Performance analysis of a D-sequence based Direct Sequence CDMA system

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The performance of a D-sequence based Direct Sequence CDMA system over an additive white Gaussian noise (AWGN) channel is analyzed. The results indicate a superior performance for this system over a PN sequence based CDMA system when a set of D-sequences with zero or near-zero crosscorrelation is employed. Results are presented for a set of six synchronous users, where each user's signal uses a different chip length.

Keywords: *Spread Spectrum, CDMA, Information Theory, Pseudo Noise codes*

INTRODUCTION: Code Division Multiple Access (CDMA) is a communication technique in which a user's signal bandwidth is spread by a sequence unique to the user, which is uncorrelated with the signal. These sequences have certain useful properties like (i) sharp two-valued autocorrelation functions, (ii) balance in number of ones and zeros and (iii) cross correlation which is as low as possible. Typical candidates for these sequences include shift register or m-sequences, Pseudo Noise (PN) sequences, Gold sequences [1] and Kasami sequences [2]. Although PN sequences possess properties (i) and (ii), they have relatively high cross correlation which results in interference between signals from different users at a user's mobile station. This drawback presents problems in power control, reliable symbol estimation, and a possibility of signal outage. To overcome these problems, the IS-95 CDMA standard [3] employs Walsh codes along with PN codes, as the former have zero cross correlation. However, when the Walsh codes are received out of synchronization, as happens for users, non-zero crosscorrelation results.

Chen et al have proposed a CDMA system [4] based on complementary codes which are orthogonal to each other. In this letter we present a new CDMA system based on a family of sequences called *D-sequences* obtained by the division of unity by a prime number [5]-[7]. The advantage of these sequences lies in their excellent out-of-synchronization crosscorrelation properties. We show that since many pairs of these D-sequences have a cross correlation of zero and there are other pairs that are nearly zero, their employment as spreading sequences in the next generation of CDMA is worthy of exploration.

D-sequences: D-sequences are periodic sequences produced by the decimal expansion of the inverse of a prime number q in a modulo r division. The sequence $\{c_i\}$ is generated according to the rule [7]:

$$c_i = s\ [r^i \bmod q] \bmod r$$

$$q \bmod r \equiv -k \equiv -1/s$$

where s may be ignored, since it only rotates the sequence elements. This is to be contrasted with the PN-sequences, which are generated by expanding the inverse of a primitive polynomial. It follows that if the base chosen for the D-sequence is r = 2, then the sequence created will be a series of 1s and 0s. D-sequences, however, do not have sharp two-valued autocorrelation [6]. The autocorrelation can be as high as 33% during certain shifts of the sequence, but the remaining shifts other than zero produce very small or zero autocorrelation.

We have relied on search techniques to find sets of D-sequences with zero or near-zero crosscorrelation with the objective of using it for a CDMA system.

RESULTS: The total interference power at a user's matched filter receiver is given by the sum of Multiple Access Interference (MAI) power due to other user's signals and (AWG) Noise power. A measure of MAI may be obtained by the level of cross correlation between the different user's sequences and the average signal to interference ratio *(SIR)* is given by:

$$\text{SIR} = \frac{\text{Useful Signal Power } (E_s)}{\text{MAI + Noise Power}}$$

We chose D-sequences whose *average mutual cross correlation* was under 5% for a sequence length of the least common multiple of their respective periods. The average SIR in decibels (dB) is plotted versus signal-to-noise ratio *(SNR)* and results are depicted in Fig. 1. Binary phase shift keying (BPSK) modulation is assumed, therefore e*nergy per bit* (E_b) equals s*ignal energy* (E_s) and SNR equals E_b/N_o. The analysis reports the results for six synchronous users, where E_s has been taken to be

10W. Table I provides information about the D-sequences we have employed in the simulation, while Table II lists the PN sequences that have been used. In our analysis, the D-sequences chosen with a base *r* greater than 2 have been reduced to binary form by taking $c_i = r^i \bmod q \bmod r \bmod 2$.

If the signal transmission and reception is assumed to be synchronous, we expect the slope of the graph in Fig. 1 to be exactly unity in the ideal case of absence of MAI, or for absolute orthogonality between the spreading sequences. Our results show that the chosen D-sequences achieve a higher SIR than the selected PN sequences. Table I presents the individual periods of the various D-sequence used by the different users, chosen to ensure zero or near-zero crosscorrelation for 500 chips. Fig. 2 shows the behavior with respect to the length of the D-sequences. In this comparison, unique sequences have been assigned to users in both cases.

We chose these particular D-sequences for their excellent orthogonality property relative to each other. Larger groups of D-sequences with zero or near-zero crosscorrelation can be found using search techniques.

Our results indicate that, in principle, sets of D-sequences which have cross correlation properties superior to those of PN sequences exist, and that these sequences, when properly designed as a family, will achieve superior performance for next generation CDMA systems.

