

EE 3140 HOMEWORK #4 SOLUTIONS

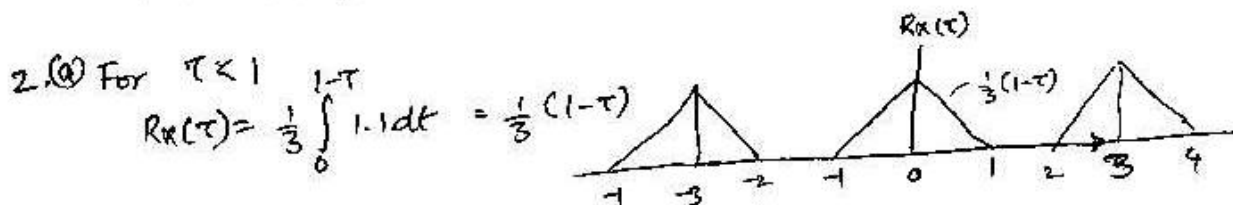
$$1.(a) R_X(0)=1; R_X(1)=0; R_X(2)=-\frac{1}{2}; R_X(3)=0; R_X(4)=\frac{1}{2}$$

$$R_X(5)=R_X(3); R_X(6)=R_X(2); R_X(7)=R_X(1)$$

$$(b) R_X(0)=1; R_X(1)=R_X(6)=-\frac{1}{7}; R_X(2)=R_X(5)=-\frac{1}{7}; R_X(3)=R_X(4)=\frac{3}{7}$$

$$(c) R_X(0)=1; R_X(1)=R_X(7)=-\frac{1}{2}; R_X(2)=R_X(6)=\frac{1}{2}; R_X(3)=R_X(5)=-\frac{1}{2}$$

$$R_X(4)=R_X(4)=\frac{1}{2}$$



(b) For $\tau < 1$

$$R_X(\tau) = \frac{1}{5} \int_0^{1-\tau} 1 \cdot 1 dt + \frac{1}{5} \int_2^{3-\tau} 2 \cdot 2 dt = \frac{1}{5} (1-\tau) + \frac{4}{5} (1-\tau) = 1-\tau$$

For $1 < \tau < 2$

$$R_X(\tau) = \frac{1}{5} \int_{2-\tau}^1 1 \cdot 2 dt = \frac{2}{5} (\tau-1)$$

For $2 < \tau < 3$

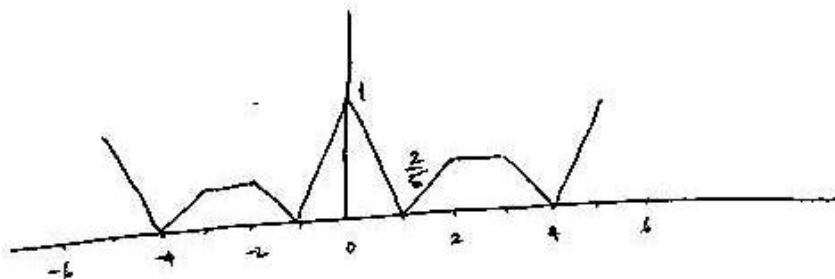
$$R_X(\tau) = \frac{1}{5} \int_0^{3-\tau} 1 \cdot 2 dt + \frac{1}{5} \int_{5-\tau}^3 2 \cdot 1 dt = \frac{2}{5}$$

For $3 < \tau < 4$

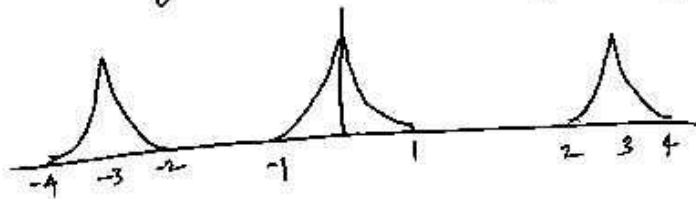
$$R_X(\tau) = \frac{1}{5} \int_2^{4-\tau} 2 \cdot 1 dt = \frac{2}{5} (4-\tau)$$

