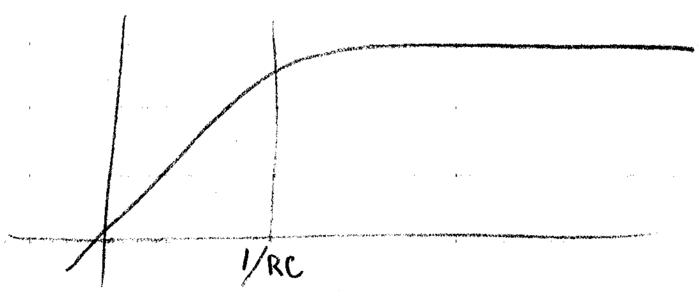




$$\frac{V_{OUT}}{V_{in}} = \frac{R}{R + 1/j\omega RC} = \frac{j\omega RC}{1+j\omega RC}$$

$$\omega_1 = \frac{1}{RC}$$

ω_1 is a breakpoint



CH 9 FREQUENCY RESPONSE AND TIME-DEPENDENT CIRUIT BEHAVIOR

C_j = depletion capacitance small signal reverse-biased junction cap.

$$C_j = k_j(\psi_o - v_D)^{-n}$$

C_d = diffusion capacitance forward biased

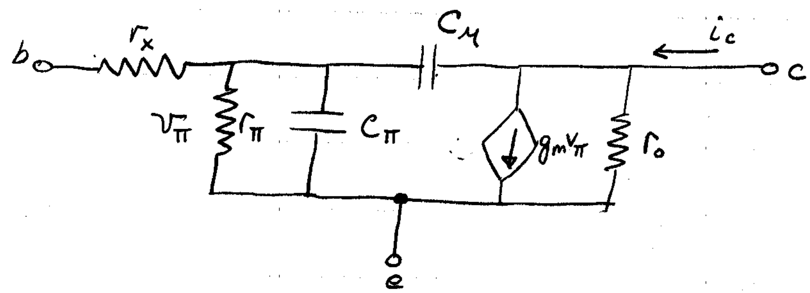
$$C_d = K_d I_s e^{-v_D/\eta V_T} \approx K_d i_D$$

9.1.4 BJT CAPACITANCE

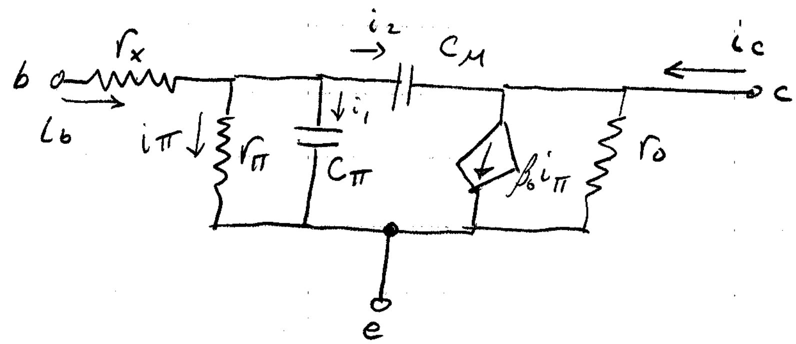
$$C_{d-be} = \frac{W^2 I_C}{2D\eta V_T}$$

W ≡ WIDTH OF BASE REGION

D ≡ MINORITY-CARRIER DIFFUSION CONSTANT.



