# Development of Astronomy between the Vedānga Jyotiṣa and Āryabhaṭa 

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In this chapter we review the development of astronomy between two specific dates, roughly from 1300 BC to AD 500 . Although this development is best understood by an examination of the Vedic and post-Vedic texts ${ }^{1}$, note that not all scientific knowledge of those early times can be assumed to have been committed to writing or, if written down, has survived. There are gaps in the sequence of ideas and these were filled in the past based on preconceived notions rather than a sound approach. New evidence of the past two decades has contradicted the old 19th-century model of the rise of the Indian civilization and the new, emerging paradigm has significant implications for the understanding of the development of astronomy in India.

Remembering that the beginnings of Indian culture have been traced to about 40000 BC in the rock art that has been found all over India, ${ }^{2}$ it is almost certain that the heavens had been studied for a long, long time. An examination of the motifs of the art of the earliest phases supports this view. ${ }^{3}$ An amulet seal from Rehman Dheri ( 2400 BC) indicates that the nakṣatra system is an old one. The seal shows a pair of scorpions on one side and two antelopes on the other (Figure 1). It has been argued ${ }^{4}$ that this seal represents the opposition of the Orion (Mṛgaśiras, or antelope head) and the Scorpio (Rohiṇ̄̄) nakṣatras. There exists another relationship between Orion and Rohiṇì, here the name of $\alpha$ Tauri, Aldebaran. The famous Vedic myth of Prajāpati as Orion, as personification of the year, desiring his daughter (Rohiṇī) (for example Aitareya Br. 3.33) represents the age when the beginning of the year shifted from Orion to Rohiṇī. For this "transgression", Rudra (Sirius, Mṛgavyāddha) cuts off Prajāpati's head. It has been suggested that the arrow near the head of one of the antelopes represents the decapitation of Orion, and this seems a very reasonable interpretation of the iconography of the seal.

The development of the early phases of Indian astronomy must be viewed in the context of a long pre-history. The Vedānga Jyotiṣa (VJ) of Lagadha ( 1300 BC ) is one of the subsidiary Vedic texts, so its contents must be considered to be roughly coeval with the Brāhmanas and other post-Vedic texts although the VJ text that has come down to us is definitely of a later period. The Purānas also contain a lot of very old material and their astronomy appears, on all counts, to be earlier than Āryabhaṭa so they provide us with clues regarding the evolution of astronomical thought.

Figure 1: A third millennium astronomical seal

It was long popular to consider the Siddhāntic astronomy of Āryabhaṭa to be based mainly on mathematical ideas that originated in Babylon and Greece. This view was inspired, in part, by the fact that two of the five pre-Āryabhata Siddhāntas in Varāhamihira's Pañcasiddhāntikā (PS), namely Romaka and Pauliśa, appear to be connected to the West through the names Rome and Paul. But the planetary model of these early Siddhāntas is basically an extension of the theory of the orbits of the sun and the moon in the VJ. Furthermore, the compilation of the PS occurred after Āryabhata and so the question of the gradual development of ideas can hardly be answered by examining it. The analysis in this Chapter is based on fresh material from the Brāhmaṇas and the Purānas that are generally accepted to predate Āryabhaṭa.

We also examine interactions between India and the West during the pre-Siddhāntic period to see if they could throw light on the transmission of scientific ideas. Examination of the new evidence shows that Siddhāntic astronomy must be seen as a natural development of the astronomy of the previous period.

We sketch the early history of the knowledge of the distance to the sun and the development of ideas related to the size and orbit of the planets. This is followed by the examination of the sun's orbit in the Brāhmaṇas and Purānic cosmology. This important material had not been analyzed before and it provides important clues to understand the astronomy of the pre-Āryabhaṭa Siddhāntas.

## 1 The size of the planetary system

The Rgveda asserts that the universe is infinite in extent (e.g. RV 1.52.13). It is suggested that the sun is at the centre of the universe (RV 1.35.7-9) as the rays of the sun are supposed to range from the earth to the heavens. More evidence is to be found in the Brāhmanas. For example, the Śatapatha Brāhmaṇa (ŚB) 6.1.10 to 6.2.4 gives us a brief account of the creation of the universe where several elements related to the physical and the psychological worlds are intertwined. Within this account the description of the physical world is quite clear. It begins with the image of a cosmic egg, whose shell is the earth (6.1.11). From another cosmic egg arises the sun and the shell of this second egg is the sky (6.2.3). The point of this story is to suggest that the universe was perceived at this point in the shape of an egg with the earth as the centre and the sun going around it below the heavens. The stars are seen to lie at varying distances with the polestar as the furthest.

The Atharvaveda 10.7 presents an image of the frame of the universe as a cosmic pillar (skambha). In this the earth is taken to correspond to the base (10.7.32), the space to the middle parts, and the heavens to the head. The sun, in particular, is compared to the eye (10.7.33). But there is no evidence that this analogy is to be taken in a literal fashion. One can be certain that in the Vedic period, the sun was taken to be less distant than the heavens.

The motions of all the heavenly bodies is considered to be uniform as in the system of circular motions of the sun and the moon in the Vedānga Jyotiṣa. But it is clear from the manner in which the notion of tithi worked that these circular motions relate to the mean positions ${ }^{5}$, and there was awareness that the actual motions deviated from the ideal positions of the mean planets.

The idea of the uniform motion implied that the relative distance of a body from the earth was determined by its period. This set up the following arrangement for the luminaries: the moon, Mercury, Venus, the sun, Mars, Jupiter, Saturn.

Since the sun is halfway in this arrangement, it is reasonable to assume that the distance to the sun was taken to be half of the distance to the heavens. The notion of the halfway distance must date from a period when the actual periods were not precisely known or when all the implications of the period values for the size of the universe were not understood. It is not clear that a purely geocentric model was visualized. It appears that the planets were taken to go around the sun which, in turn, went around the earth. One evidence is the order of the planets in the days of the week where one sees an interleaving of the planets based on the distance from the sun and the earth, respectively; this suggests that two points of focus, the earth and the sun, were used in the scheme. Further evidence comes from the fact that the planet periods are given with respect to the sun in later texts such as the one by Āryabhaṭa. It appears that the purely geocentric model may have been a later innovation.

