

## Spring Simulation

Code in `gpup/demo-2-springs.cc` simulates of a string of beads.

Purpose is to demonstrate:

- Use of coordinate class `pCoor` and vector class `pVect`.
- Simulation of a spring.

## Simulated World

- *Point masses* connected by *ideal springs*.
- First and last ball can be frozen in space.
- Balls can collide with platform but not each other.
- Each ball has its own mass.
- All springs are identical.

## Hooke's Law

Describes an ideal spring.

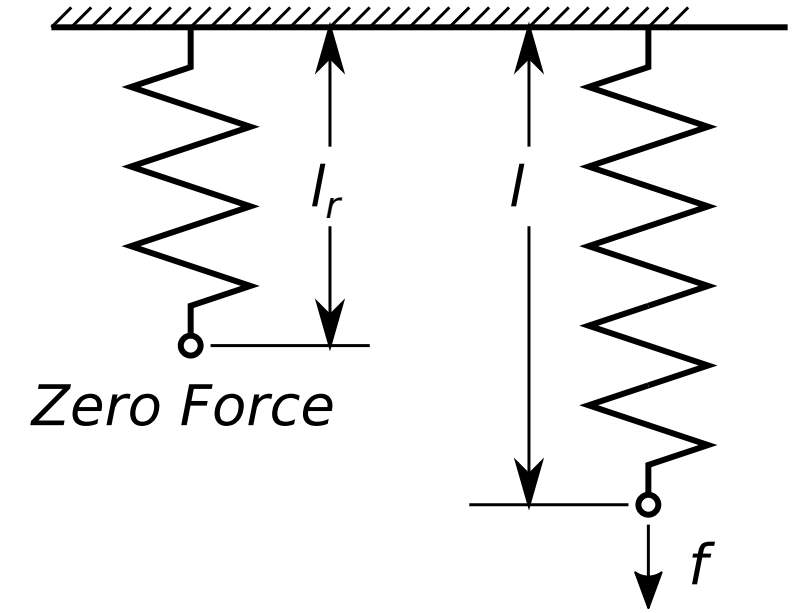
The magnitude of the force,  $f$ , is

$$f = h(l - l_r)$$

where  $h$  is the *spring constant*,

where  $l$  is the current length of the spring,

$l_r$  is the *relaxed* length of the spring.



### Computing force on ends of spring.

Consider a spring with one end at  $P_1$  and the other end at  $P_2$  ...  
 ... with relaxed length  $l_r$  and spring constant  $h$ .