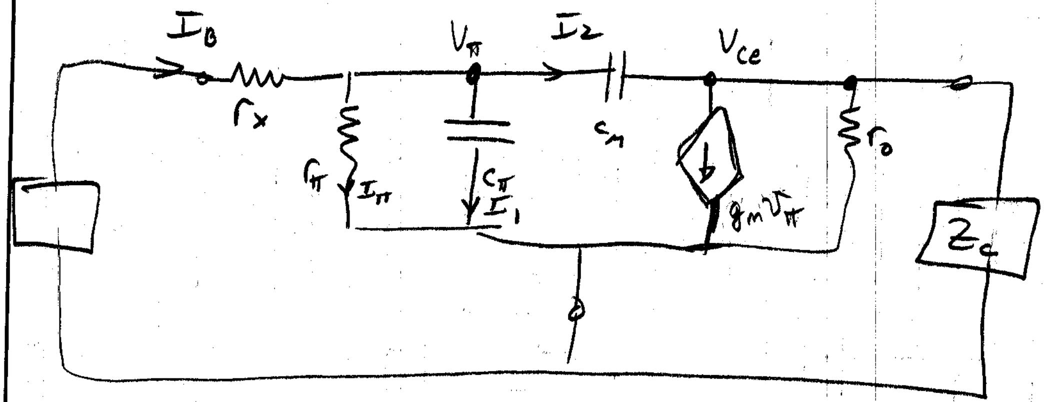
$$g_m v_\pi = g_m(r_\pi i_\pi)$$
$$= \beta_o i_\pi$$
$$\neq \beta_o i_b$$



$C_\pi = C_{be}$

$C_\mu = C_{bc} = C_{ob}$

$$i_b = i_\pi + i_1 + i_2$$



$\beta = g_m r_\pi$

$$I_b = I_\pi + I_1 + I_2$$

IN OTHER MODEL $i_c = \beta_o i_b$ or $g_m v_\pi = g_m(r_\pi i_b)$

IN THIS MODEL $g_m v_\pi = g_m(r_\pi i_\pi) \neq \beta_o i_b$

$i_b \neq i_\pi$

W/ C_π, C_μ INCLUDED i_π IS FCN OF ω.

$g_m v_\pi$ IS FCN OF ω

β_o CAN BE REPRESENTED AS FCN OF ω $\beta(\omega)$

as $\omega \uparrow$, $i_\pi \downarrow \Rightarrow \beta \downarrow$

$$I_\pi = \frac{V_\pi}{r_\pi}$$

$$I_1 = \frac{V_\pi}{1/j\omega C_\pi} = V_\pi (j\omega C_\pi)$$

$$I_2 = \frac{V_\pi - V_{CE}}{1/(j\omega C_\mu)} = (V_\pi - V_{CE}) j\omega C_\mu$$

$$I_b = \frac{V_\pi}{r_\pi} + V_\pi (j\omega C_\pi) + V_\pi (j\omega C_\mu) - V_{ce}(j\omega C_\mu)$$

$$V_\pi = \frac{I_b r_\pi + (j\omega r_\pi C_\mu) V_{ce}}{1 + j\omega r_\pi (C_\pi + C_\mu)}$$

$$V_\pi = \frac{I_b r_\pi + (j\omega r_\pi C_\mu) V_{ce}}{1 + j\omega r_\pi (C_\pi + C_\mu)}$$

$\omega \to 0$

$$V_\pi = I_b r_\pi$$

$$g_m V_\pi = \beta_0 i_b$$

ω large $\quad |j\omega C_\mu V_{ce}| << I_b$

I_2 through C_μ is small, $V_{ce} \sim (\beta_0 I_\pi)(Z_c || r_o)$

Z_c is SS Z of collector circuit.

$$|j\omega C_\mu \beta_0 I_\pi Z_c || r_o| << I_B$$

I_B IS ALWAYS Larger than I_π:

$$|\beta_0 Z_o || r_o| << \frac{1}{\omega C_\mu}$$

↑ $\beta_0 \times Z_{LOAD}$ ↑ Z of C_μ

THIS CAUSES SMALL I_2.

