EE 4610 Solutions to Exam II Nov. 2004

Problem 2

1. Let the transmitted signal be given by

$$s(t) = A_c m(t)\cos(2\pi f_c t)$$

On the receiver side, the output of the product modulator in the coherent detector is then given by

$$\begin{aligned} r(t) &= A_c A_r m(t)\cos(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{A_c A_r}{2} m(t)[\cos\phi + \cos(4\pi f_c t + \phi)] \end{aligned}$$

Upon low-pass filtering the signal $r(t)$, we get the final output of the Coherent detector, namely

$$v(t) = \frac{A_c A_r}{2} m(t)\cos\phi$$

If ϕ is constant, then $v(t) = am(t)$ where $a = \frac{A_c A_r}{2}\cos\phi$. The demodulated waveform only differs from the message $m(t)$ by a constant factor (attenuation or amplification).

Note that if $\phi = \frac{\pi}{2}$, then $v(t) = 0$, which, of course, is undesirable. If $\pi < \phi < 2\pi$, then $v(t)$ has a different sign from $m(t)$, e.g., $v(t) = -m(t)$. For audio signals, this is OK, since we hear $m(t)$ and $-m(t)$ the same. But for video signals, this needs to be corrected.

If ϕ varies with time, see answer to part (b).

2. We proceed along similar lines to part (a). In this case, $r(t)$ is given by

$$\begin{aligned} r(t) &= A_c A_r m(t)\cos(2\pi f_c t)\cos[2\pi(f_c + \Delta f)t] \\ &= \frac{A_c A_r}{2} m(t)[\cos(2\pi\Delta f t) + \cos(2\pi(2f_c + \Delta f)t)] \end{aligned}$$

Upon low-pass filtering the signal $r(t)$, the final output of the coherent detector, $v(t)$, is then given by

$$v(t) = \frac{A_c A_r}{2} m(t) \cos(2\pi \Delta f t)$$

Assuming that $\Delta f \neq 0$ is small, the amplitude of the received waveform *slowly* fluctuates, due to a *residual modulation* of the demodulated signal $v(t)$ at the frequency Δf. This is clearly undesirable and should be corrected. However, if $\Delta f = 0$, then $v(t) = r(t)$ and we get the required message signal, $m(t)$.

Problem 3

We have $f_c = 105\text{MHz}$, $A_m = 5\text{V}$, $f_m = 15\text{kHz}$, $k_f = 10\text{kHz/V}$

1. We have $\Delta f = k_f A_m = 50$ KHz. By Carson's rule,

$$B_T \approx 2\Delta f + 2f_m = 130 \text{ KHz}$$

2. $\beta = \frac{\Delta f}{f_m} = \frac{50}{15} \approx 3.33$ radians. From the universal curve,

$$B_T/\Delta f|_{\beta=3.33} \approx 3.5.$$

   This yields $B_T \approx 3.5\Delta f = 175$ KHz.

3. Here $A'_m = 2A_m = 10V$. Hence, $\Delta f' = k_f A'_m = 100\text{kHz}$. By Carson's rule,

$$B_T \approx 2\Delta f' + 2f_m = 230 \text{ KHz}.$$

   We have $\beta = \frac{100}{15} \approx 6.67$ radians. From the universal curve,

$$B_T \approx 3\Delta f' = 300 \text{ KHz}.$$

4. In this case, $f'_m = 2f_m = 30$ KHz. By Carson's rule,

$$B_T \approx 2\Delta f + 2f'_m = 160 \text{ KHz}.$$

   Also in this case, $\beta = \frac{50}{30} \approx 1.67$ radians. From the universal curve, we get

$$B_T \approx 5\Delta f = 5(50) = 250 \text{ KHz}.$$

Problem 1

1. (a)

