EE 3755, Test 1, Spring 04

3/3/04

<u>Note</u>: The test is closed books and closed notes. You are <u>not</u> allowed to use calculators. Provide all answers on exam paper. Write your name and social security number.

Name:

SSN:

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Problem 1: a) Prove that if two n-bit, signed integers are added using the 2's complement system, then the carry out of the addition should be ignored.

Problem 1 cont.

(b) Consider the 1's complement addition $X+Y$ where X and Y are the n-bit numbers $X = x_{n-1}x_{n-2}\cdots x_1 x_0$ and $Y = y_{n-1}y_{n-2}\cdots y_1 y_0$. The well known procedure for computing $X+Y$ in the 1's complement system is shown below:

Addition 1:

$$\begin{array}{rl} & x_{n-1}x_{n-2}\cdots x_0 \\ +) & y_{n-1}y_{n-2}\cdots y_0 \\ \hline c & z_{n-1}z_{n-2}\cdots z_0 \end{array}$$

carry out

Addition 2:

$$\begin{array}{rr} & z_{n-1}z_{n-2}\cdots z_0 \\ +) & c \\ \hline & w_{n-1}w_{n-2}\cdots w_0 \end{array}$$

Prove that addition 2 will never create a non zero carry out.

Problem 1 cont.

(c) Using the Booth algorithm that relies on examining <u>three</u> bits at a time, perform the signed multiplication with multiplier $=(-7)_{10}=(1001)_2$, multiplicand $=(-6)_{10}=(1010)_2$ and length $n=4$. Show all your work.

d) Consider the fixed point division of a $2n$-bit integer unsigned dividend A by an n-bit integer unsigned divisor B which should result in an n-bit quotient Q and an n-bit remainder R. Let A_1 and A_0 be respectively, the n-bit leftmost and

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Problem 1 cont:

n-bit rightmost parts of the dividend A. Prove that if $A_1 < B$, then division overflow does not occur.

Problem 1 cont.

(e) Using the division algorithm presented in class, perform the division with dividend $A = (001110)_2 = (14)_{10}$ and divisor $B = (011)_2 = (3)_{10}$. Show all your work.

C	R	Q	
X	001	110	Initialization.

(f) Consider adding two signed numbers using the 2's complement system. Let C_{in} denote the carry into the sign-bit location and C_{out} denote the carry out of the sign-bit location.
Which of the statements (i), (ii), (iii) given on next page are true and which are false?

Problem 1 cont.

(i) "If $C_{in} = C_{out}$ then neither overflow nor underflow has occured".

(ii) "If $C_{in} = 1$ and $C_{out} = 0$ an underflow has occured".

(iii) "If $C_{in} = 0$ and $C_{out} = 1$ an overflow has occured".

g) Consider dividing a positive dividend by a negative divisor.

(i) Is the produced quotient going to be positive or negative?

(ii) Is the produced remainder going to be positive or negative?

Problem 2:

a) Compute the dynamic range for a binary ($r=2$) floating point system which relies on one sign bit, 8-bit signed exponents (the 2's complement system is used for representing signed numbers) and 17-bit unsigned normalized fractions.

Problem 2 cont.

b) Consider the following two floating point numbers with 4-bit exponents in biased form and 5-bit unsigned normalized fractions:

	s_1	e_1	f_1
N_1:	0	1001	10101

	s_2	e_2	f_2
N_2:	0	1010	10001

Compute the difference $N_3 = N_1 - N_2$. Return the result in a form consisting of a normalized fraction and biased exponent. You are not allowed to use magnitude comparators for comparing e_1 with e_2 and f_1 with f_2. You must use adders for this purpose. Show all your work.

Problem 2 cont.

(c) Consider two n-bit fractions $f_1 = .a_{n-1} \cdots a_1 a_0$ and $f_2 = .b_{n-1} \cdots b_1 b_0$. Here $f_1 < f_2$. Explain how you can use the procedure for dividing a 2n-bit integer dividend by an n-bit integer divisor in order to compute $f_3 = \frac{f_1}{f_2}$, where f_3 must also be an n-bit fraction; (f_3 must be the best n-bit approximation of $\frac{f_1}{f_2}$).

Problem 2 cont.

d) Consider two binary floating point numbers A_1 and A_2 with fractions $f_1 = .11101$ and $f_2 = .11000$ respectively and 4-bit biased exponents $e_1 = (0101)_2$ and $e_2 = (1110)_2$. Do you expect an exponent overflow or exponent underflow as a result of the division operation $\frac{A_1}{A_2}$? Justify your answer.

e) Consider two binary floating point numbers A_1 and A_2 with fractions $f_1 = .10000$ and $f_2 = .10000$ respectively and 4-bit biased exponents $e_1 = (1011)_2$ and $e_2 = (1101)_2$. Do you

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Problem 2 cont.

expect an exponent overflow or exponent underflow as a result of the multiplication operation $A_1 \times A_2$? Justify your answer; (the product should be postnormalized if necessary).

f) Consider the following floating point number A. The number A has sign bit $s = 1$, biased exponent $e = 1011$ and unsigned normalized fraction $f = .111100$. Find the value of the number A.