SMALL $C_\mu \Rightarrow$ Large $\frac{1}{\omega C_\mu}$

$$V_\pi \approx \frac{I_b r_\pi}{1 + j\omega r_\pi (C_\pi + C_\mu)}$$

$$g_m V_\pi \approx \frac{g_m r_\pi I_b}{1 + j\omega r_\pi (C_\pi + C_\mu)} \equiv \beta(\omega) I_b$$

$$\beta(\omega) = \frac{g_m r_\pi}{1 + j\omega r_\pi (C_\pi + C_\mu)} = \frac{\beta_0}{1 + j\omega r_\pi (C_\pi + C_\mu)}$$

$\approx$ Become $=$ When Collector shorted to emitter

$\beta(\omega) \equiv$ short circuit s.s. current gain.

at pole $\omega_H = \dfrac{1}{r_\pi(C_\pi + C_\mu)}$ $\qquad \beta(\omega) = \beta_0$

AT $\omega > \omega_H$ THINGS FALL APART.

FIGURE OF MERIT FOR BJT :

ω when $\beta(\omega) = 1$ UNITY GAIN frequency ω_T.

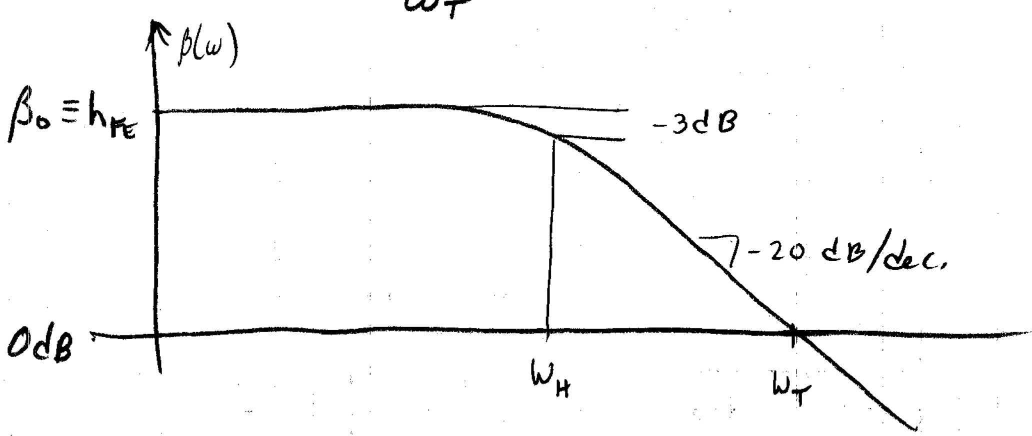
$\omega_T >> \omega_H$

$$\left|\beta(\omega)\right|_{\omega=\omega_T} = \left| \frac{\beta_0}{1 + j\omega_T r_\pi (C_\pi + C_\mu)} \right| = 1$$

$$\left[1 + \omega_T^2 r_\pi^2 (C_\pi + C_\mu)^2\right]^{1/2} \approx \omega_T r_\pi (C_\pi + C_\mu) = \beta_0$$

$$\omega_T = \frac{\beta_0}{r_\pi(C_\pi + C_\mu)} = \frac{g_m}{C_\pi + C_\mu}$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu$$



UNITY GAIN frequency ω_T: A FIGURE OF MERIT.

$$\left|\beta(\omega)\right|_{\omega=\omega_T} = \left|\frac{\beta_o}{1+j\omega_T r_\pi (C_\pi + C_\mu)}\right| = 1$$

$$j\omega_T r_\pi (C_\pi + C_\mu) = \beta_o$$

$$\omega_T = \frac{\beta_o}{r_\pi (C_\pi + C_\mu)} \equiv \boxed{\frac{g_m}{C_\pi + C_\mu} = \omega_T}$$

$$\boxed{C_\pi = \frac{g_m}{\omega_T} - C_\mu}$$

C_μ and ω_T can be measured easily.