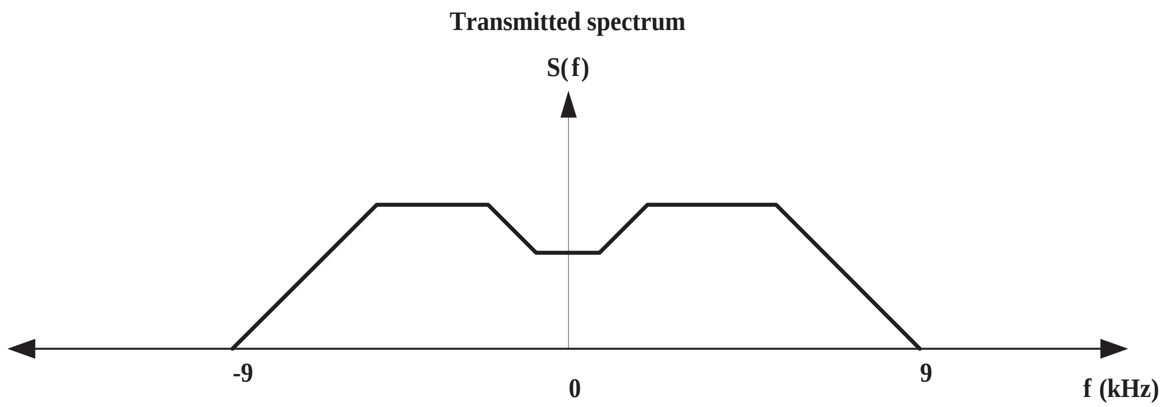
   (b)

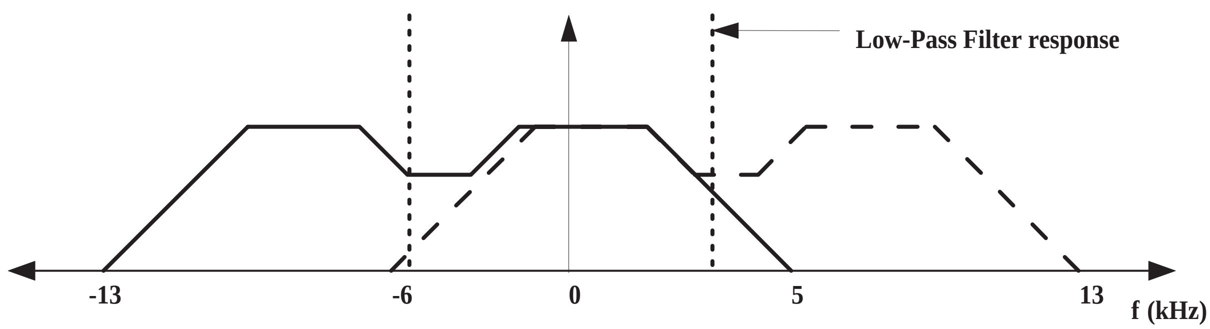
   The lowest frequency is $f = 5$ KHz.

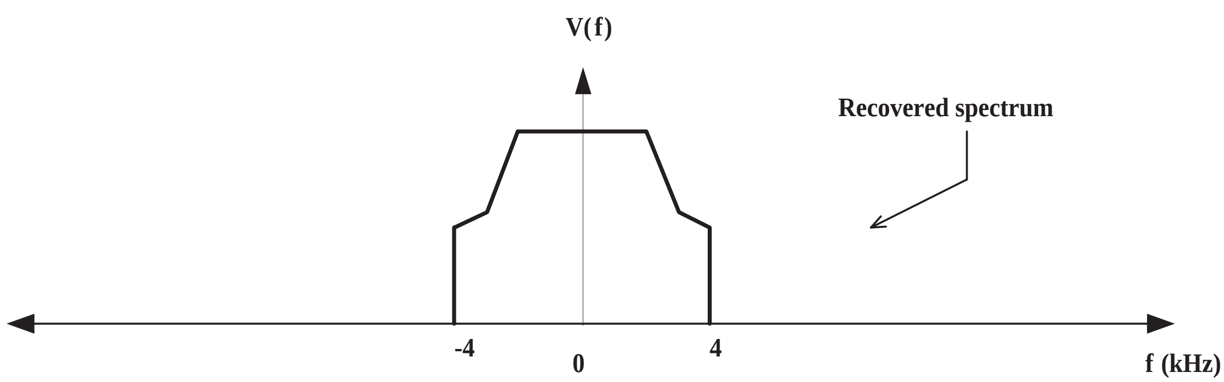
2. (a)

