

EE 3140 Homework #1 Solutions

- ① a. Sample space $S = \{1, 2, 3, 4, 5, 6\}$
 b. $A = \{2, 4, 6\}$
 c. $A^c = \{1, 3, 5\}$ Odd dots are facing up.

② $P[\{a, c\}] = P[\{a\}] + P[\{c\}] = \frac{5}{8} \quad \text{--- ①}$

$P[\{b, c\}] = P[\{b\}] + P[\{c\}] = \frac{7}{8} \quad \text{--- ②}$

$P[\{a, b, c\}] = P[\{a\}] + P[\{b\}] + P[\{c\}] = 1 \quad \text{using axiom --- ③}$

Solving ① ② and ③

$P[\{a\}] = \frac{1}{8} \quad P[\{b\}] = \frac{3}{8} \quad P[\{c\}] = \frac{4}{8}$

③ $26 \times 25 \times 24 \times 23 \times 22 \times 21 \times 20 \times 19$

④ $\binom{52}{13} \binom{39}{13} \binom{26}{13} \binom{13}{13}$

⑤^{a.} $P(A_2) = P[\text{hhhh, thhh, hthh, htht, thth, thht, hhtt, thtt}] = \frac{8}{16}$

^{b.} $P[A_1 \cap A_3] = P[\text{hhhh, htht, htth, htth}] = \frac{4}{16}$

^{c.} $P[A_1 \cap A_2 \cap A_3 \cap A_4] = P[\text{hhhh}] = \frac{1}{16}$

^{d.} $P[A_1 \cup A_2 \cup A_3 \cup A_4] = P[S - \{\text{tttt}\}] = \frac{15}{16}$

⑥
$$\frac{\binom{30}{k} \binom{200}{20-k}}{\binom{230}{20}}$$

$$\textcircled{7} \text{ a. } \binom{5}{3} \binom{6}{18}^3 \binom{12}{18}^2$$

$$\text{c. } \frac{5!}{2!2!1!} \binom{6}{18}^2 \binom{4}{18}^2 \binom{8}{18}^1$$

(Multinomial Probability)

$$\text{b. } \frac{\binom{6}{3} \binom{12}{2}}{\binom{18}{5}}$$

$$\text{d. } \frac{\binom{6}{2} \binom{4}{2} \binom{8}{1}}{\binom{18}{5}}$$

$$\textcircled{8} P[6 \text{ overheat}, 2 \text{ leak}, 2 \text{ misc}] =$$

$$\frac{10!}{6!2!2!} (0.4)^6 (0.4)^2 (0.2)^2$$

$$\textcircled{9} 8 \times 10^6$$

$$\textcircled{10} \begin{array}{ll} 10 \text{ desks:} & 10! \\ 12 \text{ desks:} & 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \end{array}$$

$$\textcircled{11} \text{ The sum of 3 tosses of a die is 7 if the outcomes are } (1,1,5) \text{ or } (1,2,4) \text{ or } (1,3,3)$$

$$(1,1,5) \text{ can be obtained in } \frac{3!}{2!1!} = 3 \text{ ways}$$

$$(1,2,4) \text{ can be obtained in } \frac{3!}{1!1!1!} = 6 \text{ ways}$$

$$(1,3,3) \text{ can be obtained in } \frac{3!}{1!2!} = 3 \text{ ways}$$

$$\text{Total \# of ways} = 3+6+3=12$$

$$\therefore \text{Probability} = \frac{12}{6^3}$$

$$\textcircled{12} P(A \cap B|C) = P[A|B \cap C] P[B|C]$$

$$= P[A|B \cap C] P[B|C] P[C]$$

$$\textcircled{13} \text{ a. } P(H) = P(H|\text{coin 1})P(\text{coin 1}) + P(H|\text{coin 2})P(\text{coin 2})$$

$$= P_1 \times \frac{1}{2} + P_2 \times \frac{1}{2} = \frac{1}{2}(P_1 + P_2)$$

$$\text{b. } P(\text{coin 2}|H) = \frac{P[H|\text{coin 2}]P[\text{coin 2}]}{P(H)} \text{ (using Baye's Theorem)}$$