The Pañcaviṃ́a Brāhmaṇa deals with various rites of different durations. The rites appear to have an astronomical intent as given by their durations: 1 through 40 days (excepting $12), 49,61,100$, and 1000 days; $1,3,12,36,100$, and 1000 years. The rites provide a plan for marking different portions of the year and also suggest longer periods of unknown meaning. In PB 16.8.6 we have a statement about the distance of the sun from the earth:
yāvad vai sahasraṃ gāva uttarādharā ity āhus tāvad asmāt lokāt svargo lokah

The world of heaven is as far removed from this world, they say, as a thousand earths stacked one above the other.

Caland ${ }^{6}$ translates this as "...as a thousand cows standing the one above the other." Presumably, this is because the Sanskrit word gauh has several meanings including the primary meanings of "earth" and "cow" but considering the context the translation by Caland is definitely wrong. Looking at the earliest Indian book on etymology, Yāska's Nirukta which is prior to 500 BC , the meaning of gauh, of which gāvah is plural, is given as: "[It] is a synonym of 'earth' because it is extended very far, or because people go over it... It is also a synonym of an animal (cow) from the same root." (Nirukta 2.5)

Let $R_{s}$ represent the distance between the earth and the sun, $R_{m}$ be the distance between the earth and the moon, $d_{s}$ be the diameter of the sun, $d_{m}$ be the diameter of the moon, and $d_{e}$ be the diameter of the earth. According to $P B, R_{s}<1000 d_{e}$, and we take that $R_{s} \approx 500 d_{e}$.

It was further known that the moon and the sun are about 108 times their respective diameters from the earth. This could have been easily determined by taking a pole and removing it to a distance 108 times its height to confirm that its angular size was equal to that of the sun or the moon. Or, we can say that $R_{s} \approx 108 d_{s}$ and $R_{m} \approx 108 d_{m}$.

Considering a uniform speed of the sun and the moon and noting that the sun completes a circuit in 365.24 days and the moon 12 circuits in 354.37 days, we find that

$$
R_{m} \approx \frac{354.37 \times 500}{365.24 \times 12} d_{e}
$$

or $R_{m} \approx 40 d_{e}$.
By using the relationship on relative sizes that $R_{s} \approx 108 d_{s} \approx 500 d_{e}$, we know that $d_{s} \approx 4.63 \times d_{e}$.

Assuming that the diameter of the earth was at some time in the pre-Siddhāntic period estimated to be about 900 yojanas $^{7}$, the distance to the moon was then about 36,000 yojanas and that to the sun about 450,000 yojanas. It also follows that the relative dimensions of the sun and the moon were taken to be in the ratio of $12.5: 1$. Knowing that the angular size of the sun and the moon is about 31.85 minutes, the size of the sun is then about 4,170 yojanas and that of the moon is about 334 yojanas.

A theory on the actual diameters of the sun, the moon, and the earth indicates a knowledge of eclipses. The RV 5.40 speaks of a prediction of the duration of a solar eclipse, so relative fixing of the diameters of the earth, the moon, and the sun should not come as a surprise.

Also note that the long periods of Jupiter and Saturn require that the sun be much closer to the earth than the midpoint to the heavens, or push the distance of the heavens beyond the $1000 d_{e}$ of $P B$ and perhaps also make the distance of the sun somewhat less than $500 d_{e}$. We do see these different modifications in the models from later periods.

The idea that the sun is roughly 500 or so earth diameters away from us is much more ancient than Ptolemy from where it had been assumed to have been borrowed by the Indians. This greater antiquity is in accordance with the ideas of van der Waerden, ${ }^{8}$ who ascribes a primitive epicycle theory to the Pythagoreans. But it is more likely that the epicycle theory
is itself much older than the Pythagoreans and it is from this earlier source that the later Greek and Indian modifications to this theory emerged which explains why the Greek and the Indian models differ in crucial details. ${ }^{9}$

Did the idea that $R_{s} \approx 500 d_{e}$ originate at about the time of $P B$, that is from the second millennium BC, or is it older? Since this notion is in conflict with the data on the periods of the outer planets, it should predate that knowledge. If it is accepted that the planet periods were known by the end of the third millennium BC, then this knowledge must be assigned an even earlier epoch. Its appearance in $P B$, a book dealing primarily with ritual, must be explained as a remembrance of an old idea. We do know that $P B$ repeats, almost verbatim, the Rgvedic account of a total solar eclipse.

It is certain that the synodic periods were first computed because the longest period, the 780 days of Mars, is not too much larger than twice the sun's period. With Mars as the furthest body in a primitive model, the sun's distance will have to be reduced to about 0.47 of the furthest point. In order to accommodate the stars, the sun will be brought even nearer. When the sidereal orbits of the planets were understood, sometime in the Vedic period, the space beyond the sun had to be taken to be vast enough to accommodate the orbits of Jupiter and Saturn. The non-circular motions of the planets would require further changes to the sizes of the orbits and these changes represent the continuing development of this phase of Indian astronomy.

The theory that $R_{s} \approx 500 d_{e}$ was so strongly entrenched that it became the basis from which different Greek and later Indian models emerged. Ptolemy considers an $R_{s}$ equal to $600 d_{e}$ whereas A ryabhaṭa assumes it to be about $438 d_{e}$. Thus the Greek and the later Indian modifications to the basic idea proceeded somewhat differently.

The ideas regarding the distance of the sun hardly changed until the modern times. The contradictions in the assumption that the luminaries move with uniform mean speed and the requirements imposed by the assumed size of the solar system led to a gradual enlargement of the models of the universe from about twice that of the distance of the sun in $P B$ to one $4.32 \times 10^{6}$ times the distance of the sun by the time of A$r y a b h a t ̣ a$. This inflationary model of the universe in $A A$ makes a distinction between the distance of the sky (edge of the planetary system) and that of the stars which is taken to be a much smaller sixty times the distance of the sun. "Beyond the visible universe illuminated by the sun and limited by the sky is the infinite invisible universe" this is stated in a commentary on $A A$ by Bhāskara I writing in 629 AD. The Purāṇic literature, part of which is contemporaneous with Āryabhaṭa, reconciles the finite estimates of the visible universe with the old Rgvedic notion of an infinite universe by postulating the existence of an infinite number of universes.

