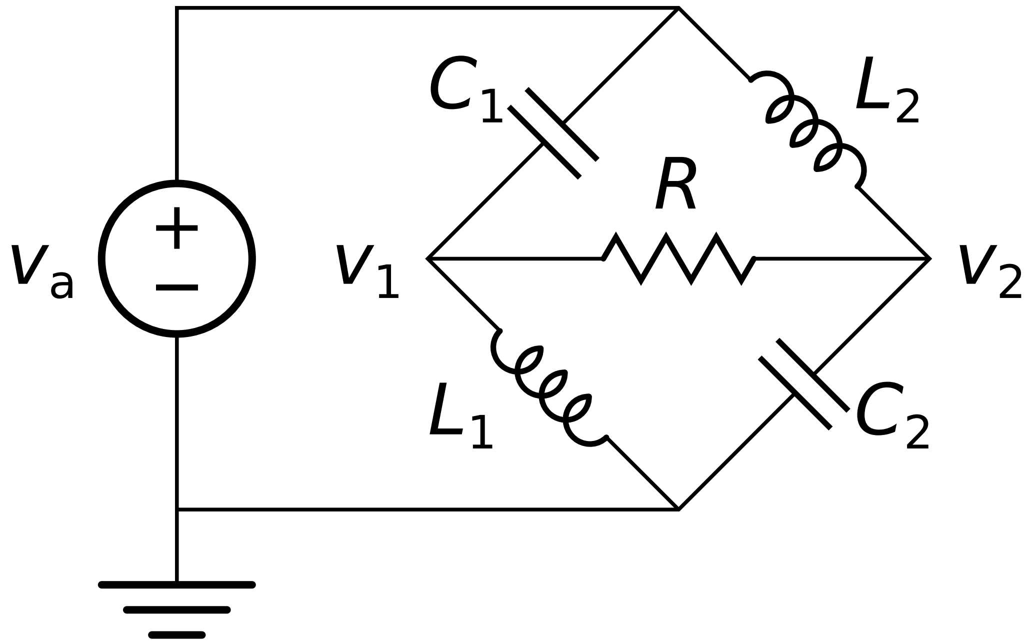
Contents

Classical RLLCC Node Analysis Example



First write KCL node equations:

$$C_1p(v_1 - v_a) + \frac{1}{R}(v_1 - v_2) + \frac{1}{L_1p}v_1 = 0 \qquad \text{Node } v_1$$

$$\frac{1}{L_2p}(v_2 - v_a) + \frac{1}{R}(v_2 - v_1) + C_2pv_2 = 0 \qquad \text{Node } v_2$$

For put them in normal form:

$$\left(C_1p + \frac{1}{R} + \frac{1}{L_1p}\right)v_1 + \left(-\frac{1}{R}\right)v_2 = C_1pv_a \qquad \text{Node } v_1$$

$$\left(-\frac{1}{R}\right)v_1 + \left(C_2p + \frac{1}{R} + \frac{1}{L_2p}\right)v_2 = \frac{1}{L_2p}v_a \qquad \text{Node } v_2$$

Solving by Hand:

Raw Node v_1 : $C_1p(v_1 - v_a) + \frac{1}{R}(v_1 - v_2) + \frac{1}{L_1p}v_1 = 0$ Node v_2 : $\frac{1}{L_2p}(v_2 - v_a) + \frac{1}{R}(v_2 - v_1) + C_2pv_2 = 0$

Solving for v_2 in the v_1 equation yields: $v_2 = RC_1pv_1 - RC_1pv_a + v_1 + \frac{R}{L_1p}v_1$.

Substituting this v_2 into the v_2 equation:

$$(RC_1C_2)p^2v_1 + (C_1 + C_2)pv_1 + \left(\frac{RC_1}{L_2} + \frac{RC_2}{L_1}\right)v_1 + \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\frac{1}{p}v_1 + \frac{R}{L_1L_2}\frac{1}{p^2}v_1 = \left(RC_1C_2p^2 + C_1p + \frac{RC_1}{L_2} + \frac{1}{L_2p}\right)v_a$$

To make things easier $C_1 \to C_2 \to C$ and $L_1 \to L_2 \to L$:

$$RC^2p^2v_1 + 2Cpv_1 + \frac{2RC}{L}v_1 + \frac{2}{Lp}v_1 + \frac{R}{L^2p^2}v_1 = \left(RC^2p^2 + Cp + \frac{RC}{L} + \frac{1}{Lp}\right)v_a$$

Next, put in normal form. (Here multiply by $\frac{p^2}{RC^2}$.):

$$\left(p^4 + \frac{2}{RC}p^3 + \frac{2}{LC}p^2 + \frac{2}{RLC^2}p + \frac{1}{(LC)^2}\right) v_1 = \left(p^4 + \frac{1}{RC}p^3 + \frac{1}{LC}p^2 + \frac{1}{RLC^2}p\right) v_a$$

The characteristic equation:

$$p^4 + \frac{2}{RC}p^3 + \frac{2}{LC}p^2 + \frac{2}{RLC^2}p + \frac{1}{(LC)^2} = 0$$

And its solution:

$$\left(p + \sqrt{-\frac{1}{LC}}\right)\left(p - \sqrt{-\frac{1}{LC}}\right)\left(p + \frac{1}{RC} + \sqrt{\frac{1}{(RC)^2} - \frac{1}{LC}}\right)\left(p + \frac{1}{RC} - \sqrt{\frac{1}{(RC)^2} - \frac{1}{LC}}\right) = 0$$

The complementary (transient) solution (assuming all four roots are distinct):

$$v_1(t) = K_1 e^{a_1 t} + K_2 e^{a_2 t} + K_3 e^{a_3 t} + K_4 e^{a_4 t}$$

$$= K_1 e^{-jt\frac{1}{\sqrt{LC}}} + K_2 e^{jt\frac{1}{\sqrt{LC}}} + K_3\, e^{-t\frac{1}{RC}}\; e^{t\left((RC)^{-2}-(LC)^{-1}\right)^{1/2}} + K_4\, e^{-t\frac{1}{RC}}\; e^{-t\left((RC)^{-2}-(LC)^{-1}\right)^{1/2}}$$

