EE 7620 Solutions to Problem Set # 5 11,10,08

1. Since the frequency band is $(2500, 3000)$, the system must be a bandpass sytem.

   (a) We have $R_b = 500$, $W = 1000$. Thus $R_b/W = .5$. Also

   $$\frac{E_b}{N_0} = \frac{PT_b}{N_0} = \frac{P}{R_b N_0} = \frac{P}{500 \times 2 \times 10^{-5}} = 100P$$

   i. Any PSK system satisfies the R_b/W requirement. BPSK and QPSK require the least power. $\frac{E_b}{N_0} = 100P = 9.12$. Thus $P = .0912$.

   ii. BFSK, 4FSK, 8FSK can be used. 8FSK uses the least power. For 8FSK, $\frac{E_b}{N_0} = 100P = 7.07$. Thus $P = .07$.

   iii. We can use any ASK system. BASK uses the least power and we get $P = .0912$.

   iv. All QASK systems are feasible. The least power is used by 4QASK which is the same as QPSK for which $P = .0912$.

   v. $\frac{E_b}{N_0} = 100P = \frac{2^{R_b/W} - 1}{R_b/W} = .8284$ Thus $P = .0082$.

   (b) We have $R_b/W = 3.5$. Also

   $$\frac{E_b}{N_0} = \frac{PT_b}{N_0} = \frac{P}{R_b N_0} = \frac{P}{500 \times 2 \times 10^{-5}} = \frac{100P}{7}$$

   i. 16PSK system satisfies the R_b/W requirement. Then $\frac{E_b}{N_0} = 100P/7 = 52.4$. Thus $P = 3.66$.

   ii. FSK can not be used.

   iii. 16ASK can be used. Then $100P/7 = 177.8$. Thus $P = 12.44$.

   iv. 16QASK system is feasible. Then $100P/7 = 20.8$. Thus $P = 1.45$.

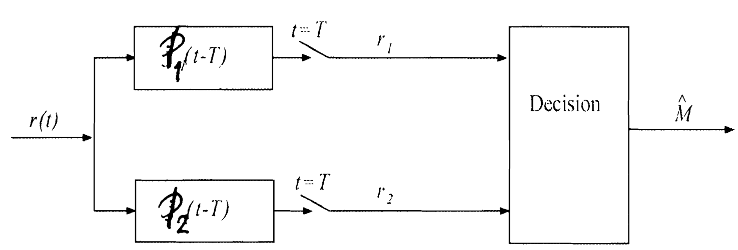
   v. $\frac{E_b}{N_0} = 100P/7 = \frac{2^{R_b/W} - 1}{R_b/W} = 2.9468$ Thus $P = .2063$.

2. (a) By examining the signal set we see that we can let $\phi_1(t) = \frac{s_2(t)}{||s_2(t)||}$ and $\phi_2(t) = \frac{s_3(t)}{||s_3(t)||}$, where $||s_1(t)|| = ||s_2(t)|| = a\sqrt{\frac{T}{2}}$. The vector representation of the signals are given by

   $$\mathbf{s}_0 = (0,0), \ \ \mathbf{s}_1 = (a\sqrt{\frac{T}{2}}, a\sqrt{\frac{T}{2}}), \ \ \mathbf{s}_2 = (a\sqrt{\frac{T}{2}}, 0), \ \ \mathbf{s}_3 = (0, a\sqrt{\frac{T}{2}})$$

   (b) $T = 2T_b = 2/R_b = 2$ msec. $W = 1/T = 500$ Hz.

   (c)



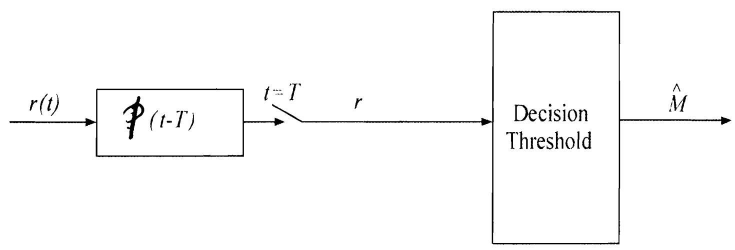
where

$$\hat{M} = \begin{cases} m_0 & r_1 \le \frac{a}{2}\sqrt{\frac{T}{2}}, r_2 \le \frac{a}{2}\sqrt{\frac{T}{2}} \\ m_1 & r_1 > \frac{a}{2}\sqrt{\frac{T}{2}}, r_2 > \frac{a}{2}\sqrt{\frac{T}{2}} \\ m_2 & r_1 > \frac{a}{2}\sqrt{\frac{T}{2}}, r_2 \le \frac{a}{2}\sqrt{\frac{T}{2}} \\ m_3 & r_1 \le \frac{a}{2}\sqrt{\frac{T}{2}}, r_2 > \frac{a}{2}\sqrt{\frac{T}{2}} \end{cases}$$