## The sizes of the planets

The ideas on planet sizes can be seen to evolve from those in the Purāṇas to the Siddhāntas. The Purānas confusingly combine two different theories, one related to the departure from the ecliptic by the moon and the other on the sidereal periods. The planets are listed in the correct sequence, supporting the view that the planet periods were known. The order of the angular sizes are correctly shown as Venus, Jupiter, Saturn, Mars, Mercury although the fractions stated are not accurate. Venus and Jupiter are taken to be $\frac{1}{16}$ th and $\frac{1}{64}$ th the
size of the moon whereas the correct fractions are $\frac{1}{20}$ and $\frac{1}{40}$. Saturn and Mars were taken to be $\frac{1}{4}$ th smaller than Jupiter and Mercury still smaller by the same fraction (VaP 53.66-67).

Table 1: The planet angular sizes in fractions of the size of the moon

| Planet | correct size | Purāna | Aryabhaṭa |
| :--- | ---: | ---: | ---: |
| Mercury | $\frac{1}{120}$ | $\frac{1}{112}$ | $\frac{1}{15}$ |
| Venus | $\frac{1}{20}$ | $\frac{1}{16}$ | $\frac{1}{5}$ |
| Mars | $\frac{1}{100}$ | $\frac{1}{84}$ | $\frac{1}{25}$ |
| Jupiter | $\frac{1}{40}$ | $\frac{1}{64}$ | $\frac{1}{10}$ |
| Saturn | $\frac{1}{80}$ | $\frac{1}{84}$ | $\frac{1}{20}$ |

By the time of Āryabhaṭa the relative sizes of the planets were better estimated (Table 1). But the angular sizes of the planets are too large by a factor of 4 excepting Mercury which is too large by a factor of 8 . Overall, the Purāna figures are more accurate and it appears that Āryabhata's overestimation by a factor of 4 may have been coloured by his ideas on optics.

## 2 The two halves of the year

The Brāhmanas recognize that the speed of the sun varies with the seasons. The year-long rites of the Brāhmaṇas were organized with the summer solstice (viṣuvant) as the middle point. There were two years: the ritual one started with the winter solstice (mahāvrata day), and the civil one started with the spring equinox (viṣuva). Vedic rites had a correspondence with the different stages of the year and, therefore, astronomy played a very significant role. These rites counted the days upto the solstice and in the latter half of the year, and there is an asymmetry in the two counts. This is an astronomical parameter, which had hitherto escaped notice, that allows us to date the rites to no later than the second millennium BC.

The Aitareya Brāhmana 4.18 describes how the sun reaches the highest point on the day called viṣuvant and how it stays still for a total of 21 days with the viṣuvant being the middle day of this period. In Pañcaviṃśa Brāhmaṇa (Chapters 24 and 25), several year-long rites are described where the viṣuvant day is preceded and followed by three-day periods. This suggests that the sun was now taken to be more or less still in the heavens for a total period of 7 days. So it was clearly understood that the shifting of the rising and the setting directions had an irregular motion.

ŚB 4.6.2 describes the rite called gavām ayana, the "sun's walk" or the "cows' walk." This is a rite which follows the motion of the sun, with its middle of the viṣuvant day.

The Yajurveda (38.20) says that the āhavanīya or the sky altar is four-cornered since the sun is four-cornered, meaning thereby that the motion of the sun is characterized by four cardinal points: the two solstices and the two equinoxes.

The year-long rites list a total of 180 days before the solstice and another 180 days following the solstice. Since this is reckoning by solar days, it is not clear how the remaining 4 or 5 days of the year were assigned. But this can be easily inferred.


Figure 2: The earth's asymmetric orbit shown in an ancient 2nd millennium altar

Note that the two basic days in this count are the viṣuvant (summer solstice) and the mahāvrata day (winter solstice) which precedes it by 181 days in the above counts. Therefore, even though the count of the latter part of the year stops with an additional 180 days, it is clear that one needs another 4 or 5 days to reach the mahāvrata day in the winter. This establishes that the division of the year was in the two halves of 181 and 184 or 185 days.

Corroboration of this is suggested by evidence related to an altar design from the Śatapatha Brāhmaṇa (ŚB 8.6) which is shown in Figure 2. This altar represents the path of the sun around the earth. The middle point, which represents the earth, is at a slight offset to the centre. This fact, and the fact that the number of bricks in the outer ring are not symmetrically placed, shows that the four quarters of the year were not taken to be symmetric.

This inequality would have been easy to discover. The Indians used the reflection of the noon-sun in the water of a deep well to determine the solstice days.

If one assumes that the two halves of the year are directly in proportion to the brick counts of 14 and 15 in the two halves of the ring of the sun, this corresponds to day counts of 176 and 189. This division appears to have been for the two halves of the year with respect to the equinoxes if we note that the solstices divide the year into counts of 181 and 184.

The apparent motion of the sun is the greatest when the earth is at perihelion and the least when the earth is at aphelion. Currently, this speed is greatest in January. The interval between successive perihelia, the anomalistic year, is 365.25964 days which is 0.01845 days longer than the tropical year on which our calendar is based. In 2000 calendar years, the date of the perihelion advances almost 35 days; in 1000 years, it advances almost a half-year ( 175 days). This means that the perihelion movement has a cycle of about 20000 years.

In the first millennium BC , the earth was at perihelion within the interval prior to the
winter solstice. Thus during this period the half of the year from the summer solstice to the winter solstice would have been shorter than the half from the winter solstice to the summer solstice. This is just the opposite of what is described in the rites of the Brāhmanas.

It is interesting that the Greeks discovered the asymmetry in the quarters of the year about 400 BC . Modern calculations show that at this time the four quarters of the year starting with the winter solstice were $90.4,94.1,92.3$, and 88.6 days long. The period from the winter solstice to the summer solstice was then 184.5 days and the perihelion occurred in mid- to late October.

The count of about 181 days from the winter to the summer solstice would be true when the perihelion occurs before the summer solstice. This will require it to move earlier than mid- to late June and no earlier than mid- to late December. In other words, compared to 400 BC , the minimum number of months prior to October is 4 and the maximum number of months is 10 . This defines periods which are from 6850 years to 17150 years prior to 400 BC.

