

EE 2720, Spring 2011

Homework # 3 solutions

EE 2720, Homework # 3 solutions (1)

Problem 1: The right side of $(T11')$ is

$$(X+Y) \cdot (X'+Z) = X \cdot Z + X' \cdot Y \quad (1)$$

The left side of ~~$(T11)$~~ $(T11')$ is

$$\begin{aligned} (X+Y) \cdot (X'+Z) \cdot (Y+Z) &= (X \cdot Z + X' \cdot Y) \cdot (Y+Z) \\ &= X \cdot Z \cdot Y + X \cdot Z \cdot Z + X' \cdot Y \cdot Y + X' \cdot Y \cdot Z \\ &= X \cdot Y \cdot Z + X \cdot Z + X' \cdot Y + X' \cdot Y \cdot Z \\ &= X \cdot Z + X' \cdot Y + X \cdot Y \cdot Z + X' \cdot Y \cdot Z \\ &= X \cdot Z + X' \cdot Y + Y \cdot Z \cdot (X+X') \\ &= X \cdot Z + X' \cdot Y + Y \cdot Z \cdot 1 = X \cdot Z + X' \cdot Y + \underbrace{Y \cdot Z} \end{aligned}$$

↑
consensus
term and
can be elimi-
nated accor-
ding to $(T11)$

$$= X \cdot Z + X' \cdot Y \quad (2)$$

Because both right and left side of $(T11')$ reduced to the same expression (which is $X \cdot Z + X' \cdot Y$), theorem $(T11')$ is valid.

EE 2720, HW # 3 solutions cont (2)

Problem 2: I will first provide the canonical sum and then the canonical product for each logic function.

$$(a) F = \sum_{x,y} (1,2) = X' \cdot Y + X \cdot Y' = \prod_{x,y} (0,3) \\ = (X+Y) \cdot (X'+Y')$$

$$(b) \prod_{A,B} (0,1,2) = \text{minterm } 3 = A \cdot B \\ = \prod_{A,B} (0,1,2) = (A+B) \cdot (A+B') \cdot (A'+B)$$

$$(c) F = \sum_{A,B,C} (3,4,6,7) = A' \cdot B \cdot C' + A \cdot B' \cdot C' \\ + A \cdot B \cdot C' + A \cdot B \cdot C = \prod_{A,B,C} (0,1,3,5) \\ = (A+B+C) \cdot (A+B+C') \cdot (A+B'+C') \cdot (A'+B+C')$$

$$(d) F = \prod_{M,N,P} (0,1,3,6,7) = \sum_{M,N,P} (2,4,5) \\ = M' \cdot N \cdot P' + M \cdot N' \cdot P' + M \cdot N' \cdot P$$

$$\text{~~XXXXXXXXXX~~ = \prod_{M,N,P} (0,1,3,6,7) \\ = (M+N+P) \cdot (M+N+P') \cdot (M+N'+P') \cdot \\ (M'+N'+P) \cdot (M'+N'+P')$$



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Problem 2 cont: (e) $F = X + Y' \cdot Z'$

$$\begin{aligned} &= X \cdot \underbrace{(Y + Y')}_{\perp} \cdot \underbrace{(Z + Z')}_{\perp} + Y' \cdot Z' \cdot \underbrace{(X + X')}_{\perp} \\ &= X \cdot (Y \cdot Z + Y \cdot Z' + Y' \cdot Z + Y' \cdot Z') + Y' \cdot Z' \cdot X \\ &\quad + Y' \cdot Z' \cdot X' \end{aligned}$$

$$\begin{aligned} &= \underset{\perp}{X} \cdot \underset{\perp}{Y} \cdot \underset{\perp}{Z} + \underset{\perp}{X} \cdot \underset{\perp}{Y} \cdot \underset{0}{Z'} + \underset{\perp}{X} \cdot \underset{0}{Y'} \cdot \underset{\perp}{Z} + \underset{\perp}{X} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'} \\ &\quad + \cancel{\underset{\perp}{X} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}} + \underset{0}{X'} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'} \end{aligned}$$

$$= \Sigma_{X, Y, Z} (0, 4, 5, 6, 7) = \Pi_{X, Y, Z} (1, 3, 3)$$

$$= (X + Y + Z') \cdot (X + Y' + Z) \cdot (X + Y' + Z')$$

(f) $F = A' \cdot B + B' \cdot C + A$

$$\begin{aligned} &= A' \cdot B \cdot \underbrace{(C + C')}_{\perp} + B' \cdot C \cdot \underbrace{(A + A')}_{\perp} \\ &\quad + A \cdot \underbrace{(B + B')}_{\perp} \cdot \underbrace{(C + C')}_{\perp} \end{aligned}$$



