EE 2720, Fall 2010

Homework # 3 solutions

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Problem 1: The right side of (T11') is

$$(X+Y)\cdot(X'+Z) = X\cdot Z + X'\cdot Y \quad (1)$$

The left side of (T11') is

$$(X+Y)\cdot(X'+Z)\cdot(Y+Z) = (X\cdot Z + X'\cdot Y)\cdot(Y+Z)$$

$$= X\cdot Z\cdot Y + X\cdot Z\cdot Z + X'\cdot Y\cdot Y + X'\cdot Y\cdot Z$$

$$= X\cdot Y\cdot Z + X\cdot Z + X'\cdot Y + X'\cdot Y\cdot Z$$

$$= X\cdot Z + X'\cdot Y + X\cdot Y\cdot Z + X'\cdot Y\cdot Z$$

$$= X\cdot Z + X'\cdot Y + Y\cdot Z\cdot(X+X')$$

$$= X\cdot Z + X'\cdot Y + Y\cdot Z\cdot 1 = X\cdot Z + X'\cdot Y + Y\cdot Z$$

consensus term and can be eliminated according to (T11)

$$= X\cdot Z + X'\cdot Y \quad (2)$$

Because both right and left side of (T11') reduced to the same expression (which is $X\cdot Z + X'\cdot Y$), theorem (T11') is valid.

**Problem 2:** I will first provide the canonical sum and then the canonical product for each logic function.

(a) $F = \Sigma_{X,Y}(1,2) = X' \cdot Y + X \cdot Y' = \Pi_{X,Y}(0,3)$
$= (X+Y) \cdot (X'+Y')$

(b) $\Pi_{A,B}(0,1,2) = \text{minterm } 3 = A \cdot B$
$= \Pi_{A,B}(0,1,2) = (A+B) \cdot (A+B') \cdot (A'+B)$

(c) $F = \Sigma_{A,B,C}(3,4,6,7) = A' \cdot B \cdot C + A \cdot B' \cdot C'$
$+ A \cdot B \cdot C' + A \cdot B \cdot C = \Pi_{A,B,C}(0,1,2,5)$
$= (A+B+C) \cdot (A+B+C') \cdot (A+B'+C') \cdot (A'+B+C')$

(d) $F = \Pi_{M,N,P}(0,1,3,6,7) = \Sigma_{M,N,P}(2,4,5)$
$= M' \cdot N \cdot P' + M \cdot N' \cdot P' + M \cdot N' \cdot P$
$= \Pi_{M,N,P}(0,1,3,6,7)$
$= (M+N+P) \cdot (M+N+P') \cdot (M+N'+P') \cdot$
$(M'+N'+P) \cdot (M'+N'+P')$

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**Problem 2 cont:** (e) $F = X + Y' \cdot Z'$

$$= X \cdot \underbrace{(Y+Y')}_{1} \cdot \underbrace{(Z+Z')}_{1} + Y' \cdot Z' \cdot \underbrace{(X+X')}_{1}$$

$$= X \cdot (Y \cdot Z + Y \cdot Z' + Y' \cdot Z + Y' \cdot Z') + Y' \cdot Z' \cdot X$$
$$+ Y' \cdot Z' \cdot X'$$

$$= \underset{1}{X} \cdot \underset{1}{Y} \cdot \underset{1}{Z} + \underset{1}{X} \cdot \underset{1}{Y} \cdot \underset{0}{Z'} + \underset{1}{X} \cdot \underset{0}{Y'} \cdot \underset{1}{Z} + \underset{1}{X} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}$$
$$+ \cancel{\underset{1}{X} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}} + \underset{0}{X'} \cdot \underset{0}{Y'} \cdot \underset{0}{Z'}$$

$$= \Sigma_{X,Y,Z}(0, 4, 5, 6, 7) = \Pi_{X,Y,Z}(1, 2, 3)$$

$$= (X + Y + Z') \cdot (X + Y' + Z) \cdot (X + Y' + Z')$$

(f) $F = A' \cdot B + B' \cdot C + A$

$$= A' \cdot B \cdot \underbrace{(C + C')}_{1} + B' \cdot C \cdot \underbrace{(A + A')}_{1}$$
$$+ A \cdot \underbrace{(B + B')}_{1} \cdot \underbrace{(C + C')}_{1}$$