$\beta(\omega) = h_{fe}(\omega) = h_{fe}$

$C_\pi \propto g_m \propto I_c$

$\beta_o = h_{fe}$

ω_H = pole frequency

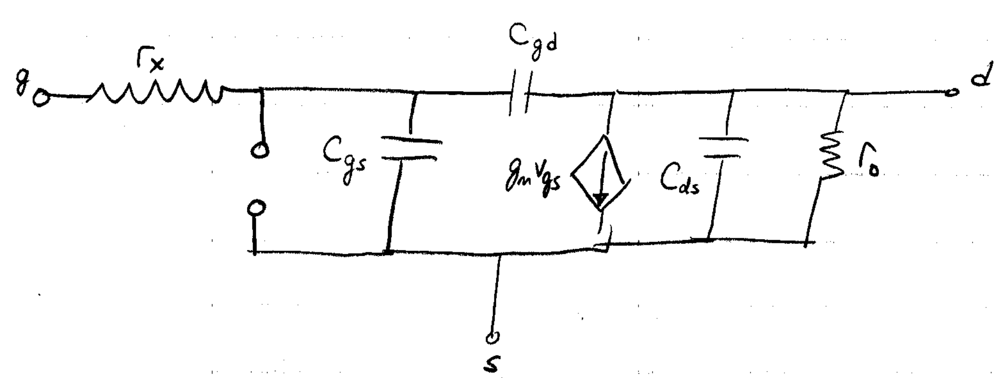
9.1.5 CAPACITANCE IN THE MOSFET

$$C = \frac{\epsilon_{ox} A}{t_{ox}}$$

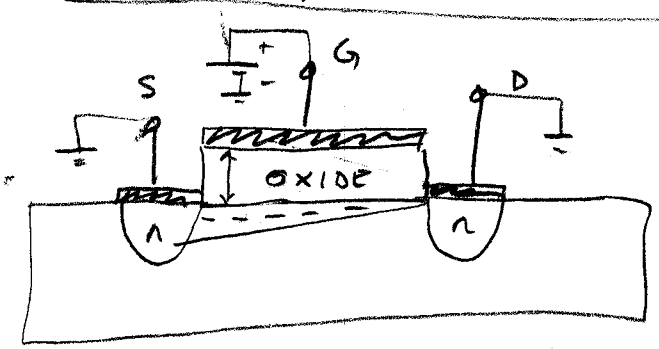
$\epsilon_{ox} \equiv$ dielectric of permitivity of oxide layer
$A \equiv$ Area of conducting plates
$t_{ox} \equiv$ THICKNESS OF OXIDE

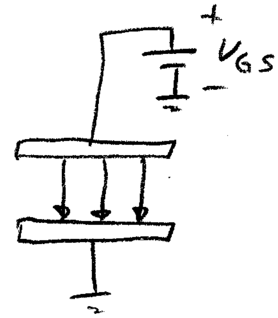
$C_{gs} = \frac{\epsilon_{ox} A}{t_{ox}}$ $\quad A = WL \quad$ for $\quad v_{DS} << v_{GS}$

$C_{gs} = \frac{2}{3}\frac{\epsilon_{ox} A}{t_{ox}}$ $\quad$ for $\quad v_{DS} \sim (v_{GS} - V_{TR})$



INTERNAL CAPACITANCE IN FET



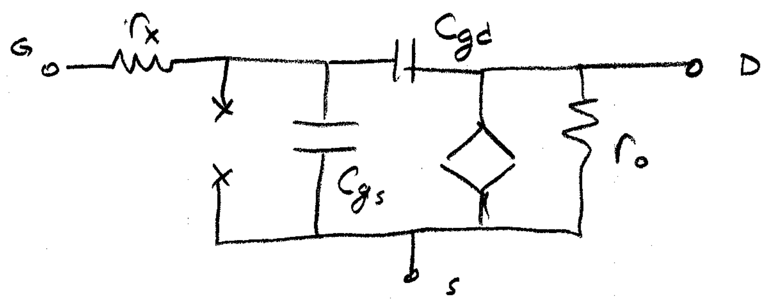


VOLTAGE APPLIED TO GATE CREATES CHARGED PLATE.
SUBSTRATE IS OTHER PLATE

$\vec{E}$ IS WEAKER NEAR DRAIN END OF CHANNEL

$C_{gs} > C_{gd}$

NO CAPACITANCE FROM DRAIN TO SOURCE



SAME MODEL FOR JFET,