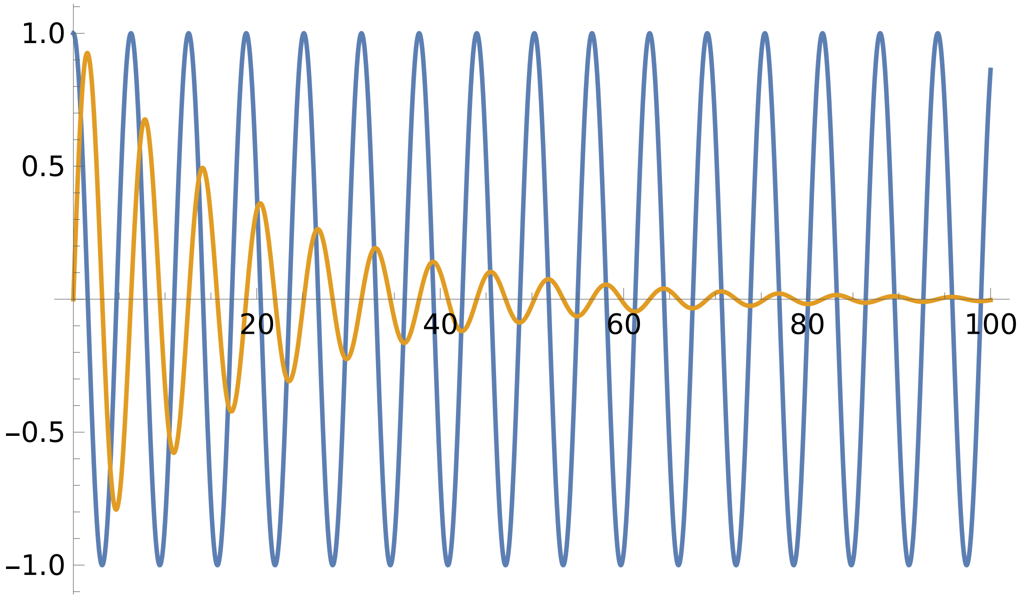
assuming that $K_1 = K_2$ and K_1 is real

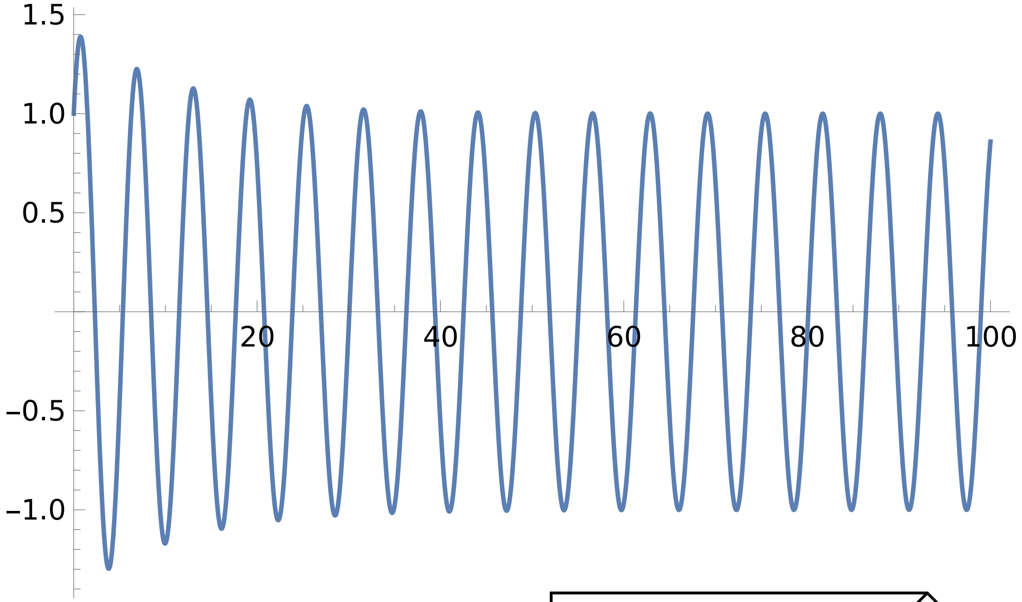
$$= K_1 \cos\left(t\frac{1}{\sqrt{LC}}\right) + K_3\, e^{-t\frac{1}{RC}}\; e^{t\left((RC)^{-2}-(LC)^{-1}\right)^{1/2}} + K_4\, e^{-t\frac{1}{RC}}\; e^{-t\left((RC)^{-2}-(LC)^{-1}\right)^{1/2}}$$

also assuming $K_3 = K_4$, K_4 is real, and $(RC)^{-2} < (LC)^{-1}$

$$= K_1 \cos\left(t\frac{1}{\sqrt{LC}}\right) + K_3\, e^{-t\frac{1}{RC}}\; \cos\left(t\sqrt{(RC)^{-2} - (LC)^{-1}}\right)$$

Plot of possible solutions.

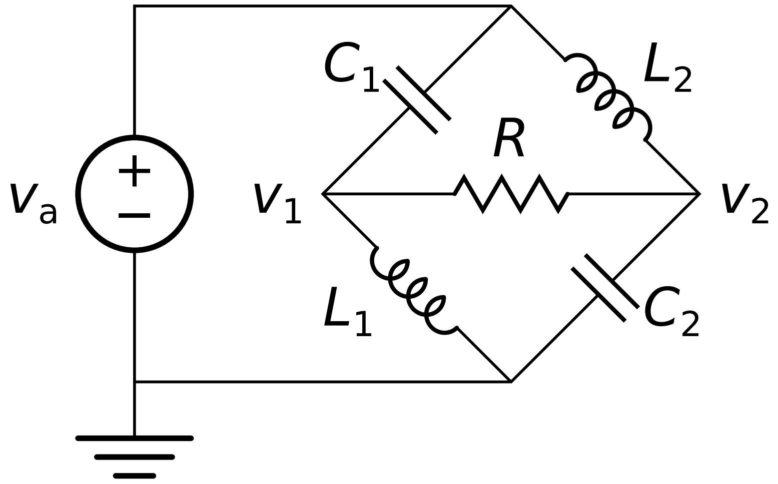




Still need to determine constants (the $K_1, \ldots$).

