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The object plane and receiver pupil planes in the simulation consisted of  $N = 2048 \times 2048$  computational grids with identical  $182\mu\text{m}$  sample spacings in both planes. The optical wavelength,  $\lambda$ , is  $1.55\mu\text{m}$ , and the range,  $L$ , from the receive pupil plane to the object is 100 meters. The numerical propagation consisted of 10 partial propagations of 10 meters each to avoid the wraparound effects described above.

The optical field in the receiver pupil plane was measured in three sub-apertures with 48mm diameters and 70mm center-to-center spacings. In Fig. 5, we display the focused image after 60 realizations in the three-aperture configuration. This is the best possible realization that can be achieved when there was no aberrations.

To simulate inter-aperture aberrations, we added random piston/tip/tilt and rotation errors to each sub-aperture. Figures 6(a), 6(b), and 6(c) show the average realizations in each sub-aperture. The first step of the algorithm is the intra-aperture correction. This is achieved as given in (3). The Zernike polynomials used are  $Z_2^{-2}, Z_2^0, Z_2^2, Z_4^2$ . The optimization is done using the *fminunc* function of MATLAB, minimizing the well-known power measure  $\sum_{u,v} |b(u,v)|^{0.5}$  [10]. The output of this step is given in Fig. 6(d), 6(e), and 6(f). As shown in Fig. 6(g), the composite at this point suffers from piston/tip/tilt and rotation errors. Next, we do the inter-aperture corrections as described in Algorithm 1. The same optimization technique and the sharpness measure mentioned above is used. Figure 6(h) shows the result with piston/tip/tilt correction only. Figure 6(i) shows the result when both piston/tip/tilt and rotation/shift corrections are done. Figure 7 shows selected zoomed regions to highlight the effectiveness of the algorithm. (Compare the recovered result in Fig. 7(d) with the best possible result given in Fig. 5.)

#### 4. Conclusions

In this paper, we present a method to correct for rotational and translational errors in addition to piston/tip/tilt errors in multi-aperture coherent imaging. For rotational correction, the pupil field is transformed from Cartesian to polar coordinates, where a rotation becomes a circular shift, which is then fixed in Fourier domain by a linear phase correction. For translational errors, the correction is done on the phase of the image plane field. As the piston/tip/tilt, rotational, and translational corrections are done on different domains, a sequential algorithm is adopted, where one domain correction is done at a time. The corrections can be done iteratively; in our experiments, two iterations (first iteration with two sub-apertures as inputs, second iteration with all sub-apertures as inputs) resulted in satisfactory satisfactory results. It is also possible to apply the idea to dynamic scenes, where frame-by-frame error correction has to be done. Finally, a straightforward extension of the method can be done address pupil magnification errors as well: Instead of transforming the pupil data to polar coordinates from Cartesian coordinates, we may transform it to log-polar coordinates, where a magnification in pupil data corresponds to a shift in the log axis. Such a shift would become a phase shift when Fourier transform is taken, and can be fixed through linear phase correction.