(d)

$$P(E) = 1 - \left[1 - Q\left(\sqrt{\frac{a^2T}{4N_0}}\right)\right]^2$$

3. (a) An O.N. basis for the signal set is given by $\phi(t) = \sqrt{\frac{2}{T}}\sin(2\pi f_c t)$ for $0 \le t \le T$. The signal vectors are $\mathbf{s}_0 = 0$, $\mathbf{s}_1 = A\sqrt{\frac{T}{2}}$, and $\mathbf{s}_0 = -A\sqrt{\frac{T}{2}}$. The receiver block diagram is plotted below.



$$\hat{M} = \begin{cases} m_2 & r < -\frac{A}{2}\sqrt{\frac{T}{2}} \\ m_1 & r > \frac{A}{2}\sqrt{\frac{T}{2}} \\ m_0 & -\frac{A}{2}\sqrt{\frac{T}{2}} \le r \le \frac{A}{2}\sqrt{\frac{T}{2}} \end{cases}$$

(b)

$$\begin{aligned} P(C|\mathbf{s}_2) &= P(R < -\frac{A}{2}\sqrt{\frac{T}{2}}|\mathbf{s}_2) = P(-A\sqrt{\frac{T}{2}} + N < -\frac{A}{2}\sqrt{\frac{T}{2}}) \\ &= P(N < \frac{A}{2}\sqrt{\frac{T}{2}})) = 1 - Q\left[\sqrt{\frac{A^2T}{4N_0}}\right] \\ P(C|\mathbf{s}_1) &= P(R > \frac{A}{2}\sqrt{\frac{T}{2}}|\mathbf{s}_1) = P(A\sqrt{\frac{T}{2}} + N > \frac{A}{2}\sqrt{\frac{T}{2}}) \\ &= 1 - Q\left[\sqrt{\frac{A^2T}{4N_0}}\right] \end{aligned}$$

$$
\begin{aligned}
P(C|\mathbf{s}_0) &= P(-\frac{A}{2}\sqrt{\frac{T}{2}} \leq R \leq \frac{A}{2}\sqrt{\frac{T}{2}}|\mathbf{s}_0) \\
&= 1-2Q\left[\sqrt{\frac{A^2T}{4N_0}}\right]
\end{aligned}
$$

(c)

$$P(C) = 1/3\,[P(C|\mathbf{s}_2)+P(C|\mathbf{s}_1)+P(C|\mathbf{s}_0)]$$

Thus

$$P(E) = 1-P(C) = \frac{4}{3}Q\left[\sqrt{\frac{A^2T}{4N_0}}\right]$$

Note that you can also get this from the general formula for ASK with $q=3$.

(d)

$$W = \frac{1}{T} = \frac{1}{\log_2 3T_b} = \frac{R_b}{\log_2 3}$$

Thus $\frac{R_b}{W} = \log_2 3 = 1.585$.

$$E_{ave} = 0+\frac{A^2T}{2}+\frac{A^2T}{2} = A^2T$$

Also $E_{ave} = \log_2 3E_b$. Thus

$$P(E) = \frac{4}{3}Q\left[\sqrt{\frac{\log_2 3E_b}{4N_0}}\right]$$

Setting $P(E) = 10^{-5}$ we get $\frac{E_b}{N_0} = 25.3$ or 14 dB.
We cab see that this system has better bandwiddth efficiency than BPSK but requires higher SNR.

4. (a) For BPSK, $R_b/W = 1$. Thus $R_b \leq W$. Also in order to get $P_b(E) \leq 10^{-5}$, we need $E_b/N_0 \geq 9.1$ Thus $\frac{PT_b}{N_0} = \frac{P}{R_bN_0} \geq 9.1$. Thus

$$R_b = \min\{W, \frac{P}{N_0}\frac{1}{9.1}\}$$

i.

$$R_b = \min\{1000, 5000/9.1\} = 549 \text{ bits/s}$$

ii.

$$R_b = \min\{10000, 1.5\times 10^5/9.1\} = 10{,}000 \text{ bits/s}$$

(b) For QPSK, $R_b/W = 2$. Thus $R_b \leq 2W$.

$$R_b = \min\{2W, \frac{P}{N_0}\frac{1}{9.1}\}$$

i.

$$R_b = \min\{2000, 5000/9.1\} = 549 \text{ bits/s}$$

ii.