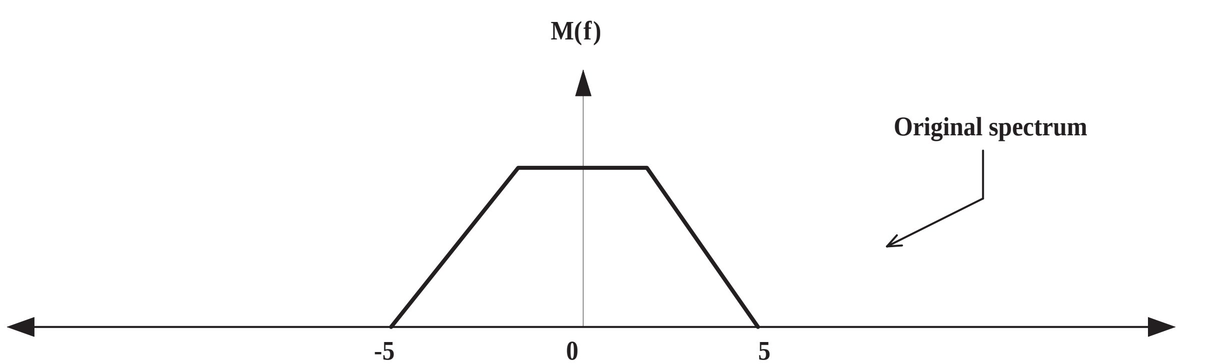
   (b)

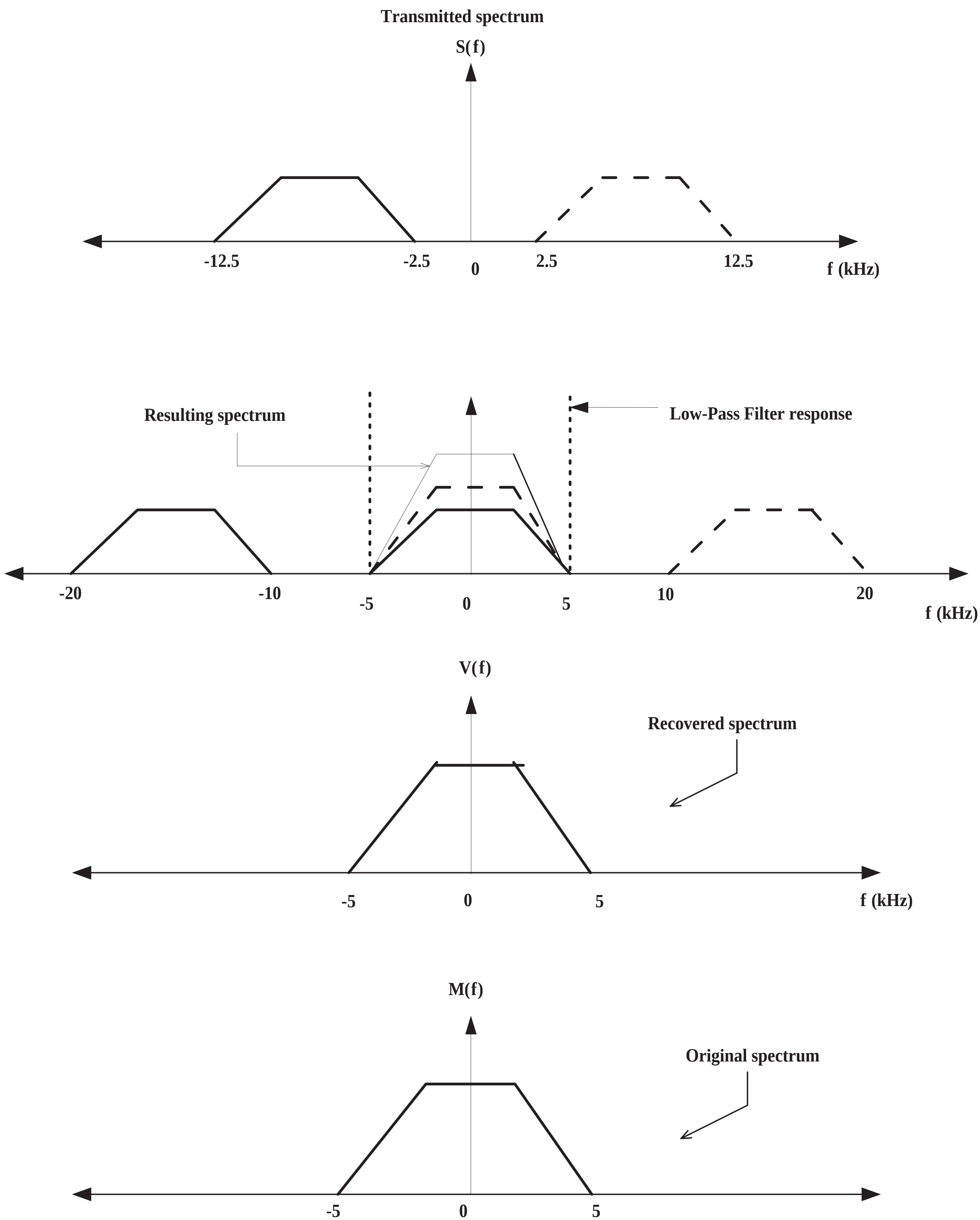
   In this case the lowest frequency to guarantee that the message can be demodulated is given by $f = 10$ KHz.

