

EE 3140 SOLUTIONS FOR TEST #2

① $\iint f_{xy}(x,y) dx dy = 1$

$$\Rightarrow 1 = \int_0^2 \int_1^3 k(x+xy) dy dx = k \int_0^2 \left[xy + \frac{y^2}{2} \right]_1^3 dx = k \int_0^2 (2x+4) dx = k [x^2+4x]_0^2 = k[12]$$

$$\Rightarrow \boxed{k = 1/12}$$

② From the figures $f_x(x) = 2x$ $0 < x < 1$; $f_y(y) = 2y$ $0 < y < 1$

$$f_z(z) = \int_{-\infty}^{\infty} f_x(x) f_y(z-x) dx$$

End points: $z-x=0 \Rightarrow x=z$
 $z-x=1 \Rightarrow x=z-1$

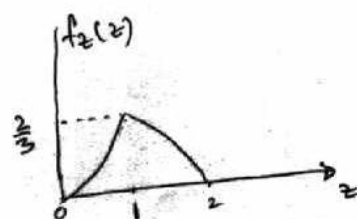
Case (i): $z < 0 \Rightarrow f_z(z) = 0$

Case (ii): $0 < z < 1 \Rightarrow f_z(z) = \int_0^z 2x \cdot 2(z-x) dx = 4 \int_0^z (zx - x^2) dx = 4 \left[\frac{zx^2}{2} - \frac{x^3}{3} \right]_0^z = 4 \left[\frac{z^3}{6} \right]$

Case (iii): $1 < z < 2 \Rightarrow f_z(z) = \int_{z-1}^1 2x \cdot 2(z-x) dx = 4 \left[\frac{zx^2}{2} - \frac{x^3}{3} \right]_{z-1}^1$

$$= 4 \left[\left(\frac{z}{2} - \frac{1}{3} \right) - \left(\frac{z(z-1)^2}{2} - \frac{(z-1)^3}{3} \right) \right]$$

Case (iv): $z > 2 \Rightarrow f_z(z) = 0$



③ Average Information: $H(X) = \sum P_i \log_2(1/P_i)$

$$H(X) = \frac{1}{4} \log_2 \frac{1}{4} + \frac{1}{4} \log_2 \frac{1}{4} + \frac{1}{4} \log_2 \frac{1}{4} + \frac{1}{8} \log_2 \frac{1}{8} + \frac{1}{16} \log_2 \frac{1}{16} + \frac{1}{16} \log_2 \frac{1}{16} = \frac{3}{4} \times 2 + \frac{1}{8} \times 3 + \frac{2}{16} \times 4 = \frac{19}{8}$$

④ $f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x,y) dy$; Here, $f_{xy}(x,y) = h$ (constant)

Since $\iint f_{xy}(x,y) dx dy = 1 \Rightarrow (2\sqrt{2})^2 h = 1 \Rightarrow h = 1/8$

$$\therefore f_x(x) = \int_{x-2}^{-x+2} \frac{1}{8} dy = \frac{1}{8} (-x+2 - x+2) = \frac{1}{4} (2-x) \quad 0 < x < 2$$

similarly $f_x(x) = \frac{1}{4} (2+x) \quad -2 < x < 0$