But not for this example . . .
. . . because it won't be necessary with Laplace transforms.

See the Laplace RLLCC section for an analysis with initial conditions.



Laplace RLC Circuit Node Analysis Example

Notes

In the time domain (the normal way) ...
... the node voltage function is $v_1(t)$ ...
... though for brevity it is usually written v_1 (the (t) is dropped).

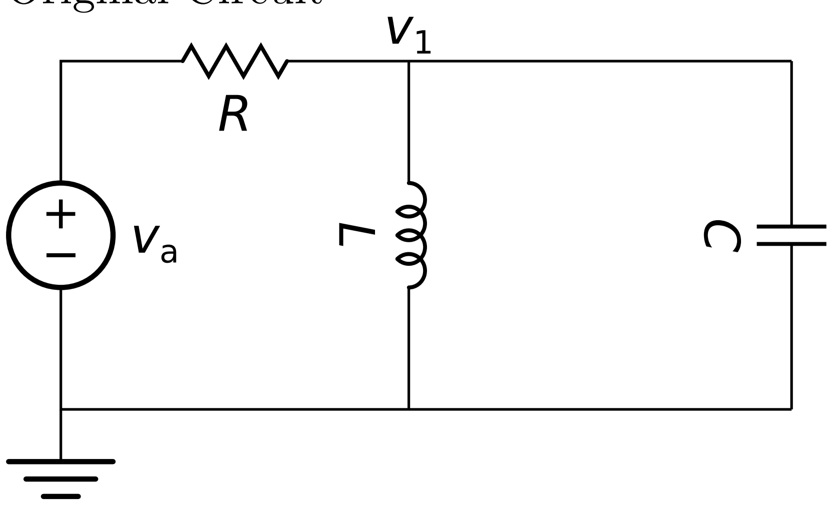
In the $\mathbf{s}$ (complex frequency) domain (the Laplace way) ...
... the node voltage function is $\mathbf{V}_1(\mathbf{s})$ ...
... though for brevity it is usually written $\mathbf{V}_1$ (the $(\mathbf{s})$ is dropped).

The initial capacitor voltage is $v_C(0^-)$ ...
... and the initial inductors current is $i_L(0^-)$.

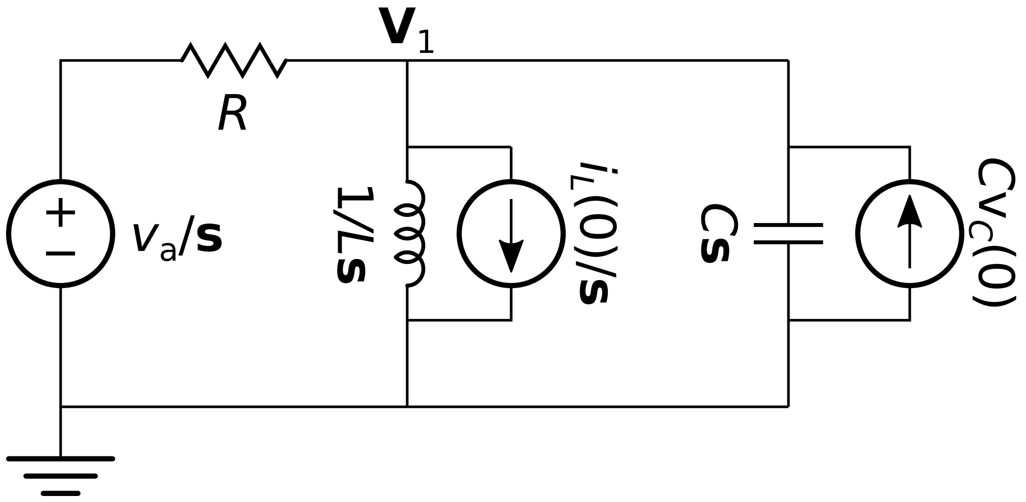
Since capacitor voltage & inductor current can't change instantly...

... $v_C(0^-) = v_C(0^+)$ and $i_L(0^-) = i_L(0^+)$ ...
... for brevity they will be shown as $v_C(0)$ and $i_L(0)$.

Original Circuit



Node Analysis Transform-Domain Circuit



Node Equation:

$$\frac{1}{R}(\mathbf{V}_1 - \frac{v_a}{\mathbf{s}}) + \frac{1}{L\mathbf{s}}\mathbf{V}_1 + \frac{i_L(0)}{\mathbf{s}} + C\mathbf{s}\mathbf{V}_1 - Cv_C(0) = 0$$

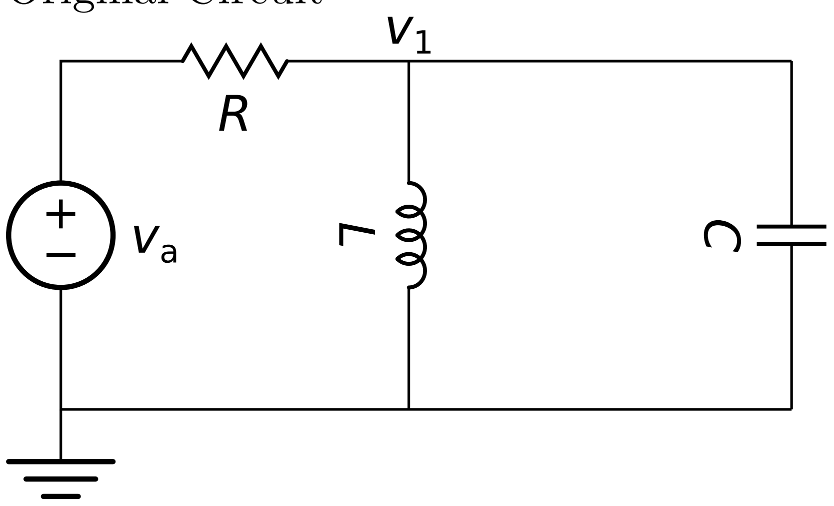
Move knowns to right-hand side, collect $\mathbf{V}_1$ coefficients on left:

$$\left(C\mathbf{s} + \frac{1}{R} + \frac{1}{L\mathbf{s}}\right)\mathbf{V}_1 = \frac{1}{R}\frac{v_a}{\mathbf{s}} - \frac{i_L(0)}{\mathbf{s}} + Cv_C(0)$$

