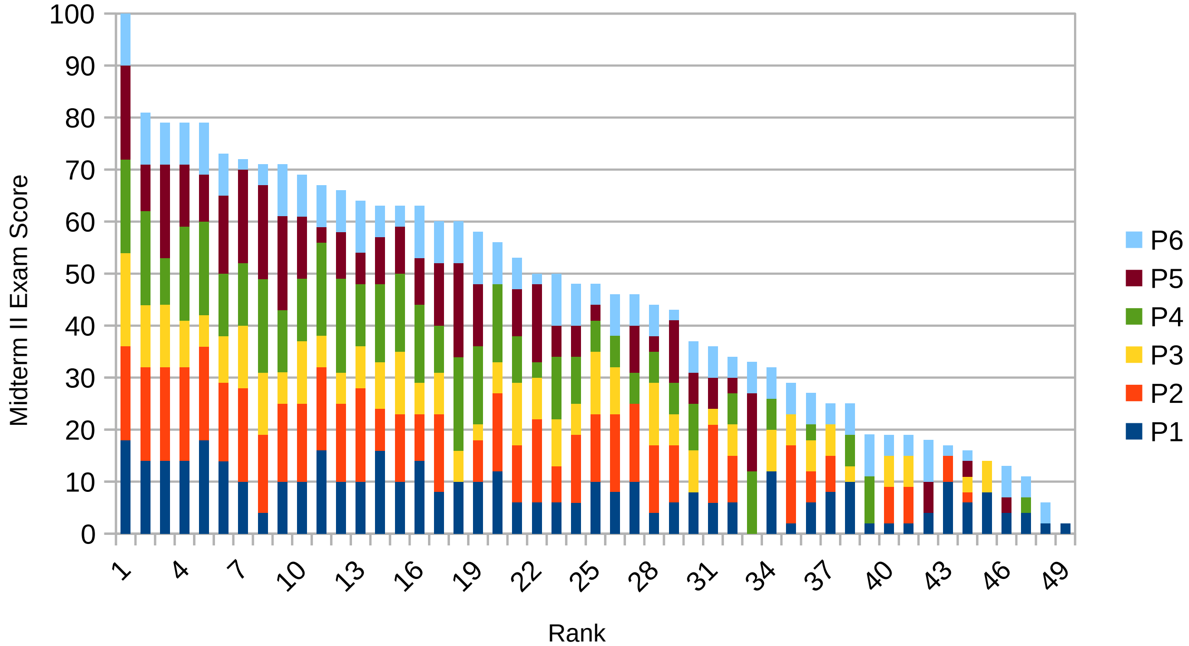
Name **Preliminary** Solution

Circuits I

LSU EE 2120

Midterm Examination II

Friday, 8 April 2022, 8:30-9:20 CDT

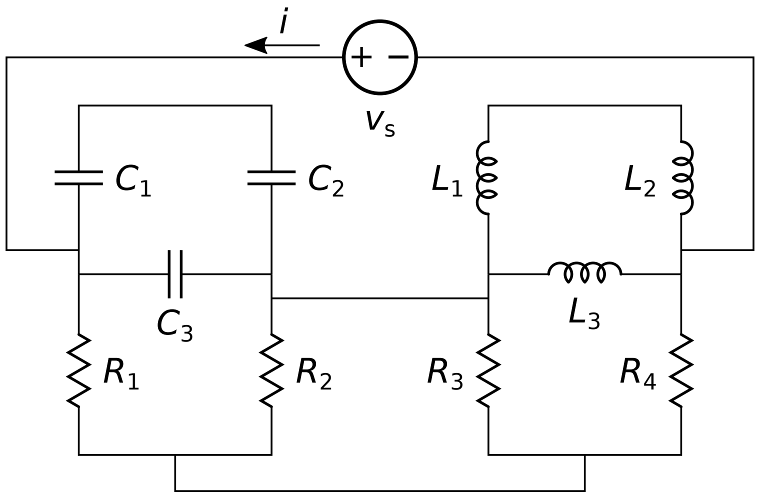


Problem 1	__________	(18 pts)
Problem 2	__________	(18 pts)
Problem 3	__________	(18 pts)
Problem 4	__________	(18 pts)
Problem 5	__________	(18 pts)
Problem 6	__________	(10 pts)
Exam Total	__________	(100 pts)

