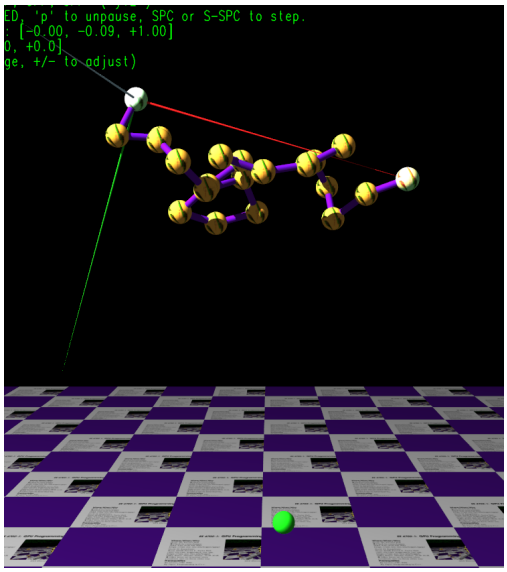
LSU EE 4702-1 **Homework 1** Solution **Due: 23 September 2020**

The solution has been checked into the repo as file `hw01-sol.cc`. A colorized version is at `https://www.ece.lsu.edu/koppel/gpup/2020/hw01-sol.cc.html`.

Problem 0: Follow the instructions on the `https://www.ece.lsu.edu/koppel/gpup/proc.html` page for account setup and programming homework work flow. Compile and run the homework code unmodified. It should initially show a string of balls compressed between two points, the first and last balls are fixed in place. The balls quickly expand and eventually will form an arc-like shape. The screenshot to the right, taken when the simulation was paused, shows the balls uncompressing, with pointy markers showing local axes, and a green ball showing the approximate place where the lowermost part of the string will be once the balls come to rest.



User Interface

Initially the arrow keys, PageUp, and PageDown can be used to move around the scene. Press (lower-case) `b` and then use the arrow and page keys to move the tail ball around. Press `l` to move the light around and `e` to move the eye (which is what the arrow keys do when the program starts). Pressing `Shift` and an arrow key will move by a larger distance (than if `Shift` were not pressed) and pressing `Ctrl` and an arrow key will move by a smaller distance. The eye can be aimed up and down by pressing `Home` and `End`, and the eye can be rotated by pressing `Insert` and `Delete`.

Pressing `p` will pause and un-pause the simulation. While the simulation is paused time can be advanced by one frame by pressing the space bar, and by one time step by pressing `Shift` and the space bar.

Press digits `1` through `4` to initialize different scenes, the program starts with scene 1. In the unmodified code scenes 1, 2, and 3 are identical: they start with the balls arranged in a line, and separated by springs at less than the relaxed distance. Once this assignment is solved scenes 1, 2, and 3 will behave differently.

Pressing `h` (head) will grab or release one end (to be precise, the ball at one end) and pressing `t` (tail) will grab or release the other end. (Actually, those keys toggle between the `OC_Locked` and `OC_Free` constraint of their respective balls.)

Press `Ctrl=` to increase the size of the green text and `Ctrl-` to decrease the size.

The `+` and `-` keys can be used to change the value of certain variables. These variables specify things such as the light intensity, spring constant, and variables that may be needed for this assignment. The variable currently affected by the `+` and `-` keys is shown in the bottom line of green text. Pressing `Tab` cycles through the different variables.

Code Generation and Debug Support

The compiler generates two versions of the code, `hw01` and `hw01-debug`. Use `hw01` to measure

performance, but use `hw01-debug` for debugging. The `hw01-debug` version is compiled with optimization turned off and with OpenGL error checking turned on. You are strongly encouraged to run `hw01-debug` under the GNU debugger, `gdb`. See the material under "Running and Debugging the Assignment" on the course procedures page.

Keys `y`, `Y`, and `Z` toggle the value of host Boolean variables `opt_tryout1`, `opt_tryout2`, and `opt_tryout3`. The user interface can also be used to modify host floating-point variable `opt_tryoutf` using the `Tab`, `+`, and `-` keys, see the previous section. These variables are intended for debugging and trying things out.