$$R_b = \min\{20000, 1.5 \times 10^5/9.1\} = 16,483 \text{ bits/s}$$

(c) For BFSK, $R_b/W = 2/3$ and $E_b/N_0 \geq 18.2$. Thus

$$R_b = \min\{2W/3, \frac{P}{N_0}\frac{1}{18.2}\}$$

i.

$$R_b = \min\{2000/3, 5000/18.2\} = 274 \text{ bits/s}$$

ii.

$$R_b = \min\{20000/3, 1.5 \times 10^5/18.2\} = 6,666 \text{ bits/s}$$

5. Using the O.N. basis functions

$$\phi_1(t) = \sqrt{2T}\cos(2\pi f_0 t), \quad \phi_2(t) = \sqrt{2T}\sin(2\pi f_0 t)$$

we get $\mathbf{s}_i = (\frac{a_i}{\sqrt{2}}, \frac{b_i}{\sqrt{2}})$, where a_i and b_i take all possible values in the set

$$\left\{\pm\frac{7A}{2}, \pm\frac{5A}{2}, \pm\frac{3A}{2}, \pm\frac{A}{2}\right\}$$

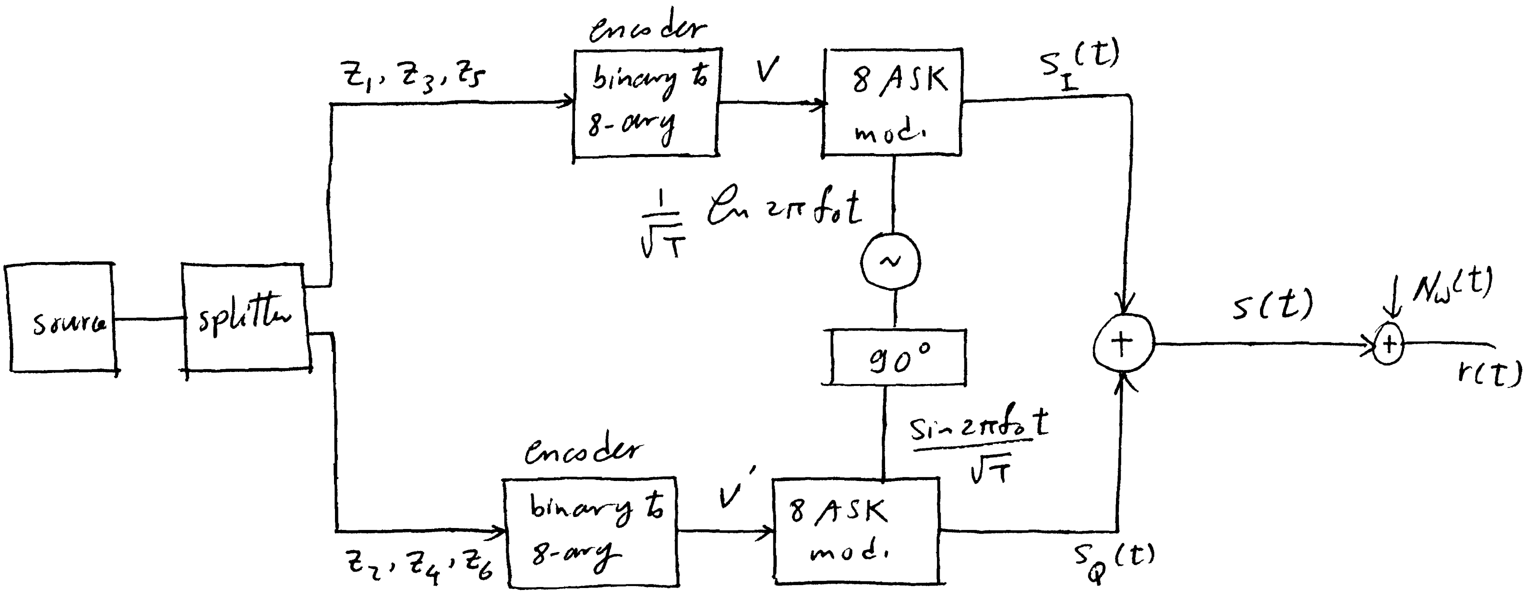
It can be seen that this is a 64QAM system.

