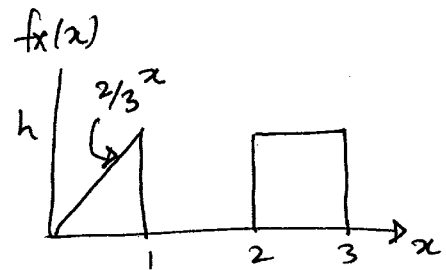


EE 3140 Test 1 Solutions



$$\textcircled{1} \quad \frac{1}{2} \cdot 1 \cdot h + 1 \cdot h = 1 \Rightarrow h = \frac{2}{3}$$

$$E(X) = \int_0^1 x \left(\frac{2}{3}x\right) dx + \int_1^3 x \left(\frac{2}{3}\right) dx$$

$$= \frac{2}{3} \cdot \frac{x^3}{3} \Big|_0^1 + \frac{2}{3} \cdot \frac{x^2}{2} \Big|_1^3 = \frac{17}{9}$$

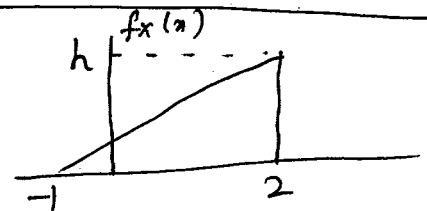
$$E(X^2) = \int_0^1 x^2 \left(\frac{2}{3}x\right) dx + \int_1^3 x^2 \left(\frac{2}{3}\right) dx = \frac{2}{3} \cdot \frac{x^4}{4} \Big|_0^1 + \frac{2}{3} \cdot \frac{x^3}{3} \Big|_1^3 = \frac{79}{18}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{79}{18} - \left(\frac{17}{9}\right)^2 = \frac{133}{162}$$

$$\textcircled{2} \quad \binom{6}{5} (0.7)^5 (0.3)^1$$

$$\textcircled{3} \quad \frac{1}{2} \cdot 3 \cdot h = 1 \Rightarrow h = \frac{2}{3} ; \text{ slope: } m = \frac{2/3}{3} = \frac{2}{9}$$

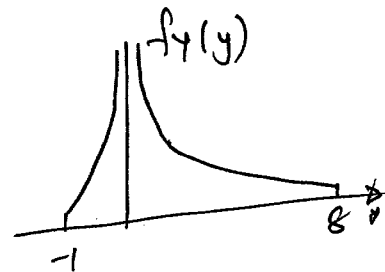
$$\therefore f_X(x) = \frac{2}{9}(x+1) \quad -1 \leq x \leq 2$$



$$y = x^3 \Rightarrow \frac{dy}{dx} = 3x^2 = 3y^{2/3}$$

$$f_Y(y) = \frac{f_X(x)}{\left| \frac{dy}{dx} \right|} = \frac{\frac{2}{9}(y^{1/3} + 1)}{|3y^{2/3}|} = \frac{2}{27} [y^{-1/3} + y^{-2/3}] ; -1 \leq y \leq 8$$

$$= 0 ; \text{ elsewhere.}$$



$$\textcircled{4} \quad P[A] = P[|X - 1/3| \leq 1] = P[-2/3 \leq X \leq 4/3] = F_X(4/3) - F_X(-2/3)$$

$$= 1 - \left[\frac{1}{3} + \frac{2}{3} \left(\frac{1}{3}\right)^2 \right] = \frac{16}{27}$$

$$\textcircled{5} \quad E(X^2) = \frac{1^2 + 1^2 + 3^2 + 0^2 + 2^2 + 6^2 + 1^2 + 4^2 + 0^2 + 2^2}{10} = \frac{72}{10}$$