$$= \frac{\frac{1}{2} P_2}{\frac{1}{2}(P_1 + P_2)} = \frac{P_2}{P_1 + P_2}$$

$$\textcircled{14} P(M) = 0.3 ; P(W) = 0.25$$

$$P(S|M) = 0.4 ; P(S|W) = 0.7$$

M: Men ; W: Women ; S: Smoke

$$P(W|S) = \frac{P(S|W)P(W)}{P(S|W)P(W) + P(S|M)P(M)} = \frac{0.7 \times 0.25}{0.7 \times 0.25 + 0.4 \times 0.3} =$$

$$\textcircled{15} P(B_1|G) = \frac{P(G|B_1)P(B_1)}{P(G|B_1)P(B_1) + P(G|B_2)P(B_2) + P(G|B_3)P(B_3)}$$

$$= \frac{1 \times \frac{1}{3}}{1 \times \frac{1}{3} + 0.1 \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3}} = \frac{2}{3}$$

$$\textcircled{16} P(A \cup B) = P(A) + P(B) - P(A)P(B) \quad \text{independent}$$

$$P(A \cup B) = P(A) + P(B) \quad \text{mutually exclusive}$$

$$\textcircled{17} P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A)P(B) - P(A)P(C) - P(B)P(C) + P(A)P(B)P(C)$$

$$= .2 + .1 + .4 - .2 \times .1 - .2 \times .4 - .1 \times .4 + .2 \times .1 \times .4 = .568$$

$$\textcircled{18} \text{ Let } K \text{ be the \# of defective items}$$

$$P(K > 1) = 1 - P(K \leq 1) = 1 - P(K=1) - P(K=0)$$

$$= 1 - \binom{n}{1} p^1 (1-p)^{n-1} - \binom{n}{0} p^0 (1-p)^n \quad \text{where } p=0.1$$

(19)

$$\binom{3}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 + \binom{3}{0} \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^3 + \binom{3}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 + \binom{3}{1} \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^2 + \dots + \binom{3}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^0$$

$\downarrow$ 1st person gets 0 Heads $\downarrow$ 2nd person gets 0 Heads $\downarrow$ 1st person gets 1 Head $\downarrow$ 2nd person gets 1 Head

$$= \sum_{i=0}^3 \left[\binom{3}{i} \left(\frac{1}{2}\right)^3 \right]^2$$

(20) (a) $P[\text{Ace in 1st draw}] = \frac{4}{52}$

(b) If the 1st draw is observed as an ace then

$$P[\text{Ace in 2nd draw} | \text{Ace in 1st draw}] = \frac{3}{51}$$

If the 1st draw is observed as not an ace then

$$P[\text{Ace in 2nd draw} | \text{No Ace in 1st draw}] = \frac{4}{51}$$

} 1st draw observed.

If the 1st draw is not observed then

$$P[\text{Ace in 2nd draw}] = P[\text{Ace in 2nd draw} | \text{Ace in 1st draw}] P[\text{Ace in 1st draw}] + P[\text{Ace in 2nd draw} | \text{No Ace in 1st draw}] P[\text{No Ace in 1st draw}]$$

$$= \frac{4}{52} \times \frac{3}{51} + \frac{4}{51} \times \frac{48}{52} = \frac{4}{52}$$

(c) $P[3 \text{ aces in 7 cards}] = \frac{\binom{4}{3} \binom{48}{4}}{\binom{52}{7}}$

$$P[2 \text{ kings in 7 cards}] = \frac{\binom{4}{2} \binom{48}{5}}{\binom{52}{7}}$$

$$P[3 \text{ aces and/or 2 kings}] = P[3 \text{ aces}] + P[2 \text{ kings}] - P[3 \text{ aces and 2 kings}]$$

$$= \frac{\binom{4}{3} \binom{48}{4}}{\binom{52}{7}} + \frac{\binom{4}{2} \binom{48}{5}}{\binom{52}{7}} - \frac{\binom{4}{3} \binom{4}{2} \binom{44}{2}}{\binom{52}{7}}$$

$$\begin{array}{c}
 \textcircled{d} \quad \binom{4}{1} \binom{48}{12} \quad \binom{3}{1} \binom{36}{12} \quad \binom{2}{1} \binom{24}{12} \quad \binom{1}{1} \binom{12}{12} \\
 \hline
 \binom{52}{13} \quad \binom{48}{13} \quad \binom{36}{13} \quad \binom{13}{13}
 \end{array}$$