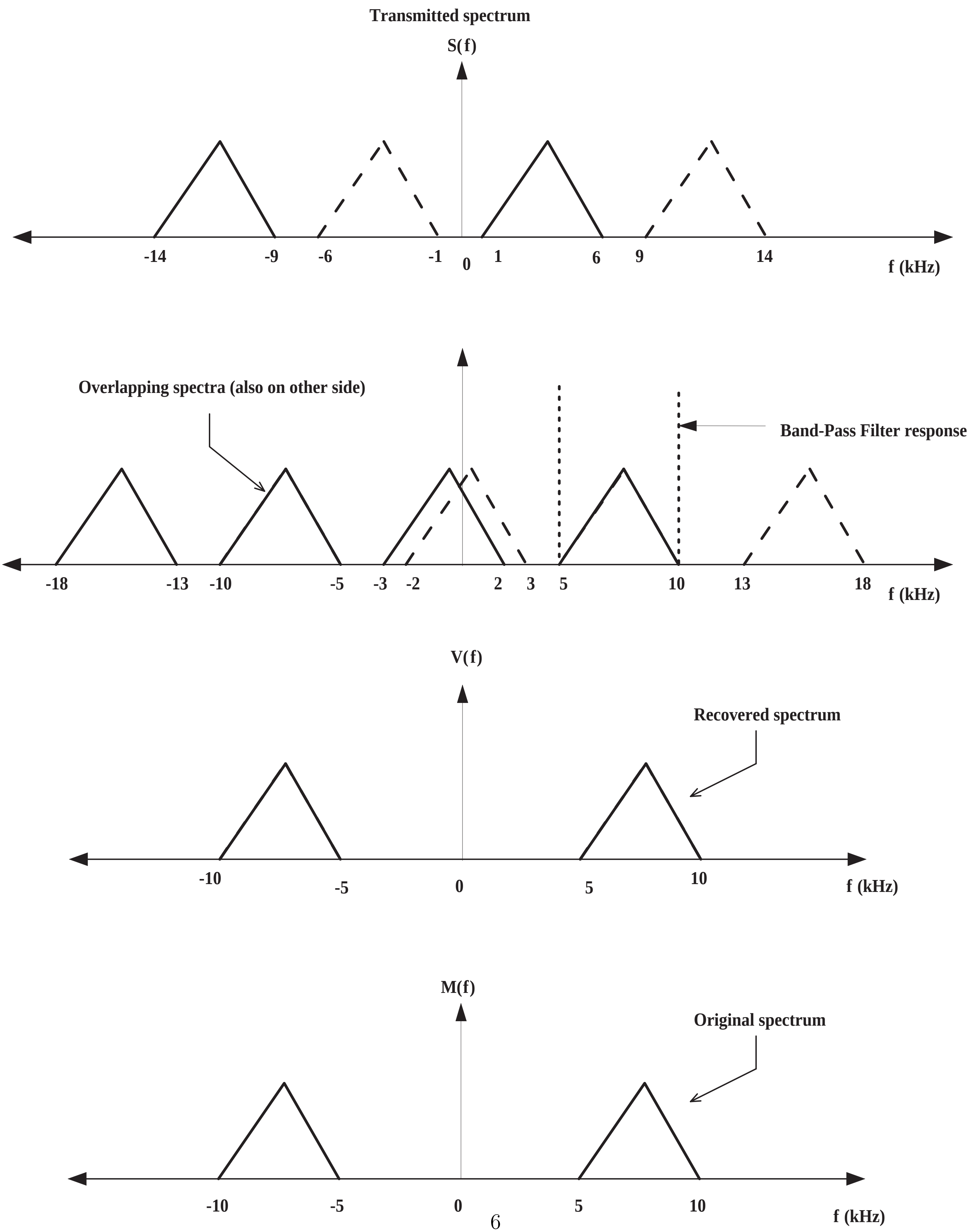

Transmitted spectrum
S(f)
-9
0
9
f (kHz)