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## EE 2720, HW#3 solutions cont. 4

### Problem 2(f) cont.:

$$= A'\cdot B\cdot C + A'\cdot B\cdot C' + A\cdot B'\cdot C + A'\cdot B'\cdot C$$

$$+ A\cdot(B\cdot C + B\cdot C' + B'\cdot C + B'\cdot C')$$

$$= \underset{0\,1\,1}{A'\cdot B\cdot C} + \underset{0\,1\,0}{A'\cdot B\cdot C'} + \underset{1\,0\,1}{A\cdot B'\cdot C} + \underset{0\,0\,1}{A'\cdot B'\cdot C}$$

$$+ \underset{1\,1\,1}{A\cdot B\cdot C} + \underset{1\,1\,0}{A\cdot B\cdot C'} + \cancel{\underset{1\,0\,1}{A\cdot B'\cdot C}} + \underset{1\,0\,0}{A\cdot B'\cdot C'}$$

$$= \Sigma_{A,B,C}(1, 2, 3, 4, 5, 6, 7) = \text{maxterm } 0 = A+B+C$$

### Problem 3: $F = (a+b)\cdot(a+c)\cdot(b+c)$

$$= (a+b+\underbrace{c\cdot c'}_{0})\cdot(a+c+\underbrace{b\cdot b'}_{0})\cdot(b+c+\underbrace{a\cdot a'}_{0})$$

$$= (a+b+c)\cdot(a+b+c')\cdot(a+c+b)\cdot(a+c+b')$$

$$\cdot(b+c+a)\cdot(b+c+a') = \underset{0\,0\,0}{(a+b+c)}\cdot\underset{0\,0\,1}{(a+b+c')}$$

$$\cdot\cancel{\underset{0\,0\,0}{(a+b+c)}}\cdot\underset{0\,1\,0}{(a+b'+c)}\cdot\cancel{\underset{0\,0\,0}{(a+b+c)}}\cdot\underset{1\,0\,0}{(a'+b+c)}$$

$$= \Pi_{a,b,c}(0, 1, 2, 4)$$

Problem 4 : (a) Algebraic approach:

$$F = A\cdot B' + C'\cdot D + E'$$
$$= \left[(A\cdot B' + C'\cdot D + E')'\right]'$$
$$= \left[(A\cdot B')'\cdot(C'\cdot D)'\cdot E\right]' \quad (1)$$

From (1) we get the following figure

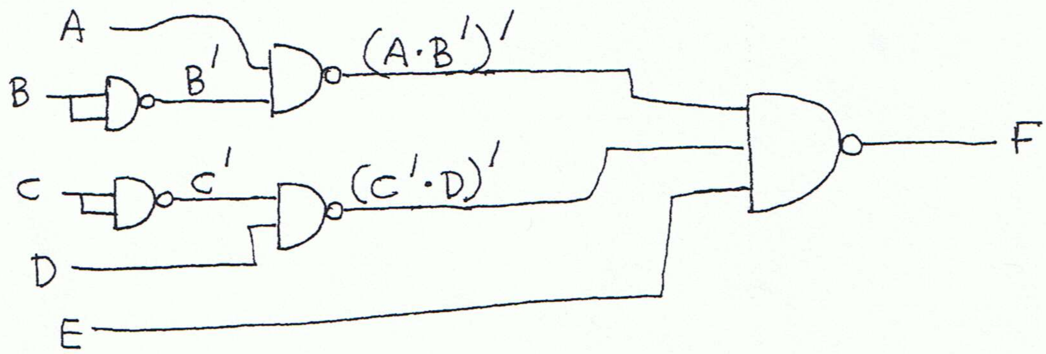


Figure 1

(b) Graphical approach: I first provide a logic circuit showing an AND-OR realization of F

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## Problem 4 cont:

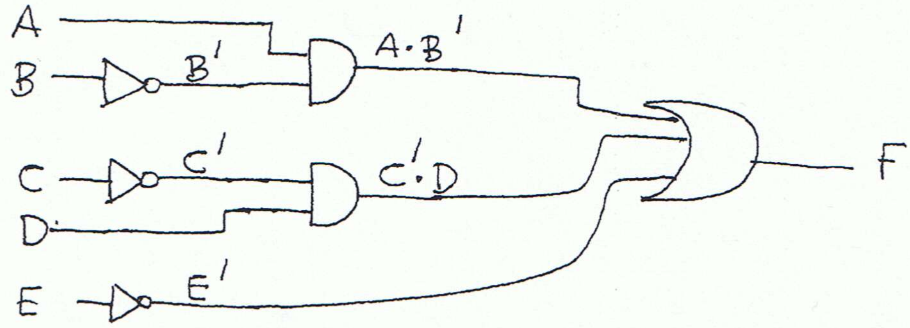


Figure 2

From the above fig. 2 one gets:

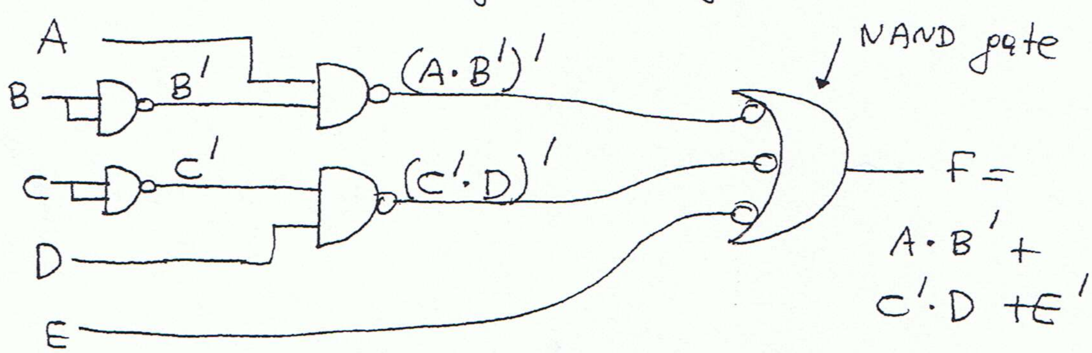


Figure 3

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Problem 5: (a) Algebraic approach:

$$F = (A+B')\cdot(C'+D)\cdot E'$$

$$= \left[\left[(A+B')\cdot(C'+D)\cdot E'\right]'\right]'$$

$$= \left[(A+B')' + (C'+D)' + E\right]' \quad (2)$$

From (2) we get the following figure.

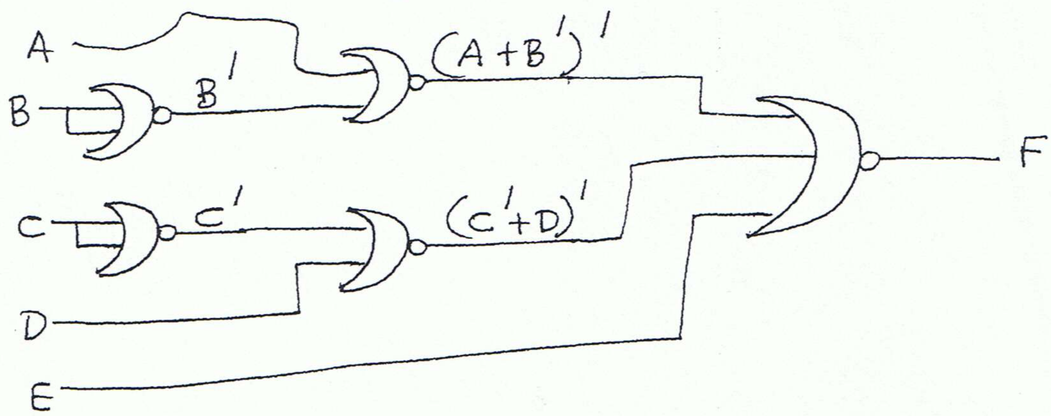


Figure 4

(b) Graphical approach: I first provide a logic circuit showing an OR-AND realization of F. This is shown on the next page

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## Problem 5 cont:

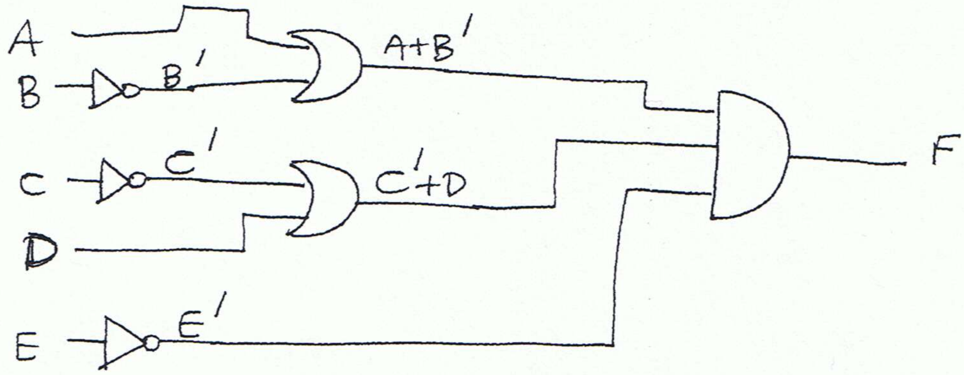
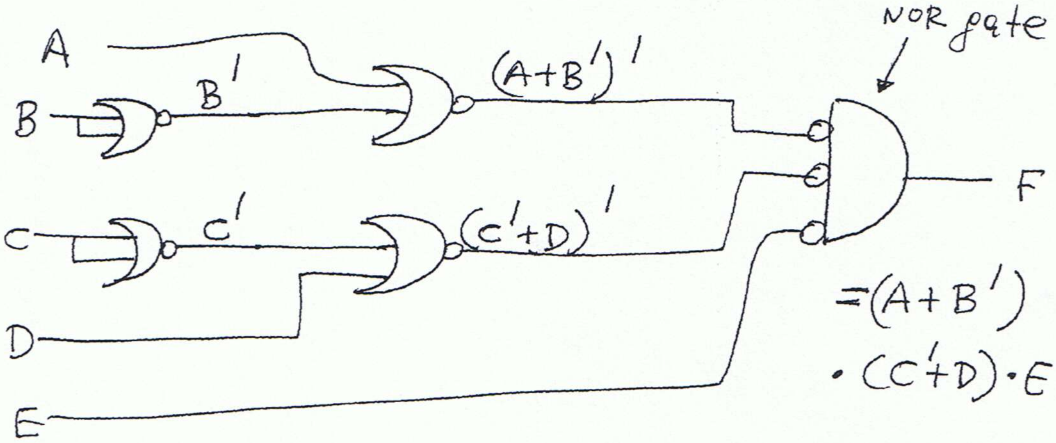



Figure 5

From the above figure 5 one gets:




$= (A+B') \cdot (C'+D) \cdot E'$

Figure 6

Problem 6:

(a) Proof of (5) or proof of $X \oplus 0 = X$

- Case $X = 0$: $0 \oplus 0 = 0$
- Case $X = 1$: $1 \oplus 0 = 1$

(b) Proof of (6) or proof of $X \oplus 1 = X'$

- Case $X = 0$: $0 \oplus 1 = 1$
- Case $X = 1$: $1 \oplus 1 = 0$

(c) Proof of (7) or proof of $X \oplus X = 0$

- Case $X = 0$: $0 \oplus 0 = 0$
- Case $X = 1$: $1 \oplus 1 = 0$

(d) Proof of (8) or proof of $X \oplus X' = 1$

- Case $X = 0$: $0 \oplus 1 = 1$
- Case $X = 1$: $1 \oplus 0 = 1$

Note: The above eqs. (5) − (8) can also be proven algebraically. How?

(e) Proof of (11) or proof of

$X \cdot (Y \oplus Z) = X \cdot Y \oplus X \cdot Z$

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Problem 6 ⓔ cont:

The right side of (11) is:

$$X\cdot Y \oplus X\cdot Z = X\cdot Y\cdot (X\cdot Z)' + (X\cdot Y)'\cdot X\cdot Z$$

$$= X\cdot Y\cdot (X'+Z') + (X'+Y')\cdot X\cdot Z$$

$$= \cancelto{0}{X\cdot Y\cdot X'} + X\cdot Y\cdot Z' + \cancelto{0}{X'\cdot X\cdot Z} + Y'\cdot X\cdot Z$$

$$= X\cdot Y\cdot Z' + X\cdot Y'\cdot Z = X\cdot (Y\cdot Z' + Y'\cdot Z)$$

$$= X\cdot (Y\oplus Z)$$ = left side of (11) $\Rightarrow$ proven.