These periods appear too early to be considered plausible and this may reflect the fact that the measurements in those times were not very accurate. Nevertheless, it means that the first millennium BC for the rites of the Brāhmaṇas is absolutely impossible.

Since the Śatapatha Brāhmaṇa has lists of teachers that go through more than fifty generations, we know that the period of the Brāhmanas was a long one, perhaps a thousand years. To be as conservative as possible, one may consider the period 2000-1000 BC as reasonable for these texts. The Vedic Samhitās are then assigned to the earlier fourth and third millennia BC.

## 3 The origins of the idea of epicycles

More than a hundred years ago, Burgess ${ }^{10}$ saw the Indians as the originators of many of the notions that led to the Greek astronomical flowering. This view slowly lost support and then it was believed that Indian astronomy was essentially derivative and it owed all its basic ideas to the Babylonians and the Greeks. It was even claimed that there was no tradition of reliable observational astronomy in India.

Using statistical analysis of the parameters used in the many Siddhāntas, Billard showed ${ }^{11}$ that the Siddhāntas were based on precise observations and so the theory of no observational tradition in India was wrong. This conclusion is reinforced by the fact that the Vedic books are according to an astronomical plan.

Earlier, it was believed that the mahāyuga/kalpa figure of $4,320,000$, which occurs in the Siddhāntas, was borrowed from the astronomy ${ }^{12}$ of the Babylonian Berossos (c. 300 $\mathrm{BC})$. But it is more logical to see it derived from the number 432,000 related to the number of syllables in the Rgveda that is mentioned in the much earlier Śatapatha Brāhmaṇa (ŚB 10.4.2).

The Siddhāntic astronomy has features which are unique to India and it represents an independent tradition. In the words of Thurston ${ }^{13}$ :

Not only did Āryabhaṭa believe that the earth rotates, but there are glimmerings in his system (and other similar Indian systems) of a possible underlying theory
in which the earth (and the planets) orbits the sun, rather than the sun orbiting the earth... The significant evidence comes from the inner planets: the period of the sighrocca is the time taken by the planet to orbit the sun.
A pure heliocentrism is to be found in the following statement in the Viṣ̣̣u Purāna 2.8:
The sun is stationed for all time, in the middle of the day... The rising and the setting of the sun being perpetually opposite to each other, people speak of the rising of the sun where they see it; and, where the sun disappears, there, to them, is his setting. Of the sun, which is always in one and the same place, there is neither setting nor rising.
It is not certain that A Aryabhata was the originator of the idea of the rotation of the earth. It appears that the rotation of the earth is inherent in the notion that the sun never sets that we find in the Aitareya Brāhmaṇa 2.7:

The [sun] never really sets or rises. In that they think of him "He is setting," having reached the end of the day, he inverts himself; thus he makes evening below, day above. Again in that they think of him "He is rising in the morning," having reached the end of the night he inverts himself; thus he makes day below, night above. He never sets; indeed he never sets.

One way to visualize it is to see the universe as the hollow of a sphere so that the inversion of the sun now shines the light on the world above ours. But this is impossible since the sun does move across the sky during the day and if the sun doesn't set or rise it doesn't move either. Clearly, the idea of "inversion" denotes nothing but a movement of the earth.

By examining early Vedic sources the stages of the development of the earliest astronomy become apparent. After the Rgvedic stage comes the period of the Brāhmaṇas in which we place Lagadha's astronomy. The third stage is early Siddhāntic and early Purāṇic astronomy.

These three stages are summarized below:

1. Rgvedic astronomy (c. 4000? - 2000 BC) Motion of the sun and the moon, naksatras, planet periods. The start of this stage is a matter of surmise but we have clues such as Vedic myths which have been interpreted to indicate astronomical events of the fourth millennium BC.
2. Astronomy of the Brāhmaṇas (2000-1000 BC) Astronomy represented by means of geometric altars; non-uniform motion of the sun and the moon; intercalation for the lunar year; "strings of wind joined to the sun." The Vedāñga Jyotiṣa (c. 1300 BC) of Lagadha must be seen as belonging to the latter part of this stage. The VJ text that has come down to us appears to be of a later era. Being the standard manual for determination of the Vedic rites, Lagadha's work must have served as a "living" text where the language got modified to a later form.
3. Early Siddhāntic and early Purānic (1000 BC - 500 AD) Here our main sources are the Śulbasūtras, the Mahābhārata, the early Purānas, the Sūrya Siddhānta and other texts. Further development of the ssighrocca and mandocca cycles, the concepts of kalpa.

It is significant that the first two stages and the beginning part of the third stage are well prior to the rise of mathematical astronomy in Babylonia and in Greece. The concepts of the sighrocca and mandocca cycles indicate that the motion of the planets was taken to be fundamentally around the sun, which, in turn, was taken to go around the earth.

The mandocca, in the case of the sun and the moon, is the apogee where the angular motion is the slowest and in the case of the other planets it is the aphelion point of the orbit. For the superior planets, the sighrocca coincides with the mean place of the sun, and in the case of an inferior planet, it is an imaginary point moving around the earth with the same angular velocity as the angular velocity of the planet round the sun; its direction from the earth is always parallel to the line joining the sun and the inferior planet.

The mandocca point serves to slow down the motion from the apogee to the perigee and speed up the motion from the perigee to the apogee. It is a representation of the nonuniform motion of the body, and so it can be seen as a direct development of the idea of the non-uniform motion of the sun and the moon.

The śsgrocca maps the motion of the planet around the sun to the corresponding set of points around the earth. The sun, with its winds that hold the solar system together, is, in turn, taken to go around the earth.

The antecedents of this system can be seen in the earlier texts. S'B 4.1.5.16 describes the sun as puṣkaramādityo, "the lotus of the sky." ŚB 8.7.3.10 says:

The sun strings these worlds [the earth, the planets, the atmosphere] to himself on a thread. This thread is the same as the wind...

This suggests a central role to the sun in defining the motions of the planets and ideas such as these must have ultimately led to the theory of the sīghrocca and the mandocca cycles.