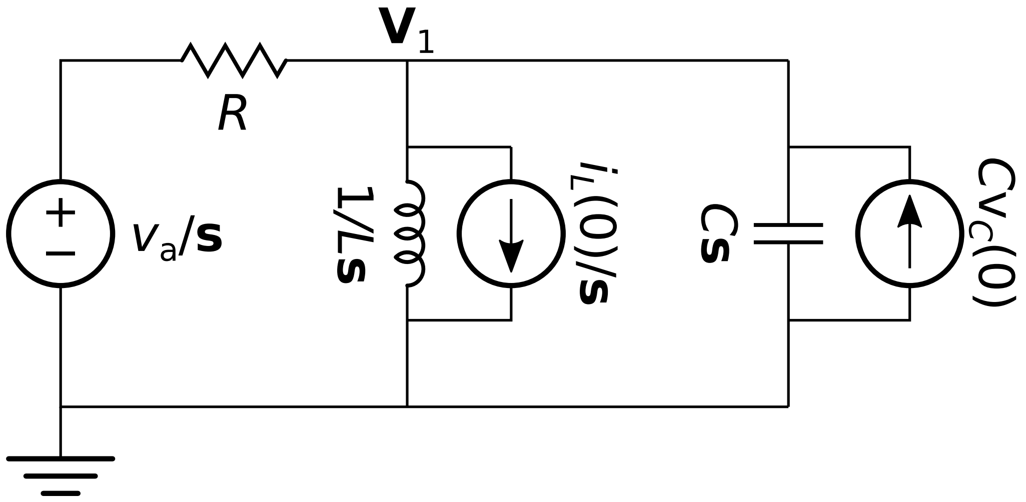
Complete normal form by multiplying by $\frac{\mathbf{s}}{C}$ (to remove $\frac{1}{\mathbf{s}}$ terms and so that coefficient of highest power of $\mathbf{s}$ is 1).

$$\left(\mathbf{s}^2 + \frac{1}{RC}\mathbf{s} + \frac{1}{LC}\right)\mathbf{V}_1 = \frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}$$

Original Circuit



Node Analysis Transform-Domain Circuit



Next, solve for $\mathbf{V}_1$:

$$\mathbf{V}_1 = \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{\mathbf{s}^2 + \frac{1}{RC}\mathbf{s} + \frac{1}{LC}}$$

then factor the denominator:

$$\mathbf{V}_1 = \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s} - a_1)(\mathbf{s} - a_2)}$$

where

$$a_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad a_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Next, manipulate $\mathbf{V}_1$ so that it is a sum of terms of the form $\dfrac{K_i}{\mathbf{s}-a_i}$:

$$\mathbf{V}_1(\mathbf{s}) \quad = \quad \frac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s}-a_1)(\mathbf{s}-a_2)} \quad = \quad \begin{cases} \frac{K_1}{\mathbf{s}-a_1} + \frac{K_2}{\mathbf{s}-a_2}, & \text{if } a_1 \neq a_2; \\ \frac{K_1}{(\mathbf{s}-a_1)^2} + \frac{K_2}{\mathbf{s}-a_1}, & \text{if } a_1 = a_2. \end{cases}$$

Time-Domain Solution:

$$v_1(t) = \begin{cases} K_1\, e^{a_1 t} + K_2\, e^{a_2 t}, & \text{if } a_1 \neq a_2; \\ K_1\, t\, e^{a_1 t} + K_2\, e^{a_1 t}, & \text{if } a_1 = a_2. \end{cases}$$

All that remains to do is to find K_1 and K_2 ...
... use partial-fraction expansion to do so.

For the case where $a_1 \neq a_2$:

$$
\begin{aligned}
K_1 &= (\mathbf{s} - a_1)\,\mathbf{V}_1 \,\Big|_{\mathbf{s}=a_1} \\
&= (\mathbf{s} - a_1)\,\frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s} - a_1)(\mathbf{s} - a_2)} \,\Bigg|_{\mathbf{s}=a_1} \\
&= \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s} - a_2)} \,\Bigg|_{\mathbf{s}=a_1} \\
&= \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1}{(a_1 - a_2)}
\end{aligned}
$$

Similarly

$$
K_2 = \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,a_2}{(a_2 - a_1)}
$$

The $\mathbf{s}$-Domain Solution for the case $a_1 \neq a_2$:

Remember that:

$$K_1 = \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\, a_1}{(a_1 - a_2)} \qquad K_2 = \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\, a_2}{(a_2 - a_1)}$$

Put the K_i's in the expression for $\mathbf{V}_1$:

$$\begin{aligned}
\mathbf{V}_1 &= \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s} - a_1)(\mathbf{s} - a_2)} \\
&= \frac{K_1}{\mathbf{s} - a_1} + \frac{K_2}{\mathbf{s} - a_2} \\
&= \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\, a_1}{(a_1 - a_2)(\mathbf{s} - a_1)} \quad + \quad \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\, a_2}{(a_2 - a_1)(\mathbf{s} - a_2)}
\end{aligned}$$

For the case where $a_1 = a_2$:

$$\mathbf{V}_1 = \frac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s}-a_1)^2} = \frac{K_1}{(\mathbf{s}-a_1)^2} + \frac{K_2}{\mathbf{s}-a_1}$$

$$\begin{aligned}
K_1 &= (\mathbf{s}-a_1)^2\,\mathbf{V}_1 \Big|_{\mathbf{s}=a_1} \\
&= (\mathbf{s}-a_1)^2\,\frac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s}-a_1)^2} \Big|_{\mathbf{s}=a_1} \\
&= \frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s} \Big|_{\mathbf{s}=a_1} \\
&= \frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1
\end{aligned}$$

$$
\begin{aligned}
K_2 &= \frac{d}{d\mathbf{s}} \left[(\mathbf{s} - a_1)^2 \, \mathbf{V}_1 \right] \Big|_{\mathbf{s}=a_1} \\
&= \frac{d}{d\mathbf{s}} \left[(\mathbf{s} - a_1)^2 \, \frac{\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s} - a_1)^2} \right] \Big|_{\mathbf{s}=a_1} \\
&= \frac{d}{d\mathbf{s}} \left[\frac{1}{RC} v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s} \right] \Big|_{\mathbf{s}=a_1} \\
&= v_C(0) \Big|_{\mathbf{s}=a_1} \\
&= v_C(0)
\end{aligned}
$$