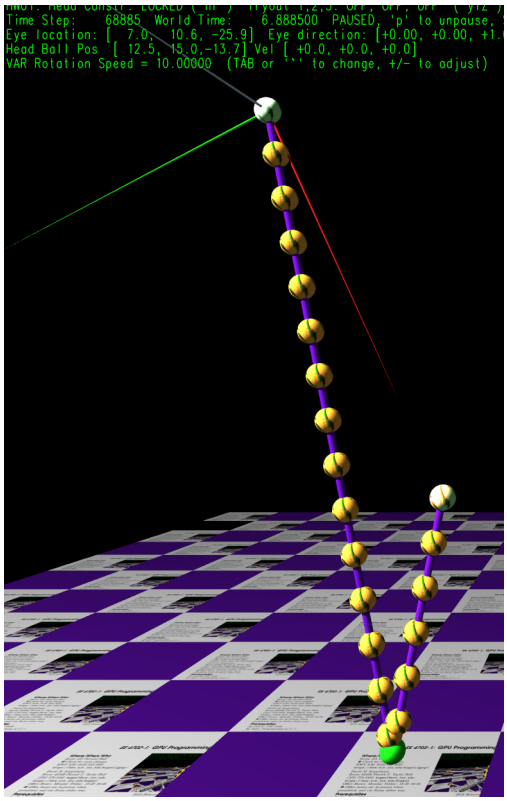
Problem 1: Scene 1 starts with the balls arranged in a line and the first and last balls locked in place. The distance between adjacent balls is set to less than the spring's relaxed distance and so the balls are quickly pushed away from each other. They will eventually come to rest in an arc. The goal of this problem is to modify the setup routine so that the initial position of the balls are chosen such that the balls are closer to equilibrium when the scene starts.

In the ideal solution to this problem the balls are arranged to form a *catenary*, the name given to the curve made by an ideal cable with two ends fixed in space. A catenary is close in shape to a parabola.

This problem will be solved with a much cruder approximation: two straight lines. The code in `ball_setup_hw01` chooses a first and last position for the balls, `first_pos` and `last_pos`, denote these P_f and P_l. Let n denote the value of `chain_length`, the number of balls, and let l_r denote the value of `distance_relaxed`. The setup routine computes `nadir_pos`, denote that P_n.

Call $L = (n-1)l_r$ the (relaxed) length of the chain. Assume for a moment that the chain were an un-stretchable string, with the ends still fixed at P_f and P_l. Take a pencil and pull the string in some direction until it is taut. (That is, until the string forms a straight line from P_f to the pencil tip and P_l to the pencil tip.) Draw a dot. Pull in some other direction and draw a dot. Repeat ∞ times. Perhaps you've guessed that shape drawn this way is an ellipse. Points P_f and P_l are the foci. Since an ellipse is defined on a plane, we need a normal. The normal used is parallel to the xz plain. The point P_n is on this ellipse, and it is chosen so that it is the point on the ellipse with the minimum y value.

To help understanding these concepts the location of P_f, P_l, P_n, and the axes in the ellipse's local space can be visualized. To do so pause the simulation (using `p`) in scene 1, 2, or 3. (Switch scenes using `1`, `2`, etc.) The ellipse local x, y, and z axes are shown by red, green, and blue needles. Positions P_f, P_l, and P_f are shown by white, red, and green featureless spheres (which may be hidden by balls). See the screenshot to the right and the code below `hw01.markers_show` in `ball_setup_hw01` to see which colors are assigned.



P_n is chosen such that $\|\overrightarrow{P_f P_n}\| + \|\overrightarrow{P_n P_l}\| = L$. This means that the first several balls can be arranged on the line from P_f to P_n and the remaining balls can be arranged on the line from P_n to P_l, while keeping the distance between adjacent balls to l_r with possibly one exception. The exception occurs when no ball is exactly at P_n. In the exception case consider the two balls closest to P_n. The distance from the first of these balls, to P_n to the second of these balls will be l_r, but

the distance between these two balls will be less than
l_r.

(a) Modify the code in `ball_setup_hw01` so that the balls are arranged in a straight line starting from `first_pos` reaching exactly or close to `nadir_pos`, and ending at `last_pos`. The distance between adjacent balls must be `distance_relaxed` except for the exception condition described above.

No ball should be placed below `nadir_pos`. The first ball must be at `first_pos` and the last ball must be at `last_pos`. See the screenshot above.

If this is solved correctly the two balls near `last_pos` should twitch (because they are closer than their relaxed distance). Initially the two lowest balls should be near the green marker (the location of `nadir_pos`) and the white and red spherical markers should not be visible.

To help debug your solution pause the simulation using `p`, then start the scene, `1`. Compare your placement of the balls to the marker balls.

Consider the loop that computes positions in the unmodified code:

```
pNorm dfn(first_pos,nadir_pos), dln(last_pos,nadir_pos);
for ( int i=0; i<chain_length; i++ )
  {
    Ball* const ball = &balls[i];
    pCoor pos = first_pos + i * first_last_dist / ( chain_length-1 ) * ax;
```

The balls need to be arranged on a line either starting at `first_pos` and toward `nadir_pos` or on a line from `nadir_pos` to `last_pos`. The balls on each line will be spaced exactly `distance_relaxed` apart.

A simple way to solve the problem is to compute the value of `i` for the first ball on the line from `nadir_pos` to `last_pos`. That's called `i_dir_change` in the solution (below). It's computed by dividing the distance from `first_pos` to `nadir_pos` by `distance_relaxed` and rounding up.