The theory that the sun was the "lotus" [the central point] of the sky and that it kept the worlds together by its "strings of wind" may have given rise to the heliocentric tradition in India. The offset of the sun's orbit evolved into the notion of mandocca and the motions of the planets around the sun were transferred to the earth's frame through the device of the sīghrocca.

The Brāhmanas consider non-circular motion of the sun and, by implication, of the moon and that the sun is taken to be about 500 earth diameters away from the earth. Analysis of Rgvedic astronomy has shown that planet periods had been determined. Logically, the next step would be to characterize the details of the departure from the circular motion for the planets. In VaP 53.71 it is stated that the planets move in retrograde (vakra) motion.

Although the extant Sūrya Siddhānta (SS) is a late book, it preserves old pre-Siddhāntic ideas on the motions of the planets:

Forms of time, of invisible shape, stationed in the zodiac (bhagana), called the conjunction (śz̄ghocca), apsis (mandocca), and node (pata), are causes of the motion of the planets.
The planets, attached to these beings by cords of air, are drawn away by them, with the right and lift hand, forward and backward, according to nearness, toward their own place.

A wind, called provector (pravaha) impels them toward their own apices (ucca); being drawn away forward and backward, they proceed by a varying motion.

The so-called apex (ucca), when in the half-orbit in front of the planet, draws the planet forward; in like manner, when in the half-orbit behind the planet, it draws it backward.
When the planets, drawn away by their apices, move forward in their orbits, the amount of the motion so caused is called their excess (dhana); when they move backward, it is called their deficiency (rna). (SS 2.1-5)

The idea of the sizes was directly related to the deviation from the ecliptic. The motions were defined to be of eight different kinds:

Owing to the greatness of its orb, the sun is drawn away only a very little; the moon, by reason of the smallness of its orb, is drawn away much more;
Mars and the rest, on account of their small size, are, by the points of focus, called conjunction and apsis, drawn away very far, being caused to vacillate exceedingly.

Hence the excess and deficiency of these latter is very great, according to their rate of motion. Thus do the planets, attracted by those beings, move in the firmament, carried on by the wind.
The motion of the planets is of eight kinds: retrograde (vakra), somewhat retrograde (anuvakra), transverse (kuṭila), slow (manda), very slow (mandatara), even (sama), very swift (śz̃ghratara), and swift (śz̃ghra).

Of these, the very swift, the swift, the slow, the very slow, and the even are forms of the motion called direct ( $\mathrm{r} j \mathrm{u}$ ). (SS 2.9-13)

## The Early Siddhāntas

The development of astronomical ideas from the Vedānga Jyotiṣa onwards can also by studied from the information in the Jaina books, the Mahābhārata and the astronomical references in the general literature. For example, the Arthaśāstra uses a rule for telling time that is very similar to that in VJ.

The Pañcasiddhāntikā of Varāhamihira summarizes five early schools of Siddhāntic astronomy, namely Paitāmaha, Vāsiṣ̣̣ha, Romaka, Pauliśa, and Saura manly with regard to the calculation of eclipses.

Owing to the names Romaka and Pauliśa, it was assumed that the PS mostly represents Babylonian and Greek material. But such a supposition has no firm evidence to support to it. There also exists the possibility that an India-inspired astronomy could have travelled to the West before the Siddhāntic period.

The use of cycles was current during the time of Śatapatha Brāhmana. A modular arithmetic, fundamental to Siddhāntic astronomy, was in use in Vedānga Jyotiṣa. The 2,850 year luni-solar yuga of the Romaka Siddhānta (PS 1.15) is derived from the 95 -year Yājñavalkya cycle of the Śatapatha Brāhmaṇa, as it is equal to $30 \times 95$.

Summarizing, the basic features of the Siddhāntic astronomy such as non-circular orbits of the sun and the moon and the specific notions of "ropes of wind" for the the planetary system were already present in the Brāhmaṇas and they appear in a more developed form in the primitive epicycle theory of the Sūrya Siddhānta. As the retrograde motions were recognized, the orbit sizes were adjusted and made smaller.

## 4 Pre-Siddhāntic Cosmology

Early texts consider light to be like a wind. Was any thought given to its speed? Given the nature of the analogy, one would expect that this speed was considered finite. The Purānas speak of the moving jyotiścakra, "the circle of light." This analogy or that of the swift arrow let loose from the bow in these accounts leaves ambiguous whether the circle of light is the sun or its speeding rays.

We get a specific number that could refer to the speed of light in a late text by Sāyaṇa (c. 1315-1387), prime minister in the court of Emperors Bukka I of the Vijayanagar Empire and Vedic scholar. In his commentary on the fourth verse of the hymn 1.50 of the Rgveda on the sun, he says ${ }^{14}$

## tathā ca smaryate yojanānạ̣̄ sahasre dve dve śate dve ca yojane ekena nimiṣārdhena kramamāna

Thus it is remembered: [O sun] you who traverse 2,202 yojanas in half a nimeṣa.
The same statement occurs in the commentary on the Taittirīya Brāhmaṇa by Bhatṭa Bhāskara (10th century?), where it is said to be an old Purānic tradition.

The figure could refer to the actual motion of the sun but, as we will see shortly, that is impossible. By examining parallels in the Purānic literature, we see it as an old tradition related to the speed of [sun]light.

The units of yojana and nimeṣa are well known. The usual meaning of yojana is about 9.1 miles as in the Arthaśāstra where it is defined as being equal to 8,000 dhanu or "bow," where each dhanu is taken to be about 6 feet. Aryabhata, Brahmagupta and other astronomers used smaller yojanas but such exceptional usage was confined to the astronomers; we will see that the Purānas also use a non-standard measure of yojana. As a scholar of the Vedas and a non-astronomer, Sāyaṇa would be expected to use the "standard" Arthaśāstra units.

The measures of time are thus defined in the Purānas:

$$
\begin{aligned}
& 15 \text { nimeṣa }=1 \text { kāṣthā } \\
& 30 \text { kāṣthā }=1 \text { kalā } \\
& 30 \text { kalā }=1 \text { muhūrta } \\
& 30 \text { muhūrta }=1 \text { day-and-night }
\end{aligned}
$$

A nimeṣa is therefore equal to $\frac{16}{75}$ seconds.