The $\mathbf{s}$-Domain Solution for the case $a_1 = a_2$:

We just found that: $K_1 = \frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1 \qquad K_2 = v_C(0)$

Put theses K_i's in the expression for $\mathbf{V}_1$:

$$\mathbf{V}_1 = \frac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,\mathbf{s}}{(\mathbf{s} - a_1)^2}$$

$$= \frac{K_1}{(\mathbf{s} - a_1)^2} + \frac{K_2}{\mathbf{s} - a_1}$$

$$= \frac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1}{(\mathbf{s} - a_1)^2} \quad + \quad \frac{v_C(0)}{(\mathbf{s} - a_1)}$$

Summary of Laplace RLC Results

s-Domain Voltage:

$$\mathbf{V}_1(\mathbf{s}) = \begin{cases} \dfrac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1}{(a_1-a_2)(\mathbf{s}-a_1)} \quad + \quad \dfrac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_2}{(a_2-a_1)(\mathbf{s}-a_2)}, & \text{if } a_1 \neq a_2; \\ \dfrac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1}{(\mathbf{s}-a_1)^2} \quad + \quad \dfrac{v_C(0)}{(\mathbf{s}-a_1)}, & \text{if } a_1 = a_2. \end{cases}$$

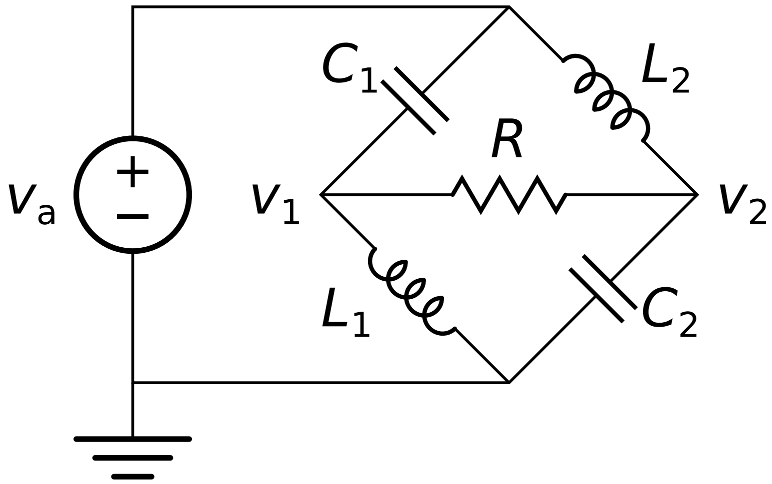
Time-Domain Voltage:

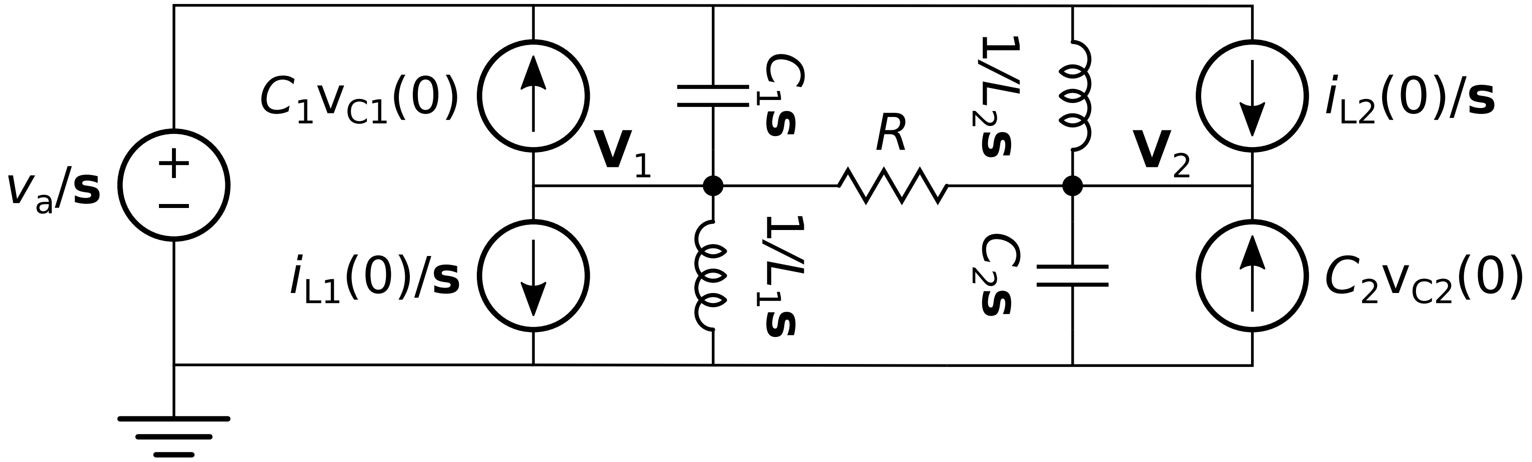
$$v_1(t) = \begin{cases} \dfrac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_1}{(a_1-a_2)}e^{a_1 t} \quad + \quad \dfrac{\frac{1}{RC}v_a - \frac{i_L(0)}{C} + v_C(0)\,a_2}{(a_2-a_1)}e^{a_2 t}, & \text{if } a_1 \neq a_2; \\ \left(\dfrac{1}{RC}v_a - \dfrac{i_L(0)}{C} + v_C(0)\,a_1\right) t\,e^{a_1 t} \quad + \quad v_C(0)\,e^{a_1 t}, & \text{if } a_1 = a_2. \end{cases}$$

where

$$a_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \qquad a_2 = -\frac{1}{2RC} - \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

