Fault Tolerance in Quantum Computation

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Abstract:

The practicality of fault tolerant quantum computing proposals is investigated by considering the entanglement of local and global errors.

Introduction

The theory of quantum computers assures us that quantum algorithms can solve certain problems far faster than classical computers. But many problems associated with the implementation of quantum computing have not yet been solved. We do not have hardware implementation of gates, and we do not have a practical way of achieving error correction to deal with the inevitable system faults and noise within the system.

The quantum error correction models assume that it is in principle possible to build a fault tolerant system. A device which works effectively even when its elementary components are imperfect is said to be a fault-tolerant. It is taken that under certain conditions related to the noise, it should be possible to do compute with arbitrary accuracy if the error level of the components is above a certain threshold, which is variously estimated to lie between 10^{-10} and 10^{-4}. The current inability to build quantum circuits is laid on the gap between the requirements and what is available and it is argued that efforts should be made to narrow "from both the experimental and the theoretical side" [1]. Some problems related to uncertainty are described in [2,3], whereas specific models of error-correction are in [4,5]. The problems of post-selection and local fault-tolerance are in [6,7,9]. There are arguments in [8] and the background papers [10,11] that general error-correction is not possible.

Given the conflicting assessments regarding the future of fault-tolerant quantum computing, we examine the assumptions in the quantum error model in this note. This could help determine the likely progress in the field in the near future.

Error models

Errors in a quantum circuit are likely to have several sources. Some of these are:

- Errors in the control circuitry (which is handled classically) of the quantum hardware

- Decoherence is the decay of coherences in a quantum system due to the coupling to an environment with many uncontrolled degrees of freedom

• Entangled states

• Errors in the timing

Some of these errors are local, whereas others are global. Whereas the circuit errors may be local, their implications would be global.

Decoherence is problematic not only because it limits the time in which the computation should be completed to as small as fraction of a second [5], it can entangle the qubits under processing with each other in an undesirable manner, and also with the environment.

The fact that the errors are not all local (or cannot be so modeled realistically) makes quantum error correction an especially hard problem.

In classical systems also there are certain "global" sources of noise, but they can be treated using the method of scrambling of data as is done effectively in turbo codes. Such scrambling is appropriate for communications, and clearly it cannot be used if the purpose is computation where the location of the data bits is of significance.

Decoherence is viewed as responsible for the transition from quantum mechanics to classical mechanics. Braun has recently claimed [12] that the decoherence of "superpositions of many-qubit code words in a quantum memory is governed by a generalized Hamming distance between the code words, with a time dependent metric tensor that is specific for the heat bath. The decoherence metric allows for the complete characterization of the decoherence of all possible superpositions of code-words, and for an optimization of the over-all decoherence."

Novais and Baranger examine [13] decoherence of a quantum computer in an environment which is non-Markovian and spatially correlated. Deriving the non-unitary time evolution of the system and environment in the presence of quantum error correction codes, they provide a program to quantify decoherence and show that the effects of correlation can be mitigated to some extent, *but not completely*.

Error correction of the bit-flip channel

Computational errors are internal or externally induced. Classical computers have the capacity to perform well because the non-linearly (clamping or hard-limiting) of the computation process makes it possible to eliminate small errors, subsequent to which the larger bit errors can be eliminated using error-correction coding. Small errors cannot be eliminated in quantum computing as it is a linear process, and it rules out operations analogous to clamping and hard-limiting.

Random unitary transformation errors can occur in initializing a qubit. In a quantum register the errors cannot be grouped in a systematic way, as they have various causes, not all of which manifest locally. It is the nonlocality of errors that makes their representation using higher dimensional code word space is unrealistic.

The error in the qubit could be a bit flip error or a phase shift error or a combination of both the errors. Some correction codes like three qubit bit flip code and the three bit phase flip code can be used to protect the qubits against these errors. Suppose we send qubits through a channel which leaves the qubits untouched with the probability 1-p, and flips the qubits with a probability p. that is, with probability p, the sate $|\psi>$ is taken to the state $X|\psi>$, where X is the usual Pauli X operator, or bit flip operator. This channel is called the bit flip channel. The bit flip code can be used to protect qubits against the effect of noise from this channel.

Suppose we encode the single qubit state $a|0> + b|1>$ in three qubits as $a|000> + b|111>$. A convenient way of writing this encoding is

$$|0> \rightarrow |000>$$
$$|1> \rightarrow |111>$$

It is understood that superpositions of basis states are taken to corresponding superpositions of encoding states. Each of the three qubits $a|000> + b|111>$ is passed through an independent copy of the bit flip channel. Should a bit flip occur on one or fewer of the qubits then a two way error correction code is used to recover the correct quantum states called three bit flip code method.

1.) error detection or syndrome diagnosis:

We perform a measurement which tells us what error, if any, occurred on the quantum state. The measurement is called the error syndrome. For the bit flip channel there are four syndromes, corresponding to the four projection operators.

P0= $|000><000| + |111><111|$ no error
P1= $|100><001| + |011><011|$ bit flip on qubit one
P2= $|010><001| + |101><101|$ bit flip on qubit two
P3= $|001><001| + |110><110|$ bit flip on qubit three

2.) Recovery:

Given the error from syndrome diagnosis we can determine the procedure to recover the initial state. For example, if the error syndrome was 1, indicating a bit flip on the first qubit, then we flip that qubit again, recovering the original state with perfect accuracy.

The syndrome measurement does not cause any change in the state, it contains only information about what error has occurred and does not allow to infer us anything about the value of *a* or *b*. It contains no information about the state is protected or not.

Bu the bit flip channel is an unrealistic idealization of the physical reality. In practice, we must deal with the unknown transformation

$$a|000\rangle + b|111\rangle \rightarrow U\,(a|000\rangle + b|111\rangle)$$

where U is an arbitrary unitary transformation if the noise process does not involve reduction of the wavefunction, and a non-unitary transformation if it does.

Quantum Zeno and anti-Zeno Effects

The Quantum Zeno and anti-Zeno effects may be invoked to look at errors in quantum computation [14]. Interaction with a quantum state slows down its evolution, and suppresses certain states. But it does so along a pre-defined "direction" in which the observations are being made. One would expect that the interaction of the environment with the quantum register would likewise affect the evolution of the state. But this "distorted" evolution would be impossible to correct for since the very effect of the environment is unknown and random. Without observing the system, we cannot know which direction the evolution of the wavefunction must be suppressed.

In [15] it is shown that the short time behavior of environment induced decoherence is responsible for the occurrence of either the Zeno or the anti-Zeno effect, suggesting once again how difficult it is to deal with decoherence. Slowed down, or directed, evolution cannot obviously be corrected by local gates.

Conclusions

A non-local error, if viewed as an unwanted unitary operation, would need a non-local operation to be corrected. Perhaps some of these could in principle be handled by the quantum error-correction models. But errors that cannot be viewed as unwanted unitary operations (since they involve the reduction of the state due to entanglement with the environment) can obviously not be corrected using local operators.

The problematic nature of errors in quantum computing makes clear that one cannot use the classical information theoretic approaches for its treatment. This note is mainly meant to highlight the limitations of the currently used ideas and we do not propose any alternative approach.

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