De and Vartak have in recent books ${ }^{15}$ argued that this statement refers to the speed of light. Converted into modern units, it does come very close to the correct figure of 186,000 miles per second!

Such an early knowledge of this number doesn't sound credible because the speed of light was determined only in 1675 by Roemer who looked at the difference in the times that light from Io, one of the moons of Jupiter, takes to reach the earth based on whether it is on the near side of Jupiter or the far side. Until then light was taken to travel with infinite velocity. There is no record of any optical experiments that could have been performed in India before the modern period to measure the speed of light.

Maybe Sāyaṇa's figure refers to the speed of the sun in its supposed orbit around the earth. But that places the orbit of the sun at a distance of over 2,550 million miles. The correct value is only 93 million miles and until the time of Roemer the distance to the sun used to be taken to be less than 4 million miles. The Indian astronomical texts place the sun only about half a million yojanas from the earth. We show that this figure is connected to Purānic cosmology and, therefore, it belongs, logically, to the period of pre-Siddhāntic astronomy.

## Physical ideas in early literature

The philosophical schools of Sāṃkhya and Vaiśeṣika tell us about the old ideas on light. ${ }^{16}$ According to Sāmkhya, light is one of the five fundamental "subtle" elements (tanmātra) out of which emerge the gross elements. The atomicity of these elements is not specifically mentioned and it appears that they were actually taken to be continuous.

On the other hand, Vaiśesika is an atomic theory of the physical world on the nonatomic ground of ether, space and time. The basic atoms are those of earth (prthivi), water ( $\bar{a} p a s$ ), fire (tejas), and air (vāyu), that should not be confused with the ordinary meaning of these terms. These atoms are taken to form binary molecules that combine further to form larger molecules. Motion is defined in terms of the movement of the physical atoms and it appears that it is taken to be non-instantaneous.

Light rays are taken to be a stream of high velocity of tejas atoms. The particles of light can exhibit different characteristics depending on the speed and the arrangements of the tejas atoms.

Although there existed several traditions of astronomy in India, only the mathematical astronomy of the Siddhāntas has been properly examined. Some of the information of the non-Siddhāntic astronomical systems is preserved in the Purānas.

The Purānic astronomy is cryptic, and since the Purānas are encyclopaedic texts, with several layers of writing, presumably by different authors, there are inconsistencies in the material. Sometimes, speculative and the empirical ideas are so intertwined that without care the material can appear meaningless. The Purānic geography is quite fanciful and this finds parallels in its astronomy as well.

We can begin the process of understanding Purānic astronomy by considering its main features, such as the size of the solar system and the motion of the sun. But before we do so, we will speak briefly of the notions in the Siddhāntas.
$\bar{A} r y a b h a t ̣ a ~ i n ~ h i s ~ \overline{~ r ~ r a b a b a t ̣ \imath ̄ y a ~(A A) ~ d e a l s ~ w i t h ~ t h e ~ q u e s t i o n ~ o f ~ t h e ~ s i z e ~ o f ~ t h e ~ u n i v e r s e . ~}$

He defines a yojana to be $8,000 n r$, where a $n r$ is the height of a man; this makes his yojana $\left(y_{a}\right)$ approximately 7.5 miles. Or $y_{s} \approx \frac{6}{5} y_{a}$, where $y_{s}$ is the standard Arthaśāstra yojana. AA 1.6 states that the orbit of the sun is $2,887,666.8$ yojanas and that of the sky is 12,474,720,576,000 yojanas.

There is no mention by Āryabhata of a speed of light. But the range of light particles is taken to be finite, so it must have been assumed that the particles in the "observational universe" do not penetrate to the regions beyond the "orbit of the sky." This must have been seen in the analogy of the gravitational pull of the matter just as other particles fall back on the earth after reaching a certain height.

The orbit of the sky is $4.32 \times 10^{6}$ greater than the orbit of the sun. It is clear that this enlargement was inspired by cosmological ideas.

The diameters of the earth, the sun, and the moon are taken to be $1,050,4,410$ and 315 yojanas, respectively. Furthermore, $A A 1.6$ implies the distance to the sun, $R_{s}$, to be 459,585 yojanas, and that to the moon, $R_{m}$, as 34,377 yojanas. These distances are in the correct proportion related to their assumed sizes given that the distances are approximately 108 times the corresponding diameters.

Converted to the standard Arthaśāstra units, the diameters of the earth and the sun are about 875 and 3,675 yojanas, and the distance to the sun is around 0.383 million yojanas.

## Purānic cosmology

The Purānic material is closer to the knowledge of the Vedic times. Here we specifically consider Vāyu Purāṇa (VaP), Viṣṇu Purāṇa (ViP), and Matsya Purāṇa (MP) ${ }^{17}$. VaP and ViP are generally believed to be amongst the earliest Purāṇas and at least 1,500 years old. Their astronomy is prior to the Siddhāntic astronomy of Āryabhaṭa and his successors.

The Purānas instruct through myths and this mythmaking can be seen in their approach to astronomy also. For example, they speak of seven underground worlds below the orbital plane of the planets and of seven "continents" encircling the earth. One has to take care to separate this imagery, that parallels the conception of the seven centres of the human's psycho-somatic body, from the underlying cosmology of the Purānas that is their primary concern in their jyotisa chapters. The idea of seven regions of the universe is present in the Rgveda 1.22.16-21 where the sun's stride is described as saptadhāman, or taking place in seven regions.

The different Purānas appear to reproduce the same cosmological material. There are some minor differences in figures that may be a result of wrong copying by scribes who did not understand the material. Here we mainly follow ViP.

ViP 2.8 describes the sun to be 9,000 yojanas in length and to be connected by an axle that is $15.7 \times 10^{6}$ yojanas long to the Mānasa mountain and another axle 45,500 yojanas long connected to the pole star. The distance of 15.7 million yojanas between the earth and the sun is much greater than the distance of 0.38 or 0.4375 million yojanas that we find in the Siddhāntas and other early books. This greater distance is stated without a corresponding change in the diameter of the sun.