Laplace RLLCC Circuit Node Analysis Example





Raw Node Equations:

$$C_1\mathbf{s}\left(\mathbf{V}_1 - \frac{v_a}{\mathbf{s}}\right) + C_1 v_{C_1}(0) + \frac{1}{R}(\mathbf{V}_1 - \mathbf{V}_2) + \frac{1}{L_1\mathbf{s}}\mathbf{V}_1 + \frac{1}{\mathbf{s}} i_{L_1}(0) = 0 \qquad \text{Node } \mathbf{V}_1$$

$$\frac{1}{L_2\mathbf{s}}\left(\mathbf{V}_2 - \frac{v_a}{\mathbf{s}}\right) - \frac{1}{\mathbf{s}} i_{L_2}(0) + \frac{1}{R}(\mathbf{V}_2 - \mathbf{V}_1) + C_2\,\mathbf{s}\,\mathbf{V}_2 - C_2\,v_{C_2}(0) = 0 \qquad \text{Node } \mathbf{V}_2$$

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Normal Form Step 1:

Move knowns to right-hand side, collect terms multiplying unknowns ($\mathbf{V}_1$ and $\mathbf{V}_2$).

$$\left(C_1\mathbf{s} + \frac{1}{R} + \frac{1}{L_1\mathbf{s}}\right)\mathbf{V}_1 + \left(-\frac{1}{R}\right)\mathbf{V}_2 = C_1 v_a - C_1 v_{C_1}(0) - \frac{1}{\mathbf{s}} i_{L_1}(0) \qquad \text{Node } \mathbf{V}_1$$

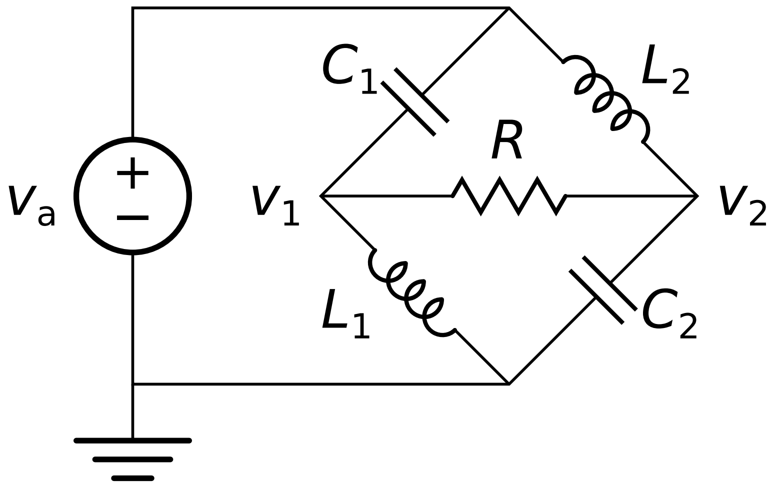
$$\left(-\frac{1}{R}\right)\mathbf{V}_1 + \left(C_2\mathbf{s} + \frac{1}{R} + \frac{1}{L_2\mathbf{s}}\right)\mathbf{V}_2 = \frac{1}{L_2\mathbf{s}^2} v_a + C_2 v_{C_2}(0) + \frac{1}{\mathbf{s}} i_{L_2}(0) \qquad \text{Node } \mathbf{V}_2$$

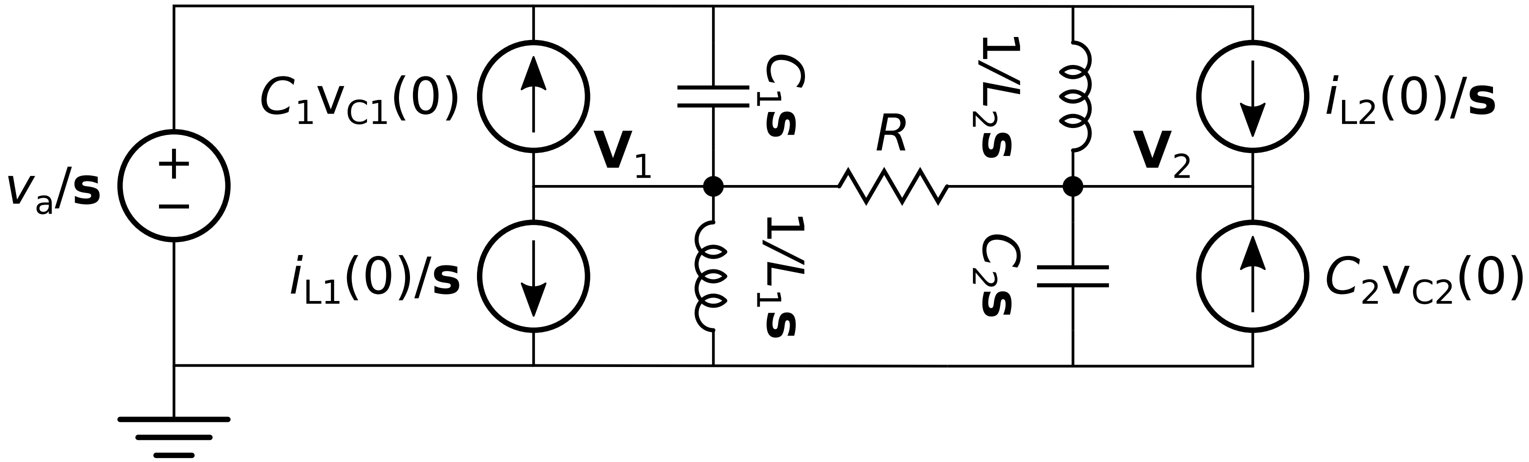
Normal Form Step 2:

Multiply by $\mathbf{s}$ until there are no more $\frac{1}{\mathbf{s}}$ terms.

$$\left(C_1\mathbf{s}^2 + \frac{1}{R}\mathbf{s} + \frac{1}{L_1}\right)\mathbf{V}_1 + \left(-\frac{1}{R}\mathbf{s}\right)\mathbf{V}_2 = C_1\mathbf{s} v_a - C_1\mathbf{s} v_{C_1}(0) - i_{L_1}(0) \qquad \text{Node } \mathbf{V}_1$$

$$\left(-\frac{1}{R}\mathbf{s}^2\right)\mathbf{V}_1 + \left(C_2\mathbf{s}^3 + \frac{1}{R}\mathbf{s}^2 + \frac{1}{L_2}\mathbf{s}\right)\mathbf{V}_2 = \frac{1}{L_2} v_a + C_2\mathbf{s}^2 v_{C_2}(0) + \mathbf{s} i_{L_2}(0) \qquad \text{Node } \mathbf{V}_2$$





