

EE 2720, Fall 2010

Homework # 2 solution,

Note: Ignore solution of problem 5
since you didn't have to do
problem 5.

EE 2720 Homework # 2 solutions

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Problem 1:

$$\begin{array}{r} 011001 \leftarrow \text{positive number} \\ +) 011011 \leftarrow \text{positive number} \\ \hline 0110100 \leftarrow \text{negative number} \end{array}$$

overall carry out to be ignored \rightarrow sign bit = 1 \Rightarrow we got a negative result. Here an overflow occurred. Remember

ber that the Dynamic range (DR) of a 6-bit two's-complement system is $DR = [-2^5, 2^5 - 1] = [-32, +31]$ and $X + Y = +25 + 27 = +52 > 31$ which \Rightarrow overflow.

Problem 2:

$$\begin{array}{r} 101100 \leftarrow \text{negative number} \\ +) 110001 \leftarrow \text{negative " } \\ \hline 101101 \leftarrow \text{positive result} \end{array}$$

ignore carry out \rightarrow sign bit = 0 \Rightarrow positive result. Here an underflow occurred.

As said in prob. 1 the Dynamic Range DR is $[-32, +31]$ and $X + Y = (-20) + (-15) = -35 < -32 \Rightarrow$ underflow.

Problem 3: Here the two numbers X and Y are of different signs and we therefore have to perform (magnitude of X) - (magnitude of Y) = (10101) - (11111)

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$$= (10100) + (\text{two's compl. of } (11111))$$

$$= 10101$$

$$\begin{array}{r} +) 00001 \\ \hline 010110 \end{array}$$

$\hookrightarrow c=0 \Rightarrow \text{result} < 0$ or

(magnitude of X) - (magnitude of Y) < 0
 or magnitude of $X < \text{magnitude of } Y$

Thus

• sign bit of $X+Y$ = sign bit of $Y=1$

and magnitude of $X+Y$

$$= \text{two's compl. of } (10110) = 01010$$

$$\text{Therefore } X+Y = 101010_2 = -1010$$

Problem 4:

$$\begin{array}{r} 1100 \text{ multiplicand} \\ \times) 1111 \text{ multiplier} \\ \hline 0011 \\ 00110 \\ 001100 \\ 0011000 \\ \hline 1010100 \end{array} \rightarrow \text{product} = 180_{10}$$

Problem 5:

$$\begin{array}{r}
 1001 \text{ multiplicand} \\
 \times 1010 \text{ multiplier} \\
 \hline
 0000 \\
 + 1001 \\
 \hline
 110010 \\
 + 0000 \\
 \hline
 1110010
 \end{array}$$

ignore this since you didn't have to do Pro 5.

$$\begin{array}{r}
 1110010 \\
 + 0111 \leftarrow \text{shifted and negated multiplicand} \\
 \hline
 10101010
 \end{array}$$

ignore carry out

product = 0101010₂ = +42₁₀.

Problem 6:

$$\begin{array}{r}
 8 \\
 + 7 \\
 \hline
 15
 \end{array}$$

$$\begin{array}{r}
 1000 \\
 + 0111 \leftarrow \text{correction needed} \\
 \hline
 1111 \\
 + 0110 \\
 \hline
 10101 \\
 \hline
 1 \quad 5 \leftarrow \text{result} = 15
 \end{array}$$

Problem 7:

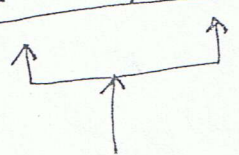
$$\begin{array}{r}
 3 \\
 + 4 \\
 \hline
 7
 \end{array}$$

$$\begin{array}{r}
 0011 \\
 + 0100 \leftarrow \text{no correction needed} \\
 \hline
 0111 \\
 \hline
 7 \leftarrow \text{result} = 7.
 \end{array}$$

Problem 8: All theorems are one-variable theorems. Just view two cases: $\begin{cases} \rightarrow X=0 \\ \rightarrow X=1 \end{cases}$ and apply the axioms. Proofs are trivial. See notes # 5 for similar proofs.

Problem 9:

X	Y	Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1



same identical columns
 $\Rightarrow (X+Y)+Z = X+(Y+Z)$
 and the theorem is proven.

Problem 10: (T10') states:

$$(X+Y) \cdot (X+Y') = X$$

Proof: $(X+Y) \cdot (X+Y') = X \cdot X + X \cdot Y' + Y \cdot X + Y \cdot Y'$
 $= X + X \cdot Y' + X \cdot Y + 0 = X + X \cdot Y' + X \cdot Y =$
 $X \cdot (1 + Y' + Y) = X \cdot 1 = X$

Problem 11: (T13') states:

$$(X_1 + X_2 + \dots + X_n)' = X_1' \cdot X_2' \cdot \dots \cdot X_n'$$

Proof: I'll first prove (T13') for $n=2$
 or I'll prove that $(X_1 + X_2)' = X_1' \cdot X_2'$ (1)
 I'll prove (1) using a truth table

$X_1 X_2$	$X_1 + X_2$	$(X_1 + X_2)'$	X_1'	X_2'	$X_1' \cdot X_2'$
00	0	1	1	1	1
01	1	0	1	0	0
10	1	0	0	1	0
11	1	0	0	0	0

↑
 identical column \Rightarrow
 eq. (1) true.

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Problem 11 cont. Assume now that theorem

(T13') is true for $n=i$, or assume that
 $(X_1 + X_2 + \dots + X_i)' = X_1' \cdot X_2' \cdot \dots \cdot X_i'$ (2)

We need to prove that the theorem is also true for $n=i+1$ or we need to

prove that

$$(X_1 + X_2 + \dots + X_i + X_{i+1})' = X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}'$$

But $(X_1 + X_2 + \dots + X_i + X_{i+1})'$

$$= [(X_1 + X_2 + \dots + X_i) + X_{i+1}]'$$

$$= (X_1 + X_2 + \dots + X_i)' \cdot X_{i+1}' \quad (\text{according to (1)})$$

$$= X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}' \quad (\text{according to (2)})$$

The proof is now completed.

Problem 12: $(X+Y) \cdot (X'+Z) = X \cdot X' + X \cdot Z$

$$+ Y \cdot X' + Y \cdot Z = 0 + X \cdot Z + X' \cdot Y + Y \cdot Z$$

$$= X \cdot Z + X' \cdot Y + Y \cdot Z$$

\swarrow consensus term and can be eliminated according to (T11)

$$= X \cdot Z + X' \cdot Y \Rightarrow \text{proven.}$$

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Problem 13: The left side of (T10) is

$X \cdot Y + X \cdot Y'$. This can be written as

$$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\substack{\uparrow \\ \text{consensus term}}}$$

$$= Y \cdot X + Y' \cdot X + X = X \cdot (Y + Y' + 1) = X \cdot 1 = X.$$

we now reached the right side of (T10) so the proof is completed

Problem 14:

$$(a) F = W \cdot X \cdot Y \cdot Z (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y' \cdot Z + W \cdot X \cdot Y \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z' + W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0.$$

$$(b) F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E = A \cdot B \cdot (1 + C' \cdot D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E = A \cdot B \cdot 1 + C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$$