Elsewhere, in VaP 50, it is stated that the sun covers 3.15 million yojanas in a muhūrta. This means that the distance covered in a day is 94.5 million yojanas. MP 124 gives the
same figure. This is in agreement with the view that the sun is 15.7 million yojanas away from the earth. The specific speed given here, translates to 116.67 yojanas per half-nimeṣa.

The size of the universe is described in two different ways, through the "island-continents" and through heavenly bodies. The geography of the Purānas describes a central continent, Jambu, surrounded by alternating bands of ocean and land. The seven island-continents of Jambu, Plakṣa, Śālmala, Kuśa, Kraunca, Śāka, and Puṣkara are encompassed, successively, by seven oceans; and each ocean and continent is, respectively, of twice the extent of that which precedes it. The universe is seen as a sphere of size 500 million yojanas.

The continents are imaginary regions and they should not be confused with the continents on the earth. Only certain part of the innermost planet, Jambu, that deals with India have parallels with real geography.

The inner continent is taken to be 16,000 yojanas as the base of the world axis. In opposition to the interpretation by earlier commentators, who take the increase in dimension by a factor of two to be only across the seven "continents," we take it to apply to the "oceans" as well. At the end of the seven island-continents is a region that is twice the preceding region. Further on, is the Lokāloka mountain, 10,000 yojanas in breadth, that marks the end of our universe.

Assume that the size of the Jambu is $J$ yojana, then the size of the universe is:

$$
\begin{equation*}
U=J\left(1+2+2^{2}+2^{3}+2^{4}+2^{5}+2^{6}+2^{7}+2^{8}+2^{9}+2^{10}+2^{11}+2^{12}+2^{13}+2^{14}\right)+10,000 \tag{1}
\end{equation*}
$$

Or,

$$
\begin{equation*}
U=32,767 J+10,000 \text { yojanas } \tag{2}
\end{equation*}
$$

If U is 500 million miles, then J should be about 15,260 yojanas. The round figure of 16,000 is mentioned as the width of the base of the Meru, the world axis, at the surface of the earth. This appears to support our interpretation.

Note that the whole description of the Purānic cosmology had been thought to be inconsistent because an erroneous interpretation of the increase in the sizes of the "continents" had been used.

When considered in juxtaposition with the preceding numbers, the geography of concentric continents is a representation of the plane of the earth's rotation, with each new continent as the orbit of the next "planet". ${ }^{18}$

The planetary model in the Purānas is different from that in the Siddhāntas. Here the moon as well as the planets are in orbits higher than the sun. Originally, this supposition for the moon may have represented the fact that it goes higher than the sun in its orbit. Given that the moon's inclination is $5^{\circ}$ to the ecliptic, its declination can be $28.5^{\circ}$ compared to the sun's maximum declination of $\pm 23.5^{\circ}$. This "higher" position must have been, at some stage, represented literally by a higher orbit. To make sense with the observational reality, it became necessary for the moon is taken to be twice as large as the sun. That this is a jumbling up of two different theories is clear from the fact that the planets are listed in the correct sequence determined by their sidereal periods.

The distances of the planetary orbits beyond the sun are as follows:

Table 2: From the earth to te Pole-star

| Interval I | yojanas |
| :--- | ---: |
| Earth to Sun | $15,700,000$ |
| Sun to Moon | 100,000 |
| Moon to Asterisms | 100,000 |
| Asterisms to Mercury | 200,000 |
| Mercury to Venus | 200,000 |
| Venus to Mars | 200,000 |
| Mars to Jupiter | 200,000 |
| Jupiter to Saturn | 200,000 |
| Saturn to Ursa Major | 100,000 |
| Ursa Major to Pole-star | 100,000 |
| Sub-total | $17,100,000$ |

Further spheres are postulated beyond the pole-star. These are the Maharloka, the Janaloka, the Tapoloka, and the Satyaloka. Their distances are as follows:

Table 3: From Pole-star to Satyaloka

| Interval II | yojanas |
| :--- | ---: |
| Pole-star to Maharloka | $10,000,000$ |
| Maharloka to Janaloka | $20,000,000$ |
| Janaloka to Tapoloka | $40,000,000$ |
| Tapoloka to Satyaloka | $120,000,000$ |
| Grand Total | $207,100,000$ |

Since the last figure is the distance from the earth, the total diameter of the universe is 414.2 million yojanas, not including the dimensions of the various heavenly bodies and lokas. The inclusion of these may be expected to bring this calculation in line with the figure of 500 million yojanas mentioned earlier.

Beyond the universe lies the limitless pradhāna that has within it countless other universes.

Purāṇic cosmology views the universe as going through cycles of creation and destruction of 8.64 billion years. The consideration of a universe of enormous size must have been inspired by a supposition of enormous age.

## Reconciling Purānic and Standard Yojanas

It is clear that the Purānic yojana $\left(y_{p}\right)$ is different from the Arthaśāstra yojana $\left(y_{p}\right)$. To find the conversion factor, we equate the distances to the sun.

$$
\begin{equation*}
0.4375 \times 10^{6} y_{s}=15.7 \times 10^{6} y_{p} \tag{3}
\end{equation*}
$$

In other words,

$$
\begin{equation*}
1 y_{s} \approx 36 y_{p} \tag{4}
\end{equation*}
$$

The diameter of the earth should now be about $875 \times 36 \approx 31,500 y_{p}$. Perhaps, this was taken to be $32,000 y_{p}$, twice the size of Meru. This understanding is confirmed by the statements in the Purānas. For example, MP 126 says that the size of Bhāratavarṣa (India) is $9,000 y_{p}$, which is roughly correct.

We conclude that the kernel of the Purāṇic system is consistent with the Siddhāntas. The misunderstanding of it arose because attention was not paid to their different units of distance.

## Speed of the sun

Now that we have a Purānic context, the statement that the sun has the speed of 2,202 yojanas per half-nimeṣa can be examined.

We cannot be absolutely certain what yojanas did he have in mind: standard, or Purāṇic. But either way it is clear from the summary of Purānic cosmology that this speed could not be the speed of the sun. At the distance of 15.7 million yojanas, the sun's speed is only 121.78 yojanas $\left(y_{p}\right)$ per half-nimesa. Or if we use the the figure from VaP, it is 116.67 . Converted into the standard yojanas, this number is only $3.24 y_{s}$ per half-nimeṣa.

