EE 3755

Spring 05

HW #1 Solutions

1 Here the two numbers X and Y are of different signs and X+Y needs to be performed. We thus perform the following subtraction:

(magnitude of X) − (magn. of Y) = (11100) − (11110)

= (11100) + 2's compl. of (11110) =

$$
\begin{array}{r}
= \quad 11100 \\
+) \quad 00010 \\
\hline
0 \quad 11110
\end{array}
$$

→ c = 0 ⇒ result < 0 ⇒ (magn. of X) − (mag. of Y) < 0

⇒ magn. of X < magn. of Y.

Therefore, sign bit of result = sign bit of Y = 1 and magnitude of (X+Y) = 2's compl. of (11110) = (00010). Thus $X+Y = (100010)_2 = (-2)_{10}$.

$\boxed{2}$ Here $n = 6$, multiplier $= (-27)_{10} =$
$= (100101)_2$; multiplicand $= (-22)_{10} = (101010)_2$
Since three bits are to be examined at a time, the field B (left of multiplier field) must be of length $6+1 = 7$ bits

	B	A	d
Initialization	0000000	100101	0
+)	1101010		
	1101010	100101	0
result of 1st cycle	1111010	101001	0
+)	1101010		
	1100100	101001	0
result of 2nd cycle	1111001	001010	0
+)	0101100		
	0100101	001010	0
	0001001	010010	1

Initialization: → 010 → add 1× mult/cand and then do 2-bit right shift

1st cycle: → 010 → add 1× mult/cand and do 2-bit right sh.

2nd cycle: → 100 → add $-2\times$ mult/cand and do 2-bit right shift

→ product =
$= (001001010010)_2 = 594 =$
$= (-22) \times (-27)$

$\boxed{3}$ Here the dividend is $A = (0000001101)_2 = (+13)_{10}$; the divisor is $B = (00101)_2 = (+5)_{10}$ and $n = 5$. Also, $-B$ = 2's complement of $B = (11011)_2$

C	R	Q	
X	00000	01101	Initialization
	00000	1101	shift R, Q left
	+) 11011		subtract B
0	11011		
	11011	11010	

1^{st} division cycle

C	R	Q	
	10111	1010	shift R, Q left
	+) 00101		add B (since previous cycle gave c=0)
0	11100		
	11100	10100	

2^{nd} division cycle

C	R	Q	
	11001	0100	shift R, Q left
	+) 00101		add B (since previous cycle gave c=0)
0	11110		
	11110	01000	

3^{rd} division cycle

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4th division cycle

R	Q
11100	1000

shift R, Q left

$$\begin{array}{r} +)\ 00101 \\ \hline 00001 \end{array} \qquad C = \boxed{1}$$

add B (since previous cycle gave c=0)

R	Q
00001	10001

5th division cycle

R	Q
00011	0001

shift R, Q left

$$\begin{array}{r} +)\ 11011 \\ \hline 11110 \end{array} \qquad C = \boxed{0}$$

subtr. B (since previous cycle gave c=1)

R	Q
11110	00010

Restore cycle

$$\begin{array}{r} +)\ 00101 \\ \hline 00011 \end{array}$$

restore remainder by adding B to it since 5th division cycle gave c=0

R	Q
00011	00010

So remainder is $R = (00011)_2 = 3$ while quotient is $Q = (00010)_2 = 2$. Double check to see that $B \times Q + R = 5 \times 2 + 3 = 13 = A$

$\boxed{4}$ Here the dividend is $A = (0000010001)_2 = (+17)_{10}$; the divisor is $B = (00011)_2 = (+3)_{10}$ and $n = 5$. Also, $-B$ = 2's complement of $B = (11101)_2$.

C	R	Q	
$\boxed{X}$	00000	10001	Initialization

1st division cycle

C	R	Q	
	00001	0001	shift R, Q left
	+) 11101		subtract B
$\boxed{0}$	11110		
	11110	00010	

2nd division cycle

C	R	Q	
	11100	0010	shift R, Q left
	+) 00011		add B (since previous cycle gave c=0)
$\boxed{0}$	11111		
	11111	00100	

3rd division cycle

C	R	Q	
	11110	0100	shift R, Q left
	+) 00011		add B (since previous cycle gave c=0)
$\boxed{1}$	00001		
	00001	01001	