Solving simultaneously and cleaning up a bit ...

$$\mathbf{V}_1 = \frac{\mathbf{s}^3(v_a - v_{C_1}(0)) + \mathbf{s}^2\left(\frac{v_{C_2}(0)}{RC_1} - \frac{v_{C_1}(0)}{RC_2} - \frac{i_{L_1}(0)}{C_1} + \frac{v_a}{RC_2}\right) + \mathbf{s}\left(\frac{i_{L_2}(0)}{RC_1C_2} - \frac{i_{L_1}(0)}{RC_1C_2} + \frac{v_a}{C_2L_2} - \frac{v_{C_1}(0)}{C_2L_2}\right) + \left(\frac{v_a}{RL_2C_1C_2} - \frac{i_{L_1}(0)}{L_2C_1C_2}\right)}{\mathbf{s}^4 + \mathbf{s}^3\left(\frac{1}{RC_2} + \frac{1}{RC_1}\right) + \mathbf{s}^2\left(\frac{1}{L_1C_1} + \frac{1}{L_2C_2}\right) + \mathbf{s}\left(\frac{1}{RL_1C_1C_2} + \frac{1}{RL_2C_1C_2}\right) + \frac{1}{L_1L_2C_1C_2}}$$

$$\mathbf{V}_2 = \frac{\mathbf{s}^4 v_{C_2}(0) + \mathbf{s}^3\left(\frac{i_{L_2}(0)}{C_2} - \frac{(v_{C_1}(0) - v_a)}{RC_2} + \frac{v_{C_2}(0)}{RC_1}\right) + \mathbf{s}^2\left(\frac{v_a}{L_2C_2} + \frac{v_{C_2}(0)}{C_1L_1} - \frac{i_{L_1}(0) - i_{L_2}(0)}{RC_1C_2}\right) + \frac{\mathbf{s}}{C_1C_2}\left(\frac{i_{L_2}(0)}{L_1} + \frac{v_a}{RL_2}\right) + \frac{v_a}{L_1L_2C_1C_2}}{\mathbf{s}^5 + \mathbf{s}^4\left(\frac{1}{RC_1} + \frac{1}{RC_2}\right) + \mathbf{s}^3\left(\frac{1}{L_1C_1} + \frac{1}{L_2C_2}\right) + \mathbf{s}^2\left(\frac{1}{RL_1C_1C_2} + \frac{1}{RL_2C_1C_2}\right) + \mathbf{s}\frac{1}{L_1L_2C_1C_2}}$$

Consider the special case $C_1 \to C_2 \to C$ and $L_1 \to L_2 \to L$:

$$\mathbf{V}_1 = \frac{\mathbf{s}^3 C^2 \left(v_a - v_{C_1(0)}\right) + \mathbf{s}^2 C \left(\frac{1}{R}(v_a - v_{C_1(0)} + v_{C_2(0)}) - i_{L_1(0)}\right) + \mathbf{s}\left(\frac{C}{L}(v_a - v_{C_1(0)}) - \frac{i_{L_1(0)}}{R} + \frac{i_{L_2(0)}}{R}\right) + \left(\frac{v_a}{LR} - \frac{i_{L_1(0)}}{L}\right)}{\left(\mathbf{s} + \sqrt{-\frac{1}{LC}}\right)\left(\mathbf{s} - \sqrt{-\frac{1}{LC}}\right)\left(\mathbf{s} + \frac{1}{RC} + \sqrt{\frac{1}{(RC)^2} - \frac{1}{LC}}\right)\left(\mathbf{s} + \frac{1}{RC} - \sqrt{\frac{1}{(RC)^2} - \frac{1}{LC}}\right)}$$

$$\mathbf{V}_2 = \frac{-\mathbf{s}^4 RL^2C^2 v_{C_2(0)} + \mathbf{s}^3 L^2 C(v_{C_1(0)} - v_{C_2(0)} - i_{L_2(0)}R - v_a) + \mathbf{s}^2(L^2(i_{L_1(0)} - i_{L_2(0)}) - RLC(v_a + v_{C_2(0)})) - \mathbf{s}L(i_{L_2(0)}R + v_a) - Rv_a}{\mathbf{s}\left(\mathbf{s} + \sqrt{-\frac{1}{LC}}\right)\left(\mathbf{s} - \sqrt{-\frac{1}{LC}}\right)\left(\mathbf{s} + \frac{1}{RC} + \sqrt{\frac{1}{(RC)^2} - \frac{1}{LC}}\right)\left(\mathbf{s} + \frac{1}{RC} - \sqrt{\frac{1}{(RC)^2} - \frac{1}{LC}}\right)}$$