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EE 2720, HW#3 solutions cont. (4)

Problem 2 (f) cont.

$$\begin{aligned}
 &= A' \cdot B \cdot C + A' \cdot B \cdot C' + A \cdot B' \cdot C + A' \cdot B' \cdot C \\
 &+ A \cdot (B \cdot C + B \cdot C' + B' \cdot C + B' \cdot C') \\
 &= \underset{0}{A'} \cdot \underset{1}{B} \cdot \underset{1}{C} + \underset{0}{A'} \cdot \underset{1}{B} \cdot \underset{0}{C'} + \underset{1}{A} \cdot \underset{0}{B'} \cdot \underset{1}{C} + \underset{0}{A'} \cdot \underset{0}{B'} \cdot \underset{1}{C} \\
 &+ \underset{1}{A} \cdot \underset{1}{B} \cdot \underset{1}{C} + \underset{1}{A} \cdot \underset{1}{B} \cdot \underset{0}{C'} + \cancel{\underset{1}{A} \cdot \underset{0}{B'} \cdot \underset{1}{C}} + \cancel{\underset{1}{A} \cdot \underset{0}{B'} \cdot \underset{0}{C'}} \\
 &= \sum_{A,B,C} (1, 2, 3, 4, 5, 6, 7) = \text{maxterm } 0 = \\
 &A+B+C
 \end{aligned}$$

Problem 3: $F = (a+b) \cdot (a+c) \cdot (b+c)$

$$\begin{aligned}
 &= (a+b+\underbrace{c \cdot c'}_0) \cdot (a+c+\underbrace{b \cdot b'}_0) \cdot (b+c+\underbrace{a \cdot a'}_0) \\
 &= (a+b+c) \cdot (a+b+c') \cdot (a+c+b) \cdot (a+c+b') \\
 &\cdot (b+c+a) \cdot (b+c+a') = \underset{0}{a} + \underset{0}{b} + \underset{0}{c} \cdot \underset{0}{a} + \underset{0}{b} + \underset{1}{c'} \\
 &\cdot \cancel{\underset{0}{a} + \underset{0}{b} + \underset{0}{c}} \cdot \underset{0}{a} + \underset{1}{b'} + \underset{0}{c} \cdot \cancel{\underset{0}{a} + \underset{0}{b} + \underset{0}{c}} \cdot \underset{1}{a'} + \underset{0}{b} + \underset{0}{c} \\
 &= \prod_{a,b,c} (9, 1, 2, 4)
 \end{aligned}$$

Problem 4: (a) Algebraic approach:

$$\begin{aligned} F &= A \cdot B' + C' \cdot D + E' \\ &= \left[(A \cdot B' + C' \cdot D + E')' \right]' \\ &= \left[(A \cdot B')' \cdot (C' \cdot D)' \cdot E \right]' \quad (1) \end{aligned}$$

From (1) we get the following figure

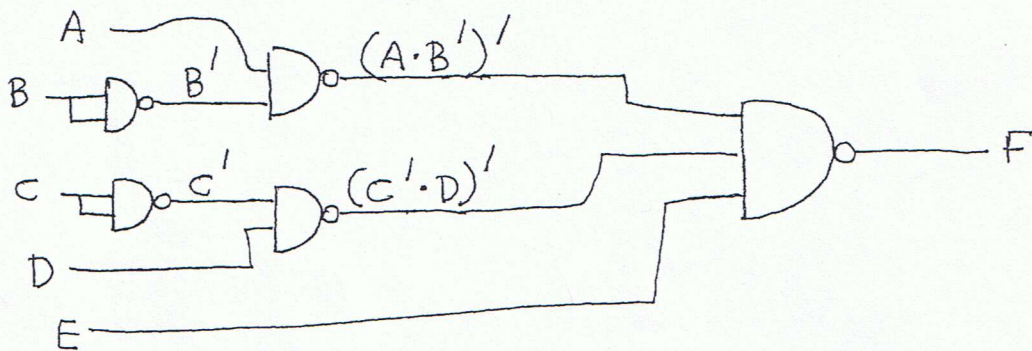
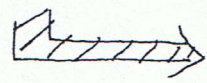


Figure 1

(b) Graphical approach: I first provide a logic circuit showing an AND-OR realization of F

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Problem 4 cont:

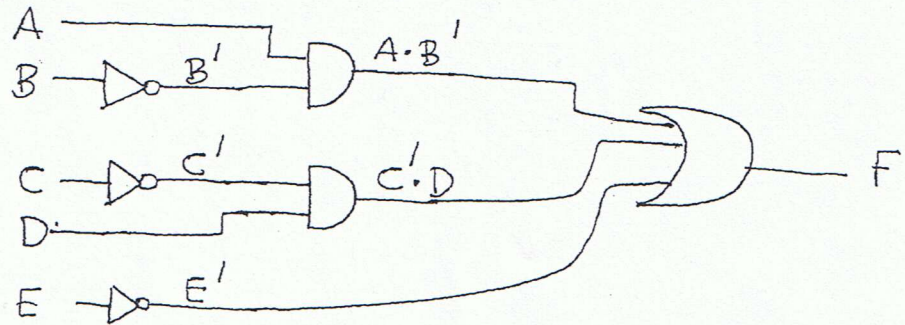


Figure 2

From the above fig. 2 one gets:

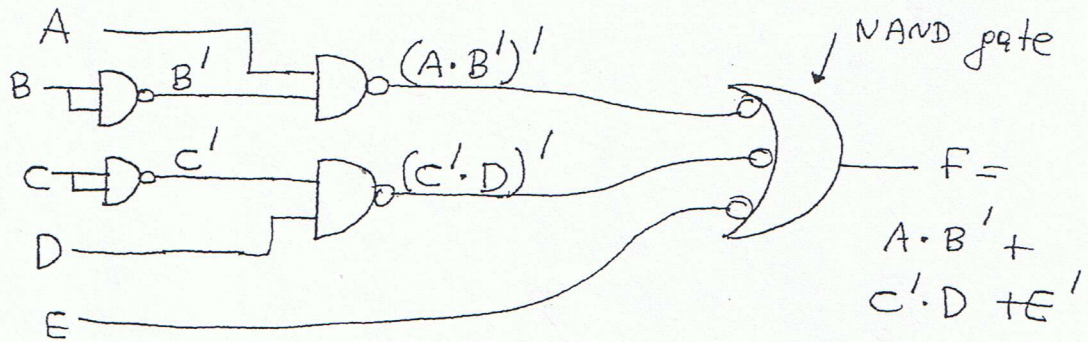
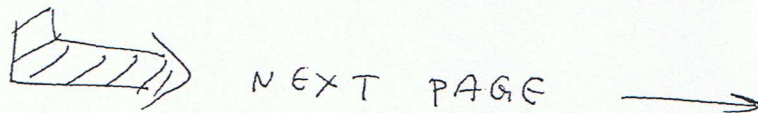



Figure 3

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Problem 5: (a) Algebraic approach:

$$\begin{aligned} F &= (A+B') \cdot (C'+D) \cdot E' \\ &= \left[\left[(A+B') \cdot (C'+D) \cdot E' \right]' \right]' \\ &= \left[(A+B')' + (C'+D)' + E \right]' \quad (2) \end{aligned}$$

From (2) we get the following figure.

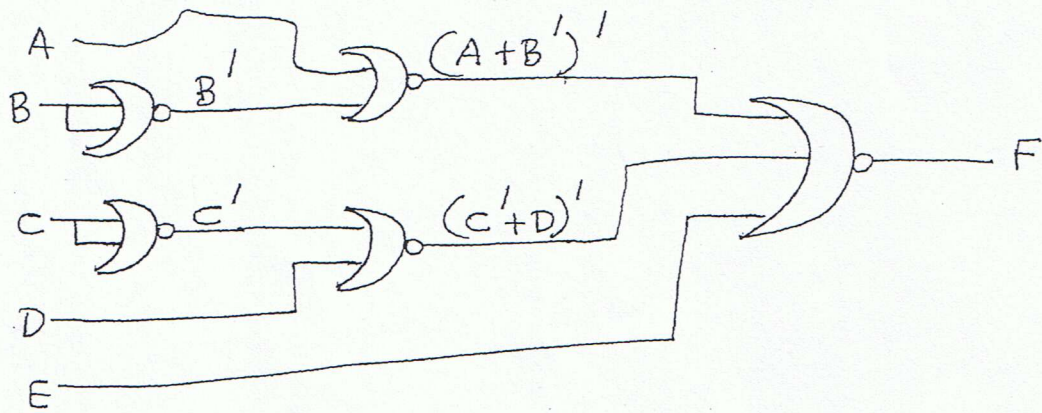


Figure 4

(b) Graphical approach: I first provide a logic circuit showing an OR-AND realization of F. This is shown on the next page

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Problem 5 cont:

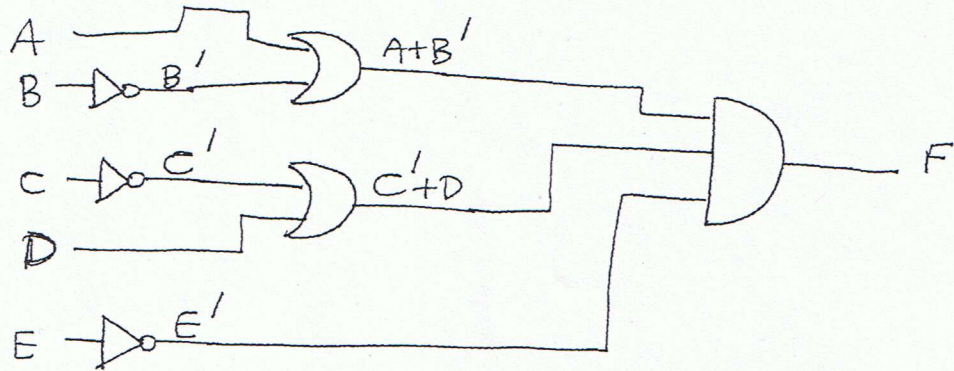


Figure 5

From the above figure 5 one gets:

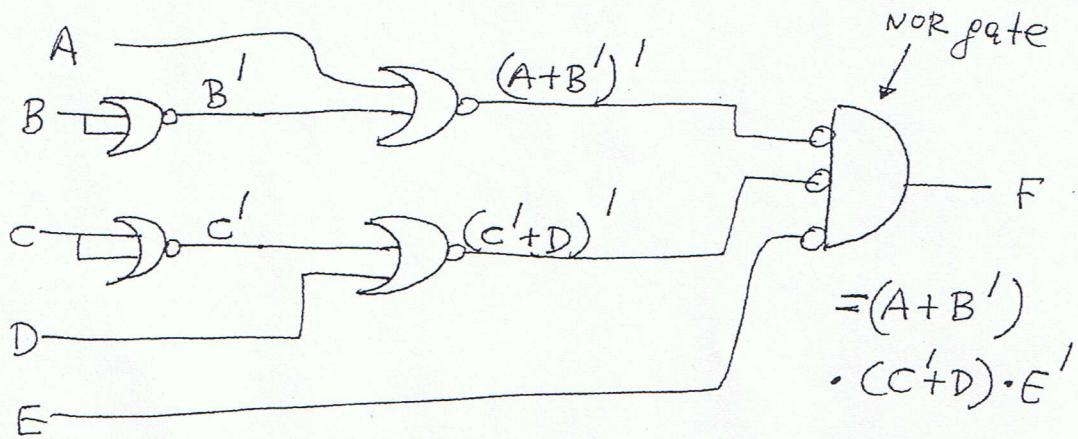


Figure 6

Problem 6:

(a) Proof of (5) or proof of $X \oplus 0 = X$

• Case $X=0$: $0 \oplus 0 = 0$

• Case $X=1$: $1 \oplus 0 = 1$

(b) Proof of (6) or proof of $X \oplus 1 = X'$

• Case $X=0$: $0 \oplus 1 = 1$

• Case $X=1$: $1 \oplus 1 = 0$

(c) Proof of (7) or proof of $X \oplus X = 0$

• Case $X=0$: $0 \oplus 0 = 0$

• Case $X=1$: $1 \oplus 1 = 0$

(d) Proof of (8) or proof of $X \oplus X' = 1$

• Case $X=0$: $0 \oplus 1 = 1$

• Case $X=1$: $1 \oplus 0 = 1$

Note: The above eqs. (5) - (8) can also be proven algebraically. How?

(e) Proof of (11) or proof of

$$X \cdot (Y \oplus Z) = X \cdot Y \oplus X \cdot Z$$



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Problem 6 (c) cont.:

The right side of (11) is:

$$\begin{aligned} X \cdot Y \oplus X \cdot Z &= X \cdot Y \cdot (X \cdot Z)' + (X \cdot Y)' \cdot X \cdot Z \\ &= X \cdot Y \cdot (X' + Z') + (X' + Y') \cdot X \cdot Z \\ &= \cancel{X \cdot Y \cdot X'} + X \cdot Y \cdot Z' + \cancel{X' \cdot X \cdot Z} + Y' \cdot X \cdot Z \\ &= X \cdot Y \cdot Z' + X \cdot Y' \cdot Z = X \cdot (Y \cdot Z' + Y' \cdot Z) \\ &= X \cdot (Y \oplus Z) = \text{left side of (11)} \Rightarrow \text{proven.} \end{aligned}$$