Alias Complimentary Solution

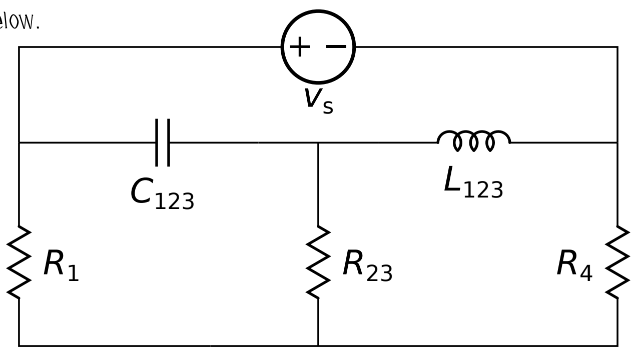
Good Luck!

Problem 1: [18 pts] Simplify the circuit below by replacing each group of parallel components by a single equivalent component and each group of series components by an equivalent component. Repeat until no further simplification is possible. In your simplified circuit show the value (resistance, capacitance, or inductance) of each equivalent component in terms of the given resistances, capacitances, and inductances.



☑ Show simplified circuit:

The simplified circuit appears below.



Grading Notes: Many students had trouble with two important concepts: identifying when components are in series or in parallel, and replacing a set of parallel or series components with a single component such that the transformed circuit is equivalent to the original. These are not difficult concepts and are also fundamental and so those who don't grasp them will be at a huge disadvantage.

☑ In terms of $R_1, \ldots, C_1, \ldots, L_1, \ldots$ show resistances, capacitances, and inductances of the components in the simplified circuit.

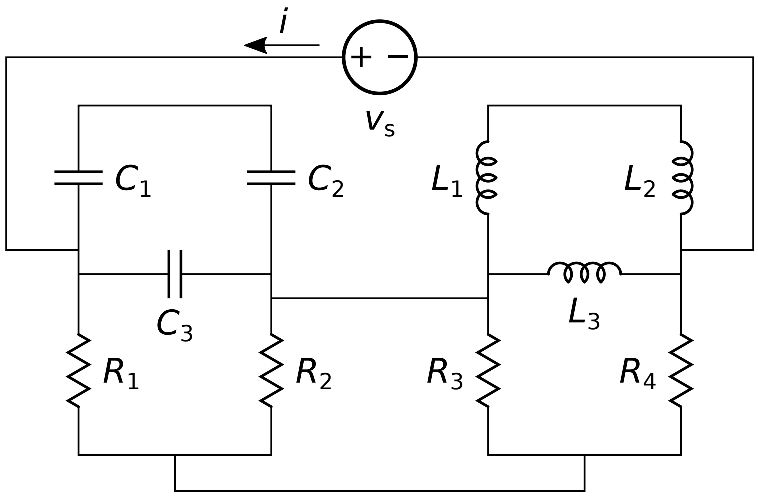
The equivalent values are:

$$C_{123} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} + C_3$$

$$L_{123} = \frac{1}{\frac{1}{L_1+L_2} + \frac{1}{L_3}}$$

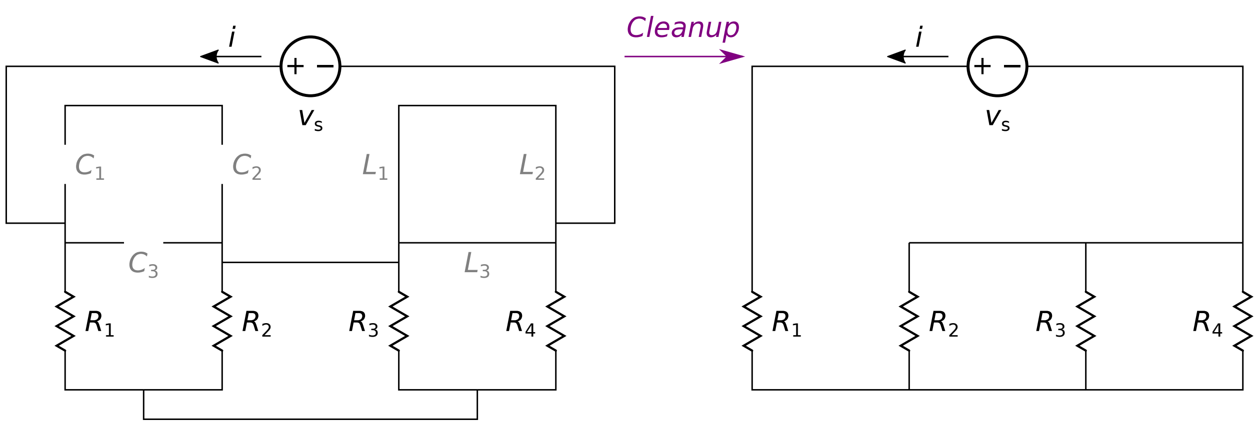
$$R_{23} = \frac{1}{\frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_2 R_3}{R_2 + R_3}$$

Problem 2: [18 pts] Show the DC steady-state equivalent of the circuit below. Show the value of i in terms of the sources and components in the DC steady-state circuit.



☑ Show DC steady-state equivalent:

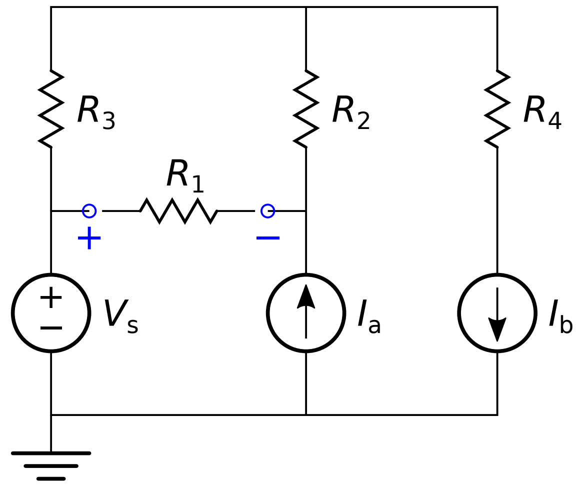
The solution appears below. Both circuits below are equivalent. The one on the left is drawn to clearly show that capacitors are replaced by opens and inductors are replaced by shorts. In the circuit on the right dead-end wires are removed and wires are straitened.



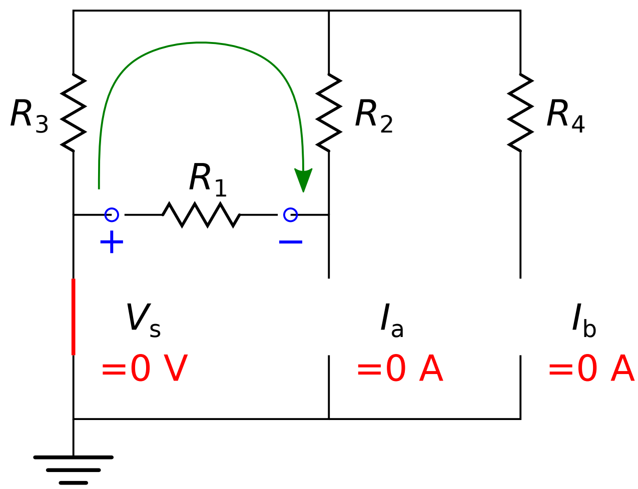
☑ In terms of $V_s, R_1, R_2, \ldots$ show i.

It should be obvious from the re-drawn equivalent circuit that R_2, R_3, and R_4 are in parallel with each other. The current is: $i = \frac{V_s}{R_1 + \frac{1}{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}}}$. Hmm, that's unreadable. Let $R_{\mathrm{eq}} = R_1 + (1/R_2 + 1/R_3 + 1/R_4)^{-1}$. Then $i = V_s/R_{\mathrm{eq}}$.

Problem 3: [18 pts] Show Thévenin and Norton equivalents of the circuit to the right at the indicated port. Resistor R_1 is not part of the equivalent circuits. Show values for the resistance, voltage source, and current source in the equivalent circuit in terms of the components below.

