

- 1) Please attempt each problem on a new page.
- 2) Show all your work clearly.

1. The following code is used to transmit a binary source with rate R_b across an AWGN channel with noise power spectral density of $\frac{N_0}{2}$.

$$\begin{aligned} \mathbf{s}_1 &= \sqrt{E_c}(+1, +1, +1, +1), & \mathbf{s}_2 &= \sqrt{E_c}(-1, -1, -1, -1) \\ \mathbf{s}_3 &= \sqrt{E_c}(+1, +1, -1, -1), & \mathbf{s}_4 &= \sqrt{E_c}(-1, -1, +1, +1) \\ \mathbf{s}_5 &= \sqrt{E_c}(+1, -1, +1, -1), & \mathbf{s}_6 &= \sqrt{E_c}(-1, +1, -1, +1) \\ \mathbf{s}_7 &= \sqrt{E_c}(+1, -1, -1, +1), & \mathbf{s}_8 &= \sqrt{E_c}(-1, +1, +1, -1) \end{aligned}$$

- (a) Find the input blocklength K , output blocklength L and the code rate.
 - (b) Diagram a complete communication system that uses this code to transmit over the AWGN channel. Your system should include an optimum receiver. Be sure to specify the operation of each block.
 - (c) Use the union bound to obtain an upper bound to block error probability.
 - (d) Find an expression for the block error probability (what kind of signal set is this?)
 - (e) Find the $\frac{E_b}{N_0}$ needed to get block error probability of 10^{-5} (you can use the results from the book).
 - (f) Repeat part (1e) using the upper bound in part (1c).
 - (g) Find R_b/W assuming assuming a baseband rectangular pulse $\phi(t)$ is used.
 - (h) Diagram an optimum hard decision receiver. Be sure to give the optimum vector decision rule.
 - (i) Find d_{\min} and the error correcting capability of this code.
 - (j) Find an upper bound to the block error probability of the hard decision receiver.
 - (k) Compute $\frac{E_b}{N_0}$ needed to get block error probability of 10^{-5} for the hard decision receiver (use the upper bound above).
 - (l) Compare the results of parts (1c), (1e) and (1f).
2. A binary *Golay* code has $L = 23$, $K = 12$ and $d_{\min} = 7$.
 - (a) Compute the $\frac{E_b}{N_0}$ needed to get $P(E) \leq 10^{-5}$ for both the soft decision and the hard decision receivers (use upper bounds for $P(E)$).
 - (b) Compute R_b/W assuming a baseband rectangular pulse $\phi(t)$ is used.