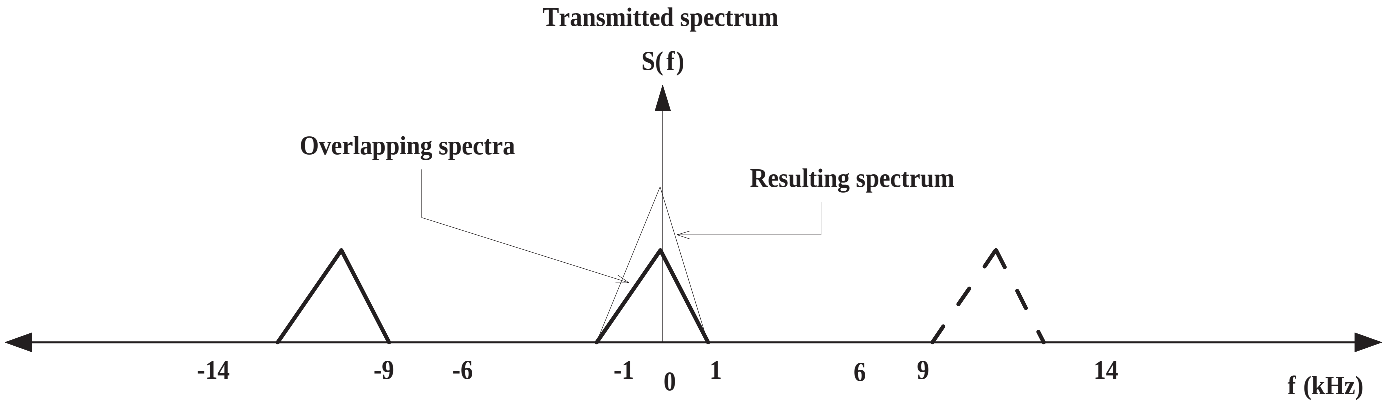

Low-Pass Filter response
-13
-6
0
5
13
f (kHz)

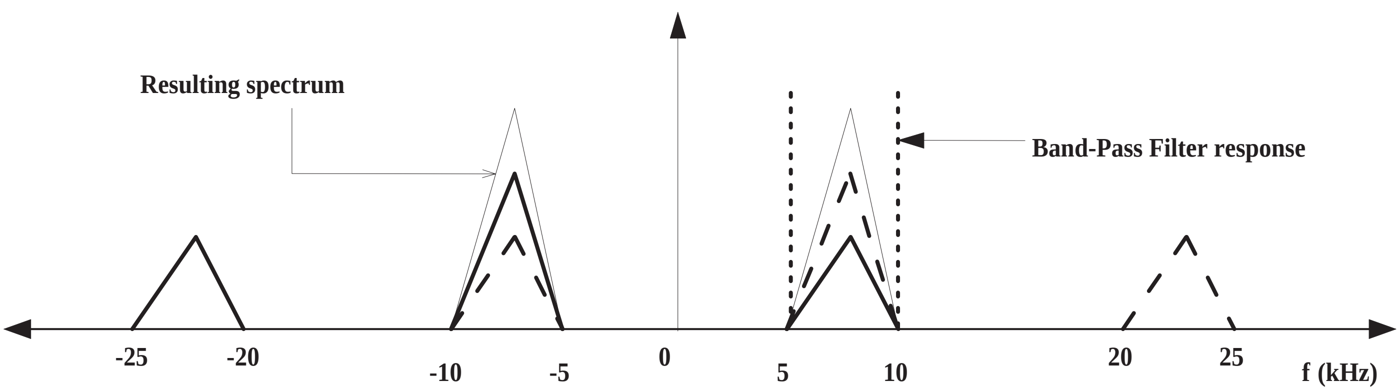

V(f)
Recovered spectrum
-4
0
4
f (kHz)