Sāyana's speed is about 18 times greater than the supposed speed of the sun in $y_{p}$ and $2 \times 18^{2}$ greater than the speed in $y_{s}$. So either way, a larger number with a definite relationship to the actual speed of the sun was chosen for the speed of light.

The Purānic size of the universe is 13 to 16 times greater than the orbit of the sun, not counting the actual sizes of the various heavenly bodies. Perhaps, the size was taken to be 18 times greater than the sun's orbit. It seems reasonable to assume, then, that if the radius of the universe was taken to be about 282 million yojanas, a speed was postulated for light so that it could circle the farthest path in the universe within one day. This was the physical principle at the basis of the Purānic cosmology.

We have seen that the astronomical numbers in the Purānas are much more consistent amongst themselves, and with the generally accepted sizes of the solar orbit, than has been hitherto assumed. The Purānic geography must not be taken literally.

We have also shown that the Sāyaṇa's figure of 2,202 yojanas per half-nimeṣa is consistent with Purānic cosmology where the size of "our universe" is a function of the speed of light. This size represents the space that can be spanned by light in one day.

It is quite certain that the figure for speed was obtained either by this argument or it was obtained by taking the postulated speed of the sun in the Purānas and multiplying that by 18 , or by multiplying the speed in standard yojanas by $2 \times 18^{2}$. We do know that 18 is a sacred number in the Purānas, and the fact that multiplication with this special number gave a figure that was in accord with the spanning of light in the universe in one day must have given it a special significance.

Is it possible that the number 2,202 arose because of a mistake of multiplication by 18 rather than a corresponding division (by 36) to reduce the sun speed to standard yojanas? The answer to that must be "no" because such a mistake is too egregious. Furthermore,

Sāyaṇa's own brother Mādhava was a distinguished astronomer and the incorrectness of this figure for the accepted speed of the sun would have been obvious to him.

If Sāyaṇa's figure was derived from a postulated size of the universe, how was that huge size, so central to all Indian thought, arrived at? A possible explanation is that the physical size of the universe was taken to parallel the estimates of its age. These age-estimates were made larger and larger to postulate a time when the periods of all the heavenly bodies were synchronized. The great numbers in the Purānas suggest that the concepts of mahāyuga and kalpa must have had an old pedigree and they can be viewed as generalizations of the notion of yuga.

The speed of light was taken to be $2 \times 18^{2}$ greater than the speed of the sun in standard yojanas so that light can travel the entire postulated size of the universe in one day. It is a lucky chance that the final number turned out to be exactly equal to the true speed. This speed of light must be considered the most astonishing "blind hit" in the history of science! But it is consistent with Purānic model of the cosmos and it is, in most likelihood, a pre-Āryabhaṭa figure.

## 5 India and the West

In astronomy it is possible that ideas arose independently in different parts of the world. Although we have argued for an essentially independent development of astronomy in the pre-Siddhāntic period, we are not in a position to completely discount all outside influences especially because considerable interaction existed in the ancient world and ideas must have travelled in all directions.

Nevertheless, a certain priority for the Vedic material has become apparent. Recent studies of Celtic material indicate that a calendar similar to the 5 -year yuga of VJ with two intercalary months was current amongst the Druids ${ }^{19}$. The connections between the Vedic and the Druidic material must predate the rise of the astronomy in Mesopotamia, because otherwise the more direct Mesopotamian theories would have won out against the complex Vedic system. The Druids also appear to have counted in months of 27 days similar to the conjoining of the moon with the naksatras of Vedic astronomy. This supports the idea of transmission from India into Europe.

This idea is further supported by a new analysis of a Rgvedic hymn on Vena ${ }^{20}$ which suggests that the seed ideas of the Venus mythologies of the Mesopotamians, the Greeks, and the later Purāṇic period are all present in the Vedic texts. More support for transmission of Indian ideas to the West comes from the field of art. A figure from the neolithic/chalcolithic period of Indian art ( 5000 BC ?) appears to be the prototype of the "Gilgamesh" or "hero" motif with a god or goddess holding back two beasts on either side. The beasts are without their front ends, so clearly the depiction is symbolic. ${ }^{21}$ David Napier has argued for a transmission of Indian motifs into Greece in the second millennium BC. ${ }^{22}$

As another example consider the Gundestrup cauldron, found in Denmark a hundred years ago. This silver bowl has been dated to around the middle of the 2 nd century BC. The iconography is strikingly Indic as clear from the elephant (totally out of context in Europe) with the goddess and the yogic figure. According to the art historian Timothy Taylor,

A shared pictorial and technical tradition stretched from India to Thrace, where the cauldron was made, and thence to Denmark. Yogic rituals, for example, can be inferred from the poses of an antler-bearing man on the cauldron and of an ox-headed figure on a seal impress from the Indian city of Mohenjo-Daro...Three other Indian links: ritual baths of goddesses with elephants (the Indian goddess is Lakshmi); wheel gods (the Indian is Vishnu); the goddesses with braided hair and paired birds (the Indian is Hariti). ${ }^{23}$

The plausibility of a transmission of Indian astronomy to the West is reinforced by Seidenberg's analysis that Indian geometry and mathematics predate Babylonian and Greek mathematics ${ }^{24}$. It is very likely that the same cultural processes that were responsible for the spread of Indian mathematics and art were also responsible for the spread of Indian astronomy during the pre-Siddhāntic period. That will explain how the thoroughly Indian methods of the Romaka and Pauliśa Siddhāntas could have returned to India from the West.

Abbreviations<br>AA Āryabhaṭīya of Āryabhaṭa<br>AV Atharvaveda<br>MP Matsya Purāṇa<br>PB Pañcaviṃśa Brahmaṇa<br>PS Pañcasiddhāntikā<br>RV Rgveda<br>ŚB Śatapatha Brāhmaṇa<br>SS Sūrya Siddhānta<br>VaP Vāyu Purāṇa<br>ViP Viṣṇu Purāṇa<br>VJ Vedāñga Jyotiṣa

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Figure 1. A third millennium astronomical seal.
Figure 2. The earth's asymmetric orbit shown in an ancient 2nd millennium altar.