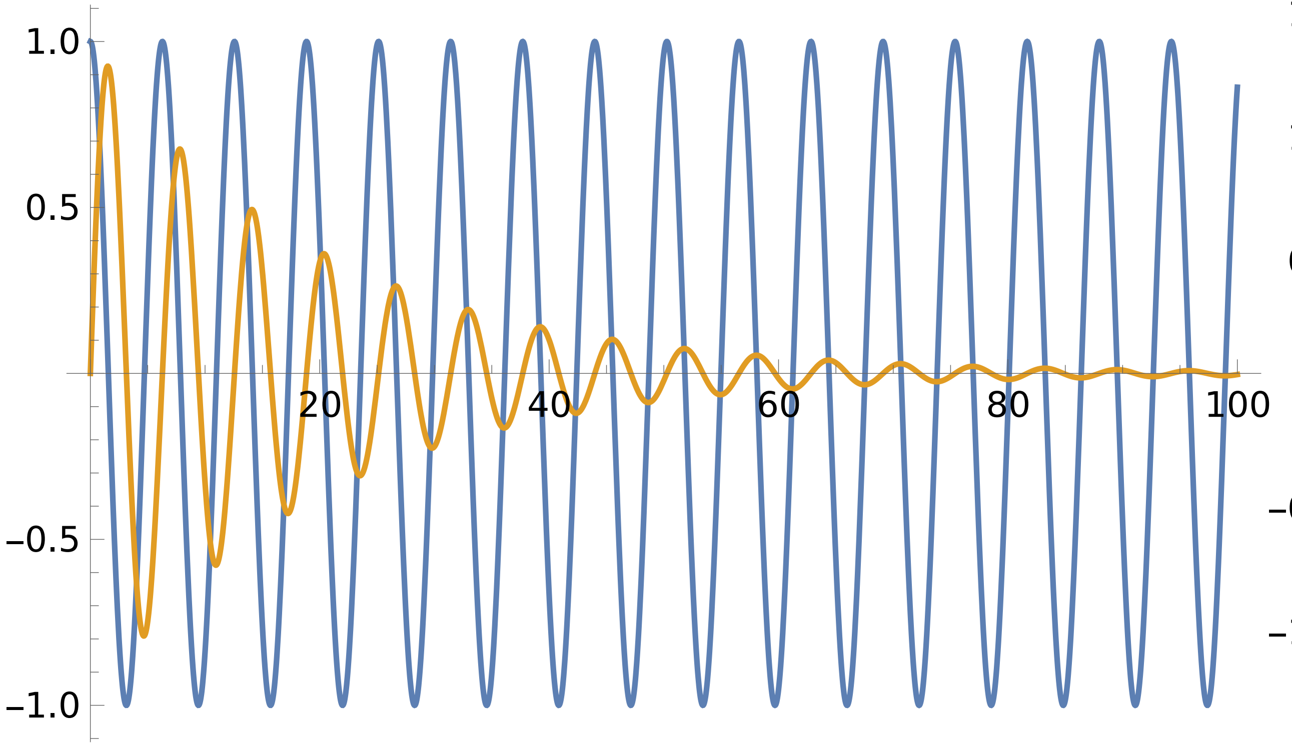
The solutions above are the same as those obtained using the classical analysis, except that there are no constants to find!

Remaining steps:

Use partial-fraction expansion to transform to $\mathbf{V}_1$ to $\frac{K_1}{\mathbf{s}-a_1} + \frac{K_2}{\mathbf{s}-a_2} + \frac{K_3}{\mathbf{s}-a_3} + \frac{K_4}{\mathbf{s}-a_4}$ or $\frac{K_1}{\mathbf{s}-a_1} + \frac{K_2}{\mathbf{s}-a_2} + \frac{K_{32}}{(\mathbf{s}-a_3)^2} + \frac{K_{31}}{\mathbf{s}-a_3}$ form ... ... and $\mathbf{V}_2$ to $\frac{K_0}{\mathbf{s}} + \frac{K_1}{\mathbf{s}-a_1} + \frac{K_2}{\mathbf{s}-a_2} + \frac{K_3}{\mathbf{s}-a_3} + \frac{K_4}{\mathbf{s}-a_4}$ or $\frac{K_0}{\mathbf{s}} + \frac{K_1}{\mathbf{s}-a_1} + \frac{K_2}{\mathbf{s}-a_2} + \frac{K_{32}}{(\mathbf{s}-a_3)^2} + \frac{K_{31}}{\mathbf{s}-a_3}$ form.

Convert to time domain (if desired).

These remaining steps are left as an exercise to the reader.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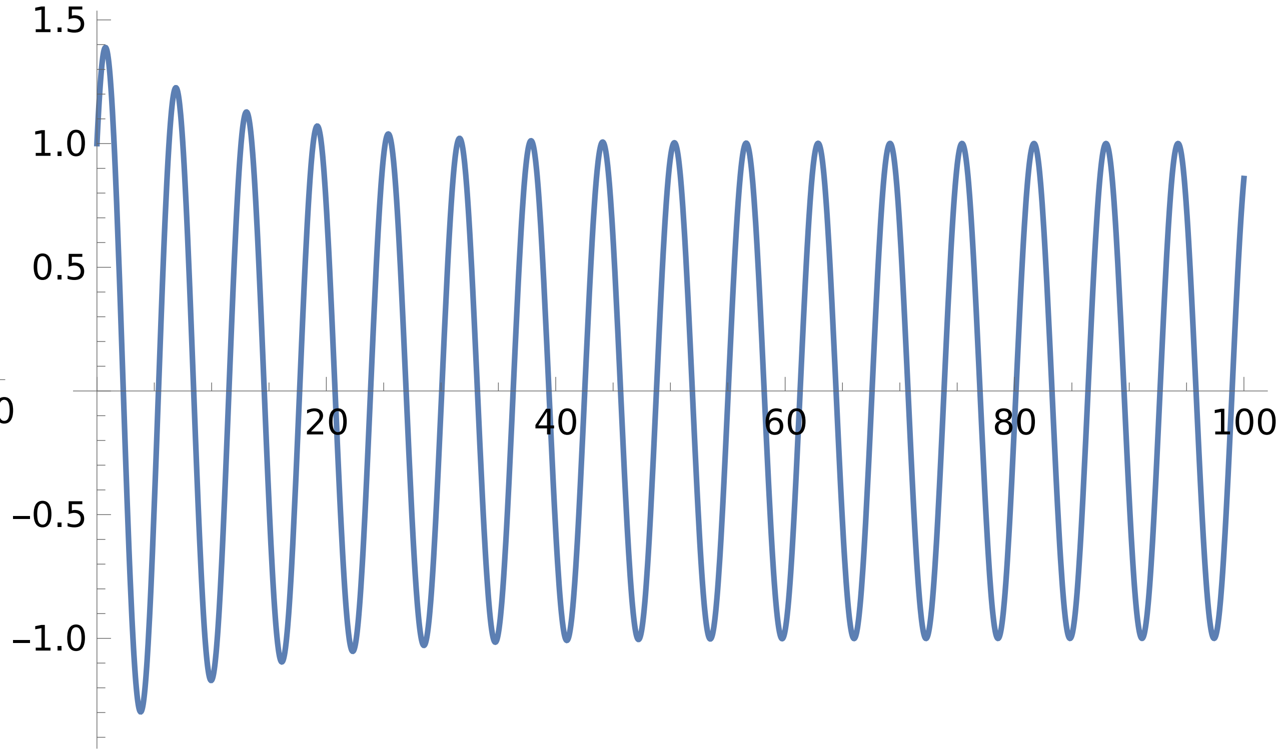
−1.0
20
40
60
80
100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