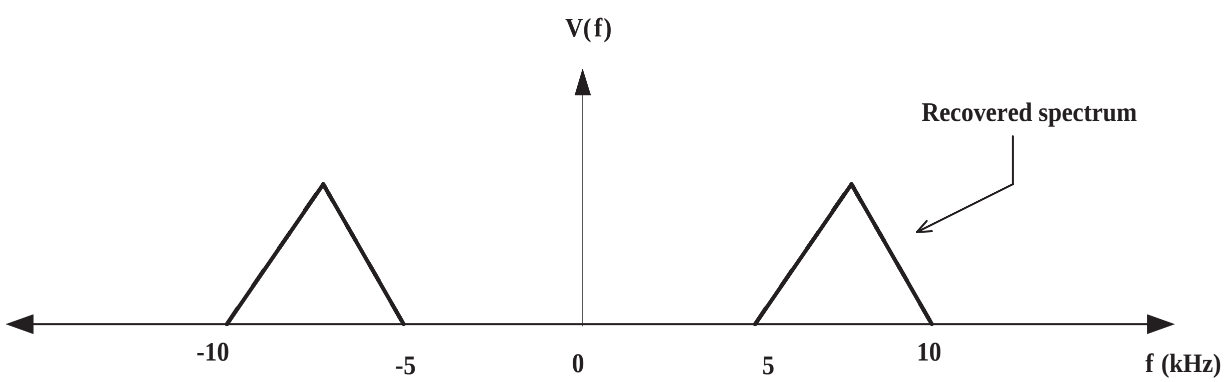

M(f)
Original spectrum
-5
0
5
f (kHz)

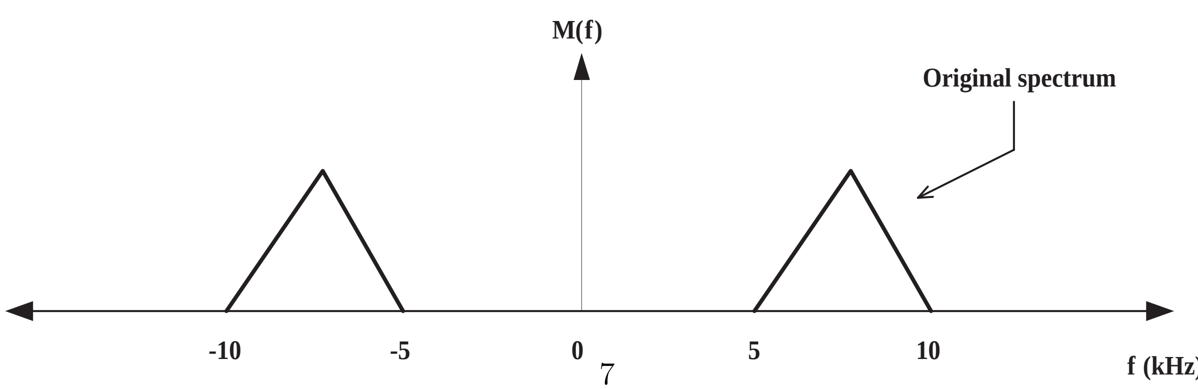

Transmitted spectrum
S(f)
-12.5
-2.5
0
2.5
12.5
f (kHz)
Resulting spectrum
Low-Pass Filter response
-20
-10
-5
0
5
10
20
f (kHz)
V(f)
Recovered spectrum
-5
0
5
f (kHz)
M(f)
Original spectrum
-5
0
5
f (kHz)


Transmitted spectrum
S(f)
-14
-9
-6
-1
0
1
6
9
14
f (kHz)
Overlapping spectra (also on other side)
Band-Pass Filter response
-18
-13
-10
-5
-3
-2
2
3
5
10
13
18
f (kHz)
V(f)
Recovered spectrum
-10
-5
0
5
10
f (kHz)
M(f)
Original spectrum
-10
-5
0
5
10
f (kHz)


Transmitted spectrum
S(f)
Overlapping spectra
Resulting spectrum
-14
-9
-6
-1
0
1
6
9
14
f (kHz)


Resulting spectrum
Band-Pass Filter response
-25
-20
-10
-5
0
5
10
20
25
f (kHz)


V(f)
Recovered spectrum
-10
-5
0
5
10
f (kHz)


M(f)
Original spectrum
-10
-5
0
5
10
f (kHz)