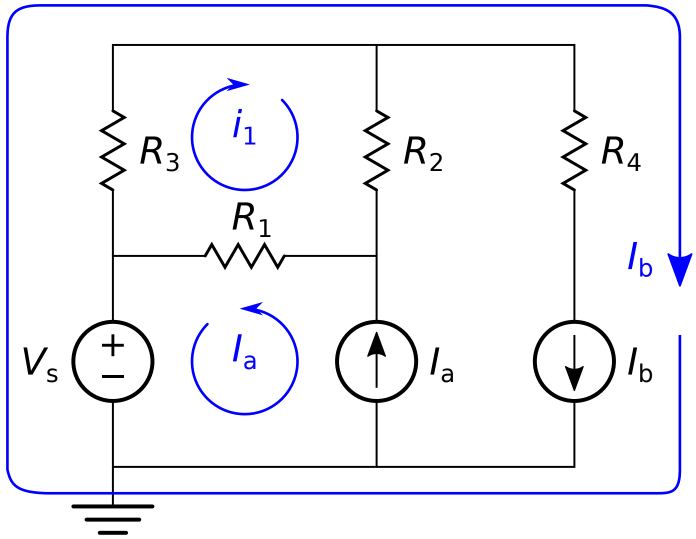


- ☑ Draw a schematic of the Thévenin equivalent circuit.
- ☑ Draw a schematic of the Norton equivalent circuit.
- ☑ In terms of V_s, I_a, I_b, and the resistances show V_{th}.
- ☑ In terms of V_s, I_a, I_b, and the resistances show I_{no}.
- ☑ In terms of V_s, I_a, I_b, and the resistances show R_{th}.



After setting sources to zero the equivalent resistance is easily read as $R_{\text{eq}} = R_3 + R_2$ see the illustration above. That equivalent resistance is both the Thévenin and Norton resistances: $\boxed{R_{\text{th}} = R_{\text{no}} = R_{\text{eq}} = R_3 + R_2}$. As everyone in this course should be able to tell at a glance, the current through R_2 is I_a. With only a bit more thought one can see that the current through R_3 is $I_b - I_a$. The Thévenin voltage is the sum of the drops from the $-$ to the $+$ terminals: $\boxed{V_{\text{th}} = -I_a R_2 - (I_b - I_a) R_3}$. With those two found we can easily compute $\boxed{I_{\text{no}} = V_{\text{th}}/R_{\text{th}}}$.

Problem 4: [18 pts] Identify mesh currents and using those write a KVL mesh equation for the familiar circuit on the right. This is similar to Homework 5 Problem 2 in that only one mesh will carry an unknown current, and that the other two meshes must be chosen to respect that I_a and I_b are current sources.



- ☑ Show a symbol and loopy arrow (↻ or ↺) for each mesh current. ☑ Only one of these should be unknown.
- ☑ Write a loop equation for the unknown current.
- ☑ Put the loop equation in normal form (known currents and voltages on the right).

The loops are shown above. The one unknown loop is i_1, the other two, I_a and I_b, are based on the current sources. To keep the solution simple it is important that each current source is next to one loop, which they are. (In contrast two loops pass through resistor R_1, the loops are i_1 and I_a.)

The loop equation for i_1 is:

$$(i_1 + I_b)R_3 + i_1 R_2 + (i_1 + I_a)R_1 = 0$$

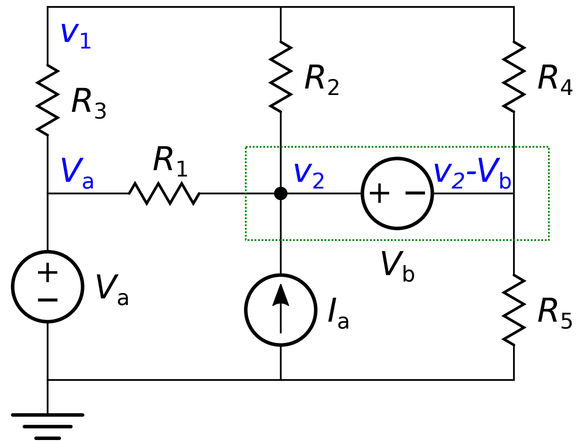
.