Then `ball->position` is set based on its distance from `first_pos` if `i < i_dir_change`, otherwise based on its distance from `last_pos`.

```
pNorm dfn(first_pos,nadir_pos), dln(last_pos,nadir_pos);
const int i_dir_change = 0.9999f + dfn.magnitude / distance_relaxed;
for ( int i=0; i<chain_length; i++ )
  {
    Ball* const ball = &balls[i];
    ball->position = i < i_dir_change
      ? first_pos +                       i   * distance_relaxed * dfn
      : last_pos  + ( chain_length - 1 - i ) * distance_relaxed * dln;
```

Grading note: many solutions were more complicated than they needed to be for two reasons. First, some solutions computed the coordinates of the balls on the line toward `last_pos` based on their distance from `nadir_pos`. That made the code a bit messier. What made the code messier still was the attempt to find this distance within the loop, in some cases using a second iterator for the balls toward `last_pos`.

(b) In this problem that twitching when a scene is started will be fixed. The solution is not ideal from a physical model point-of-view, but it makes a decent question. Modify the code so that each spring can have its own relaxed distance. Do so by adding a relaxed distance member to `Ball` and using it appropriately. The code in `ball_setup_hw01` must set this new member to `distance_relaxed` for all balls except for one. That one ball is the one that's closer to its neighbor. Set it to the appropriate distance if `opt_special_dist_relaxed` is true, otherwise set it to `distance_relaxed`. Modify the code in `time_step_cpu` so that the per-ball `distance_relaxed`

values will be used. Part of this problem is deciding whether the value of `distance_relaxed` set for a ball applies to its connection to its predecessor in the `balls` array or to its connection with its successor.

Use scene 2 for this problem. In scene 2 (and 3) `opt_special_dist_relaxed` is true.

If this is solved directly scene 2 should start without a sudden twitch. Instead the balls will gently come to rest in the correct place.

In the solution below a new member, `Ball::distance_relaxed`, was added that holds the relaxed distance of the spring connecting a ball to its predecessor. (That means `ball->distance_relaxed` for the first ball is meaningless.) For the ball closest to `nadir_pos` on the line towards `last_pos` `ball->distance_relaxed` is set to the distance to the previous ball, otherwise it is set to the global `distance_relaxed`:

```
ball->distance_relaxed =
  opt_special_dist_relaxed && i == i_dir_change
  ? pNorm(balls[i-1].position,ball->position).magnitude
  : distance_relaxed;
```

The time step routine needs to be modified so that it uses the correct `Ball::distance_relaxed` when computing `spring_stretch`. The inner loop in the time step routine finds the force between `ball` and `neighbor_ball`. Since `Ball::distance_relaxed` is the spring to the predecessor (lower numbered) ball, the code should use `Ball::distance_relaxed` from the higher-numbered ball. That is done by the code below, which has been shortened for clarity:

```
for ( int i=0; i<chain_length; i++ ) {
    Ball* const ball = &balls[i];
    ball->force = ball->mass * gravity_accel;
    for ( int direction: { -1, +1 } ) {
        const int n_idx = i + direction;  // Compute neighbor index.
        if ( n_idx < 0 or n_idx >= chain_length ) continue;
        Ball* const neighbor_ball = &balls[n_idx];
        pNorm ball_to_neighbor( ball->position, neighbor_ball->position );
        const float distance_between_balls = ball_to_neighbor.magnitude;

        const int idx_gt = max(i,n_idx);
        const float spring_stretch =
          distance_between_balls
          - balls[idx_gt].distance_relaxed * fabs(direction);
```

One common problem was not realizing that a single distance relaxed member of `Ball` would have to refer to either the ball's predecessor or successor. In those incorrect solutions the time step routine itself would determine what other ball `ball->distance_relaxed` was supposed to connect to. That was wrong because it makes `ball->distance_relaxed` something that could only be used for one particular setup.

(*c*) Modify the code in `ball_setup_hw01` so that when `opt_spin` is true (which is the case in scene 3) the balls are given an initial velocity so that they rotate rigidly around the line defined by `first_pos` and `last_pos`. To do this set the ball velocity. Note that balls closer to the line will have a lower velocity.

If this is solved correctly the balls will swing around the axis while generally keeping their shape.

To solve this find the distance from a ball to the line defined by `first_pos` and `last_pos` and multiply that by `az`, which is the normal to the plane defined by `first_pos`, `last_pos`, and `nadir_pos`. In the solution this was done in a separate loop for reasons of clarity. The velocity could have been set in the main `i` loop.

```
pNorm spin_dir = cross(first_pos,nadir_pos,last_pos);
if ( opt_spin ) for ( auto& b: balls ) {
      pVect to_b0(b.position, first_pos);
      const float dist = dot( to_b0, ay );
      b.velocity = dist * 10 * spin_dir;
    }
```