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Figure captions:

Fig.1. SIR versus SNR performance of D-sequences and PN sequences

Fig. 2. SIR versus SNR performance of D-sequences of varying lengths (N)

Table captions:

Table I: D-Sequences employed in the simulation

Table II: PN sequences employed in the simulation

Figure 1

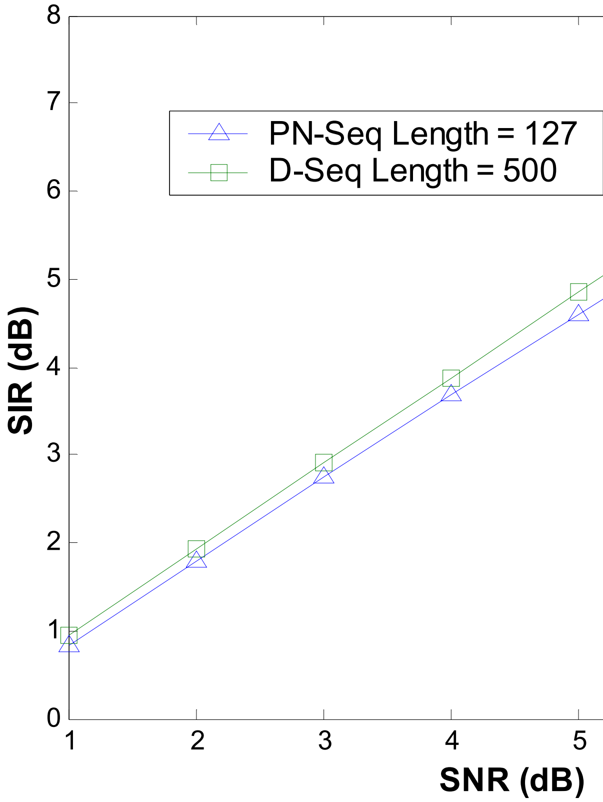

Figure 2

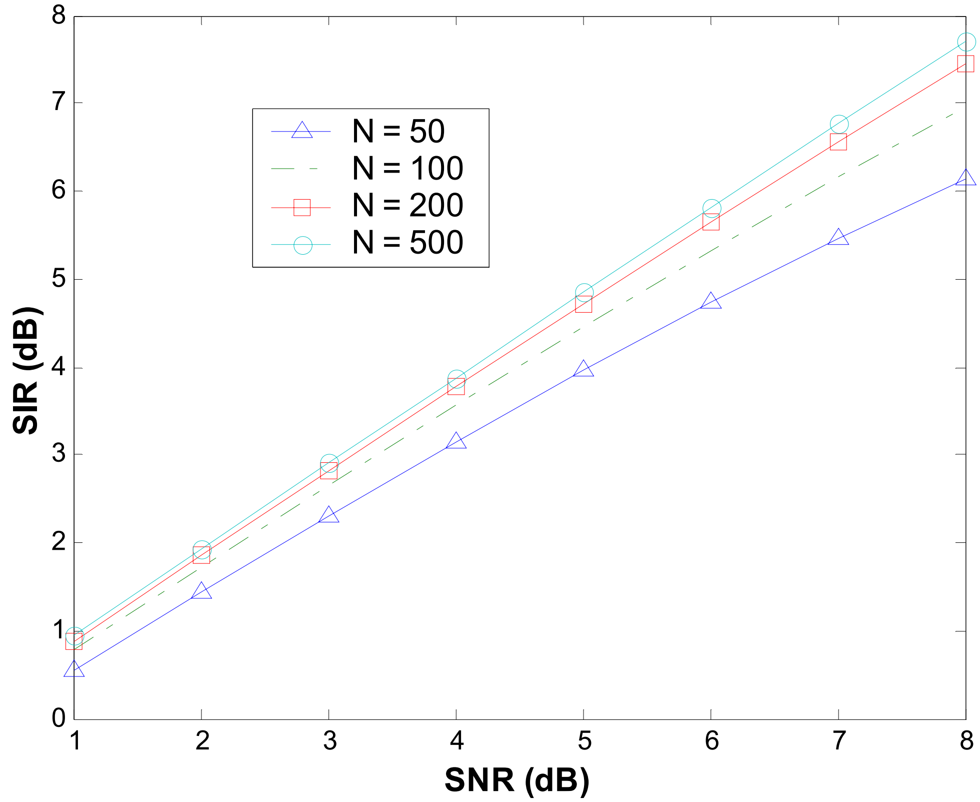

Table I

User	Prime Number	Base	Period	No. of complete bits in sequence of 500 chips
1	13	2	12	41
2	17	2	8	62
3	17	3	16	31
4	19	2	18	27
5	29	5	14	35
6	31	2	5	100

Table II

User	Shift Register tap positions	Period	Range of number of bits selected for analysis
1	[7, 1]	127	[25, 100]
2	[7, 6, 5, 4]	127	[25, 100]
3	[7, 3, 2, 1]	127	[25, 100]
4	[7, 6,4, 2]	127	[25, 100]
5	[7, 6, 3, 1]	127	[25, 100]
6	[7, 5, 4, 3, 2, 1]	127	[25, 100]