In normal form:

$$(R_3 + R_2 + R_1)i_1 = -I_b R_3 - I_a R_1$$

Problem 5: [18 pts] Label the nodes on the circuit to the right with voltages (such as $V_1, V_2, \ldots$) and write a set of KCL node equations for the circuit. There should be two equations, one of which is for a supernode. Do not write more equations than are necessary.

- ☑ Label nodes with voltages.
- ☑ Write a set of equations, no more than are necessary. **Normal form is not required.**
- ☑ There should be two equations. The only voltages they can refer to are two unknown voltages (using your voltage labels) and the two known voltages.



Node labels are shown in blue and the supernode is outlined in green. Voltage V_a is known. It is shown as a node label (in blue) because it indicates a voltage with respect to ground. (That is in contrast to the black V_a label which shows a voltage drop.) Three other nodes were labeled, v_1, v_2, and with an expression $v_2 - V_b$. The latter two nodes form a supernode because if the value of one of them, say v_2, is determined the other can be calculated using the voltage drop v_b.

Based on these labels, the node equations are:

$$\frac{v_1 - V_a}{R_3} + \frac{v_1 - v_2}{R_2} + \frac{v_1 - (v_2 - V_b)}{R_4} = 0 \qquad \text{Node } v_1$$

$$\frac{v_2 - V_a}{R_1} + \frac{v_2 - v_1}{R_2} - I_a + \frac{(v_2 - V_b) - v_1}{R_4} + \frac{v_2 - V_b}{R_5} = 0 \qquad \text{The supernode}$$

Problem 6: [10 pts] Answer the following questions.

(a) A parallel plate capacitor has a capacitance of $2.12\,\mu$F and the distance between the plates is 3 mm. Suppose that the distance between the plates can be changed without effecting the permittivity of the dielectric between them. Assuming the ☐ is large, at what distance will the capacitance be $4.72\,\mu$F?

☑ To achieve $4.72\,\mu$F distance between plates must be:

Consulting our memory or formula sheet we get $C = \epsilon A/d$. We have two capacitors, call them C_1 and C_2. For C_1 we have $C_1 = 2.12\,\mu\text{F} = \frac{\epsilon A}{3\,\text{mm}}$ and for C_2 we have $C_2 = 4.72\,\mu\text{F} = \frac{\epsilon A}{d_2}$. Neither the dielectric permittivity nor the area are known. But that's not a problem because we do know that the permittivity and area in the two capacitors are the same, and so we can solve for ϵA in each, equate the two results, and solve for d_1.

$$C_1 = 2.12\,\mu\text{F} = \frac{\epsilon A}{3\,\text{mm}} \qquad \Rightarrow \qquad \epsilon A = 2.12\,\mu\text{F} \times 3\,\text{mm}$$
$$C_2 = 4.72\,\mu\text{F} = \frac{\epsilon A}{d_2} \qquad \Rightarrow \qquad \epsilon A = 4.72\,\mu\text{F} \times d_2$$
$$2.12\,\mu\text{F} \times 3\,\text{mm} = 4.72\,\mu\text{F} \times d_2$$
$$d_2 = \frac{2.12\,\mu\text{F}}{4.72\,\mu\text{F}} 3\,\text{mm} = 1.347\,\text{mm}$$

.

☑ What needs to be assumed large for this answer to be accurate?:

The area. That's because $C = \epsilon A/d$ is accurate when A is large compared to d. Of course, the exact definition of *accurate* and *large* depends on the context.

(b) Identify the following ideal components: *Note: The word ideal was not included in the original exam.*

☑ The ideal component that stores energy in a magnetic field is:

An inductor.

☑ The ideal component that stores energy in an electric field is:

A capacitor.

☑ The ideal component that dissipates energy as heat is:

An ideal resistor. Also, any non-ideal component except maybe a superconductor. In fact, every non-ideal component, and also wire, have some resistance, capacitance, and inductance.