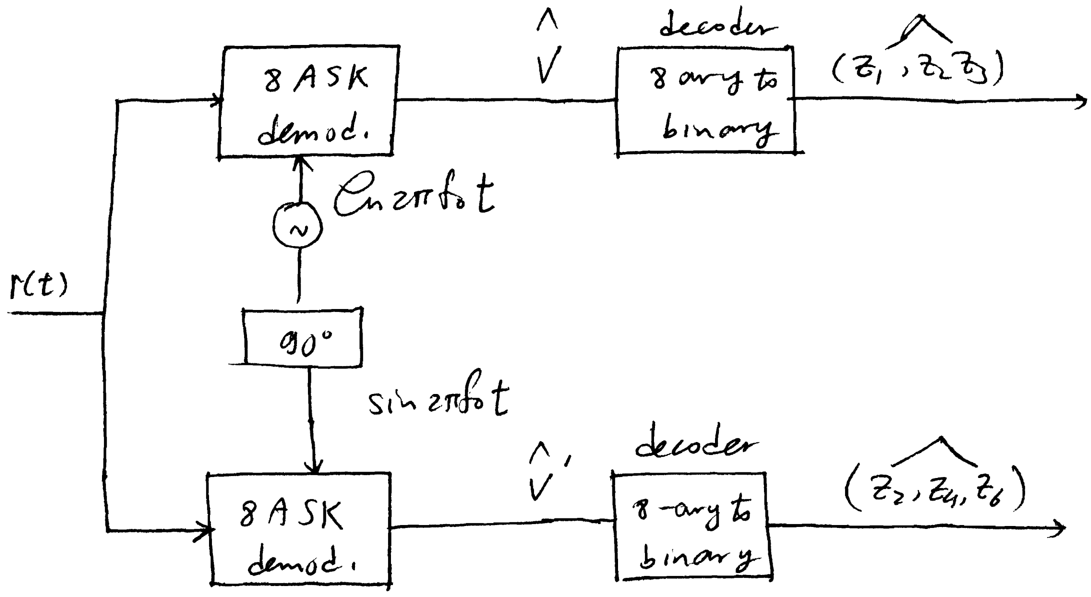
(a)

$$E_i = ||\mathbf{s}_i||^2 = \frac{a_i^2}{2} + \frac{b_i^2}{2}$$

Then

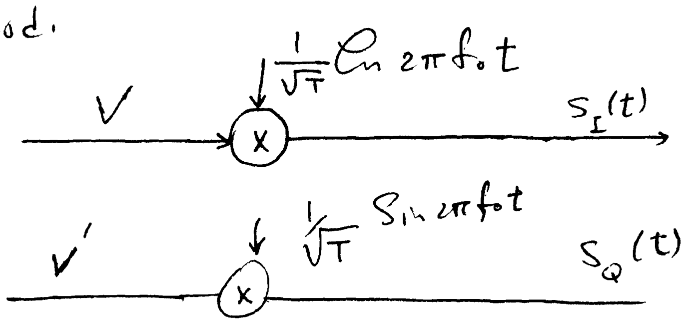
$$E_{ave} = \frac{1}{64}\sum_{i=1}^{64} E_i = \frac{21A^2}{4}$$





$$V, V' \in \left\{ \pm \tfrac{7A}{2}, \pm \tfrac{5A}{2}, \pm \tfrac{3A}{2}, \pm \tfrac{A}{2} \right\}$$

8 ASK mod.



(c) $T = T_b \log_2 q = \frac{6}{R_b} = 6msec.$

(d) Since 64QAM can be thought of as two 8ASK systems in parallel,

$$P_{64\text{QAM}}(E) = [P_{8\text{ASK}}(E)]^2$$

where

$$P_{8\text{ASK}}(E) = \frac{2(q-1)}{q} Q\left[\frac{d}{\sqrt{2N_0}}\right]$$

where $d = \frac{A}{\sqrt{2}}$. Thus

$$P_{8\text{ASK}}(E) = \frac{7}{4} Q\left[\frac{A}{2\sqrt{N_0}}\right]$$

Using Gray coding we get $P_b(E) \approx \frac{P_{8\text{ASK}}(E)}{3}$.

$$W = \frac{1}{T} = \frac{1}{8T_b} = \frac{R_b}{6}$$

Thus $R_b/W = 6$. Also $E_{ave} = \frac{21A^2}{4} = 6E_b$. Thus

$$P_b(E) = \frac{7}{12} Q\left[\sqrt{\frac{2E_b}{7N_0}}\right]$$

Setting $P_b(E) = 10^{-5}$ gives $\frac{E_b}{N_0} = 18$ dB.

8ASK demod.

$$\hat{v} = \begin{cases} 0 & r \le -3A \\ 1 & -3A < r \le -2A \\ 2 & -2A < r \le -A \\ 3 & -A < r \le 0 \\ 4 & 0 < r \le A \\ 5 & A < r \le 2A \\ 6 & 2A < r \le 3A \\ 7 & 3A < r \end{cases}$$