LSU EE 4720 **Homework 5** Solution **Due: 24 April 2009**

Problem 1: Solve Fall 2008 Final Exam Problem 3.

See exam solution,

Problem 2: Continue to consider the systems and code from Problem 3.

(a) What is the warmup time of the local predictor on branch B2?

It will take 10 B2 executions to bring the entire local history into the BHT. There are 7 distinct outcome patterns, each will take at most two outcomes to warm up. $\boxed{\text{The total warmup time is } 10 + 2 \times 7 = 24.}$

(b) What is the warmup time of the global predictor on branch B2?

When predicting B2 the contents of the GHR will look something like XXtXXXnXXX, where n and t are possible B2 outcomes and X are B1 outcomes. Only two B2 outcomes can fit. It will take two executions of B2 to bring the 2 B2 outcomes into the GHR. There are three local history patterns (TT never occurs), it will take 6 executions to warm them up (though NN won't be followed by accurate predictions). The total $\boxed{\text{warmup time is then approximately } 10 + 2 \times 6}$.

The table below shows the local outcome patterns sorted and broken at two outcomes to emphasize predictability. NT and TN are predictable since they are consistently followed by N, while NN it followed by N or T in equal proportions.

```
12 3456 7
NN NNTN NT
NN NTNN TN
NN TNNN NT
NN TNNT NN
NT NNNN TN
NT NNTN NN
TN NNNT NN
TN NTNN NN
```

Problem 3: Continuing still with Problem 3, suppose the number of iterations of the B1 loop could be 1, 2, or 3, the probability of each number of iterations is $\frac{1}{3}$ and the number of iterations is independent of everything. The patterns of B1 for an iteration of BIGLOOP can thus be N or T N or T T N.

(a) What is the accuracy of the bimodal predictor on B1. An exact solution is preferred but an approximate solution is acceptable. *Hint: Model the effect of the change of one* BIGLOOP *iteration on the counter using a Markov chain, something you may have learned about in other courses.*

Let p_i denote the probability that the BHT entry for B1 is i at the top of BIGLOOP.

Consider the change in the counter (the BHT entry) between BIGLOOP and B2. If B1 executes a 1-iteration loop, N, the counter will be decremented by 1, a 2-iteration loop will leave it unchanged (because the counter can never be 3), and a 3-iteration loop, TTN, will change it from 0 to 1, or from 1 to 2, but leave it unchanged at 2.

Based on these counter changes probability of a counter transition from 0 to 1, 1 to 0, 1 to 2, and 2 to 1 are all $\frac{1}{3}$. The rate of transitions from 0 to 1 is $\frac{1}{3}p_0$ and the rate of transitions from 1 to 0 is $\frac{1}{3}p_1$. The two must balance and so $\frac{1}{3}p_0 = \frac{1}{3}p_1$ and therefore $p_0 = p_1$. Similarly, $p_1 = p_2$. Since $p_0 + p_1 + p_2 = 1$, $p_0 = p_1 = p_2 = \frac{1}{3}$.

When the counter is zero the number of correct predictions for the 1-iteration loop is 1, for the 2-iteration loop there is 1 correct prediction, and zero correct predictions for the 3-iteration loop. The total accuracy for this case is $\frac{2}{6}$.

Similarly, when the counter is 1 the numbers of correct predictions are 1, 0, and 1, and when the counter is 2 the numbers of correct predictions are 0, 1, and 2. Since the probability of each counter value is identical, the overall $\boxed{\text{prediction accuracy is } \frac{2+2+3}{18} = \frac{7}{18}}$.

(b) How will B1's behavior impact the accuracy of the local predictor on branch B2? Show an example of execution that would result in a B2 misprediction and compute the probability of that particular execution.

Some of B1's local history patterns will match those of B2, however the subsequent outcomes may not match and so B1 will pollute B2's PHT entries.

Consider, for example, B2 pattern **nnntnntnnn**. That could be reproduced with an execution in which B1 patterns are **n n n tn n tn n n**. B2's next outcome would be a **t** but B1 in this case might have an **n**.

This particular sequence includes 9 B1 loops, and there is only one way for B1 to do this. The probability is $\left(\frac{1}{3}\right)^9$, which is pretty unlikely and is not enough to cause a misprediction of the PHT entry is already warmed up. See the next problem.

(c) Optional: Find the exact prediction accuracy of B2 on the local predictor with B1's new behavior. This may be very difficult so don't spend too much time on it.

For B1 to induce a misprediction in B2 it must mimic a B2 pattern twice before between encounters of the real B2 pattern. It is possible for B1 to produce two patterns between one of B2's because it can produce two outcomes (the ttn pattern isn't useful) in a BIGLOOP iteration where B1 always just inserts 1. The remainder of the solution is left as an exercise to the reader.