EE 2720, Spring 2011

Homework #2 solutions

EE 2720, Homework #2 solution

Problem 1:

```
   011101 ← positive number
   011110 ← positive number
 0 111011 ← negative number
```

carry out to be ignored → 0

↳ sign bit = 1 ⇒ negative result ⇒ overflow occured. Here dynamic range is DR = [−32 31]

and $X+Y = 29+30 = 59 > 31 \Rightarrow$ overflow

Problem 2:

```
   100011 ← negative number
   110000 ← negative "
 1 010011 ← positive result
```

ignore → 1

↳ sign bit = 0 ⇒ positive result ⇒ underflow

Here DR = [−32 31]

and $X+Y = -29-16 = -45 < -32$

⇒ underflow.

Problem 3: Here the two numbers are of different signs and we therefore have to perform (magnitude of X) − (magn. of Y)

$= (11001)_2 - (11110)_2$

⟹ next page

Prob. 3 cont.

$= (11001) + (\text{two's compl. of } (11110))$

$$
\begin{array}{rr}
= & 11001 \\
+) & 00010 \\
\hline
& 011011
\end{array}
$$

$\hookrightarrow c = 0 \Rightarrow$ result < 0 or

$(\text{magn. of } X) - (\text{magn. of } Y) < 0$

$\Rightarrow$ magn. of $X <$ magn. of Y

Thus

sign bit of $X+Y$ = sign bit of $Y = 1$

and mag. of $X+Y$

$=$ two's compl. of $(11011) = 00101$

Thus $X+Y = 100101_2 = -5_{10}$

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Pr. 4

$$
\begin{array}{rl}
1110 & \text{multiplicand} \\
\times)\ \underline{1111} & \text{multiplier} \\
1110 & \\
+)\ \underline{1110} & \\
101010 & \\
+)\ \underline{1110} & \\
1100010 & \\
+)\ \underline{1110} & \\
11010010 & \rightarrow \text{prod.} = 210
\end{array}
$$

Pr. 5

$$
\begin{array}{rl}
1001 = 9 & \\
+)\ \underline{1000 = 8} & \\
10001 & \leftarrow \text{correction needed} \\
+)\ \underline{0110} & \\
\underbrace{1}_{1}\,\underbrace{0111}_{7} &
\end{array}
$$

Pr 6

$$
\begin{array}{rl}
0100 = 4 & \\
+)\ \underline{0101 = 5} & \\
\underbrace{1001}_{9} & \leftarrow \text{no correction needed}
\end{array}
$$

Pr 7

$$
\begin{array}{rl}
0101 = 5 & \\
+)\ \underline{0101 = 5} & \\
1010 & \leftarrow \text{correction needed} \\
\underline{0110} & \\
\underbrace{1}_{1}\,\underbrace{0000}_{0} &
\end{array}
$$

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Problem 8: All theorems are one-variable theorems. Just view two cases: $X=0$ and $X=1$, and apply the axioms. Proofs are trivial. See notes # 5 for similar proofs.

Problem 9:

X Y Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0 0 0	0	0	0	0
0 0 1	0	1	1	1
0 1 0	1	1	1	1
0 1 1	1	1	1	1
1 0 0	1	0	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

same identical columns

$\Rightarrow (X+Y)+Z = X+(Y+Z)$

and the theorem is proven.

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Problem 10: (CT10') states:

$$(X+Y)\cdot(X+Y')=X$$

Proof: $(X+Y)\cdot(X+Y')=X\cdot X+X\cdot Y'+Y\cdot X+Y\cdot Y'$

$= X+X\cdot Y'+X\cdot Y+0 = X+X\cdot Y'+X\cdot Y=$

$X\cdot(1+Y'+Y)=X\cdot 1=X$

Problem 11: (CT13') states:

$$(X_1+X_2+\cdots+X_n)'=X_1'\cdot X_2'\cdot\cdots\cdot X_n'$$

Proof: I'll first prove (CT13') for $n=2$

or I'll prove that $(X_1+X_2)'=X_1'\cdot X_2'$ (1)

I'll prove (1) using a truth table

$X_1 X_2$	X_1+X_2	$(X_1+X_2)'$	X_1'	X_2'	$X_1'\cdot X_2'$
00	0	1	1	1	1
01	1	0	1	0	0
10	1	0	0	1	0
11	1	0	0	0	0

identical column $\Rightarrow$ eq. (1) true.

Problem 11 cont. Assume now that theorem (T13') is true for $n=i$, or assume that

$$(X_1+X_2+\cdots+X_i)' = X_1' \cdot X_2' \cdot \cdots X_i' \quad (2)$$

We need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1+X_2+\cdots X_i+X_{i+1})' = X_1' \cdot X_2' \cdots X_i' \cdot X_{i+1}'$$

But $(X_1+X_2+\cdots X_i+X_{i+1})'$

$$= [(X_1+X_2+\cdots+X_i)+X_{i+1}]'$$

$$= (X_1+X_2+\cdots+X_i)' \cdot X_{i+1}' \quad \text{(according to (1))}$$

$$= X_1' \cdot X_2' \cdots X_i' \cdot X_{i+1}' \quad \text{(according to (2))}$$

The proof is now completed.

Problem 12: $(X+Y)\cdot(X'+Z) = X\cdot X' + X\cdot Z + Y\cdot X' + Y\cdot Z = 0 + X\cdot Z + X'\cdot Y + Y\cdot Z$

$= X\cdot Z + X'\cdot Y + Y\cdot Z$ → consensus term and can be eliminated according to (T11)

$= X\cdot Z + X'\cdot Y \Rightarrow$ proven.

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Problem 13: The left side of (T10) is

$X \cdot Y + X \cdot Y'$. This can be written as

$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\text{consensus term}}$ (according to (T11)).

$= Y \cdot X + Y' \cdot X + X = X \cdot (Y + Y' + 1) = X \cdot 1 = X$. we now reached the right side of (T10) so the proof is completed

Problem 14:

(a) $F = W \cdot X \cdot Y \cdot Z \cdot (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z + W \cdot X \cdot Y' \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z'$
$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W' \cdot X \cdot Y \cdot Z$
$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0.$

(b) $F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E$
$+ C' \cdot D \cdot E$
$= A \cdot B \cdot (1 + C' D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E = A \cdot B \cdot 1$
$+ C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$