For $4 < \tau < 5$

$$R_X(\tau) = \frac{1}{5} \int_{5-\tau}^1 1 \cdot 1 dt + \frac{1}{5} \int_{7-\tau}^3 2 \cdot 2 dt = \tau-4$$



$$(c) R_X(\tau) = \frac{1}{3} \int_0^{1-\tau} t \cdot (t+\tau) dt = \frac{1}{3} \left[\frac{(1-\tau)^3}{3} + \frac{(1-\tau)^2}{2} \tau \right]$$



$$3. R_X(\tau) = E[X(t)X(t+\tau)] = \cos \omega t E[A \cdot A] = E(A^2) \cos \omega t$$

$$4. R_X(\tau) = E[A \cos(6t+\theta) \cos(3t+3\tau+\theta)]$$

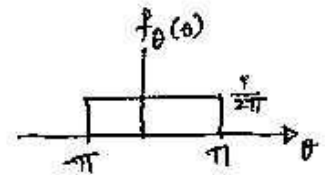
$$= \frac{A^2}{2} E[\cos(6t+3\tau+2\theta) + \cos 3\tau]$$

$$\text{Since } E[g(x)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$\text{we have, } \frac{A^2}{2} \int_{-\pi}^{\pi} [\cos(6t+3\tau+2\theta) + \cos 3\tau] \frac{1}{2\pi} d\theta$$

$$= \frac{A^2}{2\pi} \int_{-\pi}^{\pi} \cos(6t+3\tau+2\theta) d\theta + \frac{A^2}{4\pi} \int_{-\pi}^{\pi} \cos 3\tau d\theta$$

$$= 0 + \frac{A^2}{4\pi} \cos 3\tau (2\pi) = \frac{A^2}{2} \cos 3\tau$$



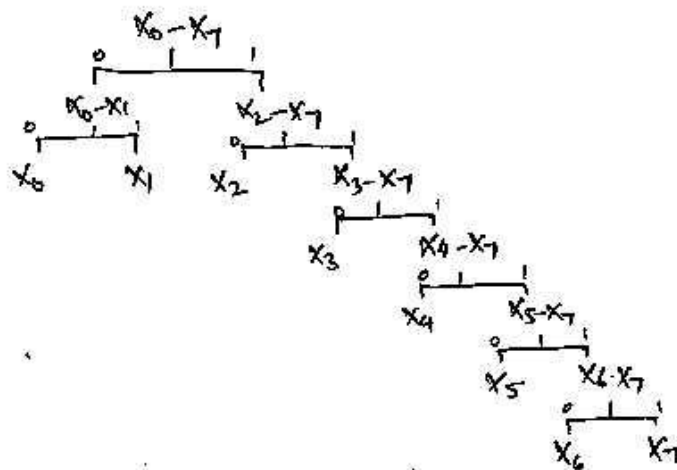
5. (a) Average length: binary: 2.12 bits
ternary: 1.4 symbols

(b) binary: 2.06 bits
ternary: 1.36 symbols

(c) binary: 2.43 bits
ternary: 1.57 symbols

6.

0.3	0.28	0.2	0.1	0.06	0.03	0.02	0.01
x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7



7. $C = 3400 \log_2 (1 + 10^{20}) = 22.638 \text{ KBPS.}$

8. $N = \frac{100 \times 10^3 \log_2 (1 + 10^{20})}{3400 \log_2 (1 + 10^{20})} = 15$

9. $S_x(f) = \mathcal{F} [A \cos 2\pi f_0 t]$
 $= \frac{A}{2} [\delta(f + f_0) + \delta(f - f_0)]$

10. $S_x(\omega) = 2 \int_0^1 A(1-\tau) e^{j\omega\tau} d\tau$

11. $R_x(0) = \frac{57}{3}$ $R_x(1) = \frac{52}{3}$ $R_x(2) = \frac{43}{3}$ $R_x(3) = \frac{37}{3}$

$$\begin{bmatrix} c(0) \\ c(1) \\ c(2) \end{bmatrix} = \begin{bmatrix} 57/3 & 52/3 & 43/3 \\ 52/3 & 43/3 & 37/3 \\ 43/3 & 37/3 & 31/3 \end{bmatrix}^{-1} \begin{bmatrix} 57/3 \\ 52/3 \\ 43/3 \end{bmatrix}$$

$$x_{n+1} = c(0)x(n) + c(1)x(n-1) + c(2)x(n-2)$$

12. # of weighing $\frac{\log_2 8}{\log_2 3} = 1.89 \therefore \underline{\underline{2 \text{ weighings}}}$