EE 7620 **Problem Set # 4** **Due: 11,03,08**

1) Please attempt each problem on a new page.
2) Show all your work clearly.

1. **Analysis of q-ary Amplitude Shift Keying**

   Consider the signal set $\mathcal{S} = \{s_0(t), s_1(t), ..., s_{q-1}(t)\}$ where

   $$s_i(t) = d\left(i + \frac{1-q}{2}\right)\sqrt{\frac{2}{T}}\cos(2\pi f_0 t) \quad \text{for} \quad 0 \le t < T \quad \text{and} \quad i = 0, 1, ..., q-1$$

   The signals are transmitted through an AWGN channel with $S_{N_W}(f) = \frac{N_0}{2}$. Messages are all equally likely.

   (a) For $q = 4$ draw a signal vector representation and the block diagram of an optimum receiver. You need to simplify the structure of your receiver as much as possible and identify the function of each block.

   (b) Show that in general, for an optimum receiver,

   $$P(E) = \frac{2(q-1)}{q} Q\left(\frac{d}{\sqrt{2N_0}}\right) \approx 2Q\left(\frac{d}{\sqrt{2N_0}}\right)$$

   (c) Find the average signal energy E_{ave} as a function of q and d.
   Hint: Helpful formulas to simplify you answers:

   $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} (2i-1)^2 = \frac{4n^3 - n}{3}$$

   (d) Find the bandwidth of the q-ary modulator.

   (e) Let $q = 2^K$. Describe how this signal set could be used to transmit binary data arriving at the rate of R_b bits/s.

   (f) Find expressions for $\frac{E_b}{N_0}$, $\frac{R_b}{W}$ and $P(E)$ (the latter as a function of $\frac{E_b}{N_0}$ and q.)

   (g) Describe an encoding rule such that $P_b(E) \approx \frac{P(E)}{K}$.

   (h) Plot $\frac{R_b}{W}$ vs. $\frac{E_b}{N_0}$ for the q-ary ASK system giving $P_b(E) = 10^{-5}$ for values of $q = 2, 4, 8, 16, 32$.

2. A communication system uses the signal set shown below in oredr to transmit equiprobable binary messages across an AWGN channel.

   $$\begin{aligned} s_0(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_0 t), \qquad 0 \le t \le T \\ s_1(t) &= \sqrt{\frac{2E}{T}}\cos(2\pi f_0 t + 2\pi h\frac{t}{T}), \qquad 0 \le t \le T \end{aligned}$$

   Assume that $f_0 >> 1$ and T, h and E are parameters.

(a) Draw the block diagram of an optimum receiver. Make sure you specify the function of each block.

(b) Evaluate the performance of the system. (Signal to noise ratio needed to get $P_b(E) = 10^{-5}$ and the bandwidth requirement.)

(c) Assuming that all other parameters are fixed, find the parameter h which minimizes the error probability. (An approximate answer suffices here.)