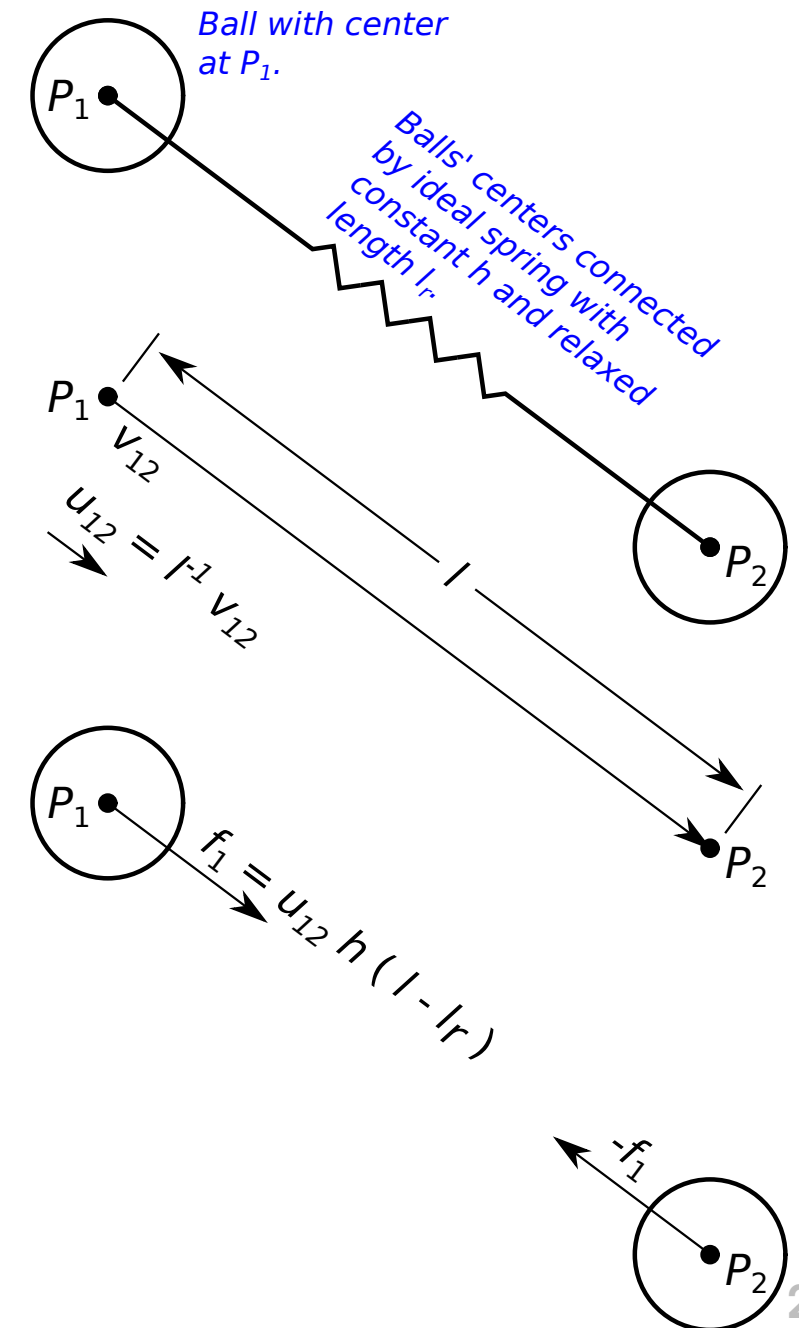
Let  $v_{12} = P_2 - P_1$ , the vector from  $P_1$  to  $P_2$ .

Let  $l = \|v_{12}\|$ , the length of vector  $v_{12}$ .

Let  $u_{12} = \frac{1}{l}v_{12}$ , the unit vector pointing from  $P_1$  to  $P_2$ .

The force on  $P_1$  is  $f_1 = h(l - l_r)u_{12}$ .

The force on  $P_2$  is  $f_2 = -f_1 = h(l - l_r)u_{21}$ .



## Variables for Physical State

### Ball class

No changes between `demo-1-simple` and `demo-2-springs` to the `ball` class were *necessary* to model the spring. But some members were added to make the simulation more interesting. The ball class in `demo-1-simple` had members only for position and velocity. Here we add members `mass` and `radius` so we can look at the effect of different masses.

### Ball Array

Variable `World::balls` holds an array of `Ball` objects. This is allocated in the `World::init` function, and the ball objects are initialized in routines `ball_setup_1`, `ball_setup_2`, etc. (The different routines put the balls in different patterns, for example, `ball_setup_2` forms them into a pendulum.)

## Spring modeling in demo-2-springs code

There is no special class used for springs. A spring is assumed between each pair of adjacent balls. (Adjacent based on the position in the `balls` array.) The length of the spring is just the distance between the respective balls. Each spring has a *relaxed length*, at which it gives no force (it will just stay at that length if no external forces act on it). Variable `World::relaxed_length` is the relaxed length for all springs.

Variable `World::opt_spring_constant` holds the spring constant ( $h$ ). To model spring friction two different spring constants are used. When a spring is being stretched (its length is larger than its relaxed length and getting larger) or compressed (its length is smaller than its relaxed length and getting smaller) spring constant `World::opt_spring_constant` is used to compute forces. When a spring is relaxing (its length is larger than its relaxed length but getting smaller, or its length is smaller than its relaxed length and getting larger) then `World::opt_spring_constant * 0.7` is used to compute forces.

## Coordinate and Vector Classes

The code uses a coordinate class `pCoor` and vector class `pVect` for the respective quantities. These classes are defined in file `coord.h`, and were written for this course. (There are many similar libraries one can use.)

File `gpup/demo-0-coor.cc` shows examples of how to use these.