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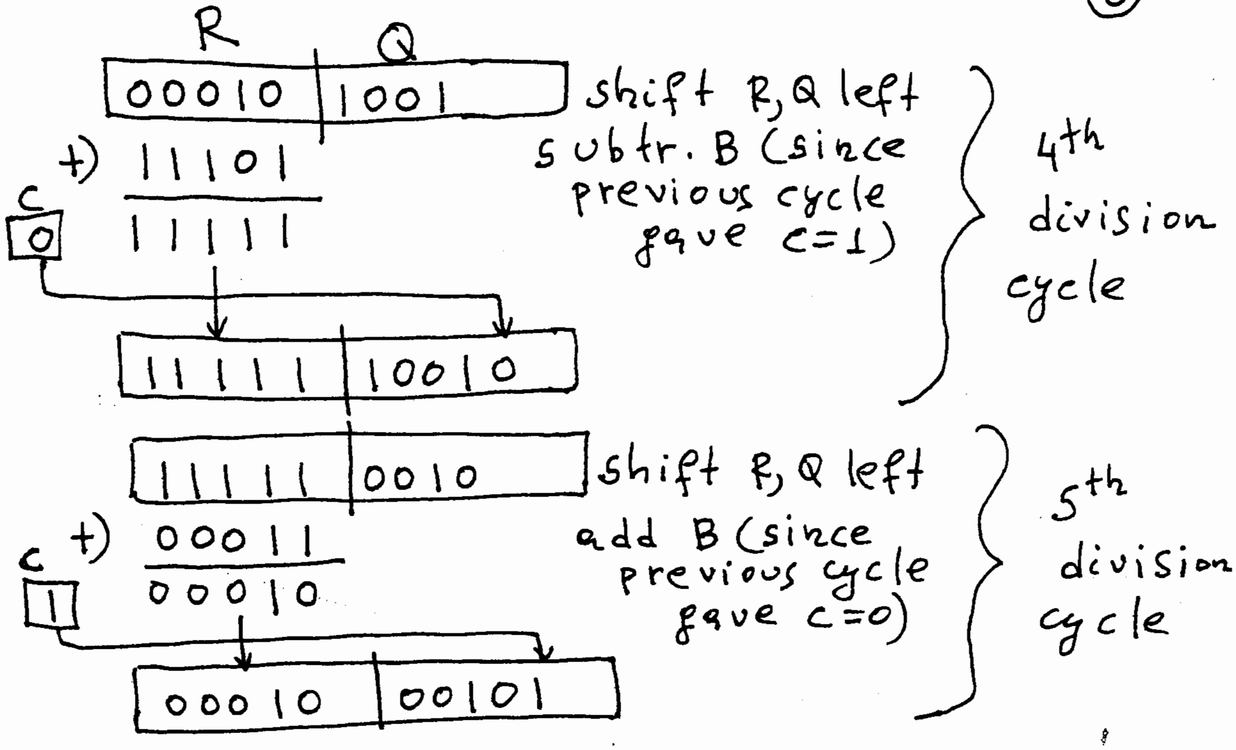

No restore cycle is necessary since the carry out of the 5th division cycle is 1. Here remainder is $R = (00010)_2 = 2$ while quotient is $Q = (00101)_2 = 5$. Double check to see that $B \times Q + R = 3 \times 5 + 2 = 17 = A$.

5 The range of the fraction is

$$\boxed{0.5 \leq f \leq 1 - 2^{-29}}$$

The range of the exponent is

$-2^{9} \leq e \leq 2^{9} - 1$ or $\boxed{-512 \leq e \leq 511}$

Thus the positive FLP dynamic range is

$$\boxed{0.5 \times 2^{-512} \leq A^{+} \leq (1 - 2^{-29}) \times 2^{511}}$$

The negative FLP dynamic range is

$$\boxed{-(1 - 2^{-29}) \times 2^{511} \leq A^{-} \leq -0.5 \times 2^{-512}}$$

6 1. Align/adjust

$e_1 - e_2 = e_1 + 2\text{'s complement of } e_2 =$

$$\begin{array}{r} = 0111 \\ +)\ 0111 \\ \hline 01110 \end{array}$$

$\hookrightarrow c = 0 \Rightarrow e_1 - e_2 < 0 \Rightarrow e_1 < e_2$ and $e_2 - e_1$

$= 2\text{'s complement of}(1110) = (0010)_2 = (2)_{10}$

Thus A_1 becomes

A_1 :

s_1	e_2	f_1'
0	1001	00111100

2. <u>Subtract fractions</u>

Since A_1 and A_2 are of different signs and $A_1 + A_2$ needs to be performed, a subtraction must take place.

$f_1' - f_2 = f_1' + 2\text{'s complement of } f_2 =$

$$
\begin{array}{rl}
 & .00111100 \\
+) & .01101110 \\
\hline
0 & .10101010
\end{array}
$$

$\hookrightarrow c = 0 \Rightarrow f_1' - f_2 < 0 \Rightarrow f_1' < f_2$. Since f_2 is the larger fraction, the result $A_3 = A_1 + A_2$ must have as a sign bit the sign bit of A_2 (negative sign).
The fraction of $A_3 = A_1 + A_2$ will be the 2's complement of $(10101010) = 01010110$
Therefore

A_3 :

1	1001	01010110

3. Postnormalize

After postnormalization we get

	s_3	e_3	f_3
A_3 :	1	1000	10101100

4. Check for exponent underflow

No exponent underflow occured.

See that $e_3 = (1000)_2 = 8 \in [0 \quad 15]$

$\boxed{7}$ (10)

(a)

$$G_5^* = G_{23} + G_{22} \cdot P_{23} + G_{21} \cdot P_{22} \cdot P_{23} + G_{20} \cdot P_{21} \cdot P_{22} \cdot P_{23}$$

(b) $P_6^* = P_{27} \cdot P_{26} \cdot P_{25} \cdot P_{24}$

(c)

$$\begin{aligned} c_{23} = {} & G_5^* + G_4^* \cdot P_5^* + G_3^* \cdot P_4^* \cdot P_5^* + G_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^* \\ & + G_1^* \cdot P_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^* + G_0^* \cdot P_1^* \cdot P_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^* \\ & + c_{-1} \cdot P_0^* \cdot P_1^* \cdot P_2^* \cdot P_3^* \cdot P_4^* \cdot P_5^* \end{aligned}$$

(d)

$$\begin{aligned} c_{26} = {} & G_{26} + G_{25} \cdot P_{26} + G_{24} \cdot P_{25} \cdot P_{26} + \\ & c_{23} \cdot P_{24} \cdot P_{25} \cdot P_{26} \end{aligned}$$