EE 2729, fall 2011

Homework #4 solution

EE 2720, Homework # 4 Solutions

Problem 1: Let the inputs of the circuit be E_3, E_2, E_1, E_0 representing the Excess-3 number. Let the outputs of the circuit be B_3, B_2, B_1, B_0 representing the BCD number. Below I show the trut table.

E_3	E_2	E_1	E_0	B_3	B_2	B_1	B_0
0	0	0	0	d	d	d	d
0	0	0	1	d	d	d	d
0	0	1	0	d	d	d	d
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	1
1	1	0	1	d	d	d	d
1	1	1	0	d	d	d	d
1	1	1	1	d	d	d	d

} d means don't care (first three rows)

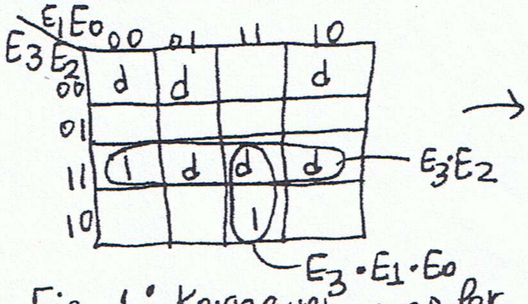
} d means don't care (last three rows)

From the above truth table, one can easily get the Karnaugh maps for the outputs B_3, B_2, B_1, B_0. These Karnaugh maps are shown on the next page.

EE 2720

HW# 4 Solutions cont.

Problem 1 cont:

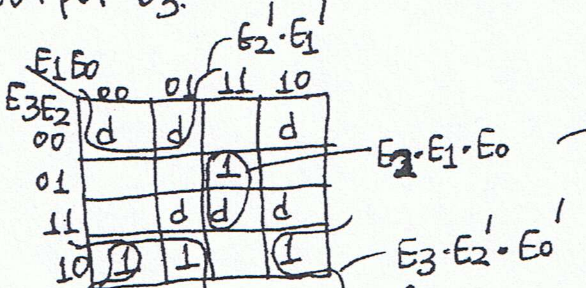


E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01				
11	1	d	d	d
10			1	

Fig. 1: Karnaugh map for output B_3.

→ From the Karnaugh map of fig. 1 we get the following expression for B_3:

$$B_3 = E_3 \cdot E_2 + E_3 \cdot E_1 \cdot E_0 \quad (1)$$

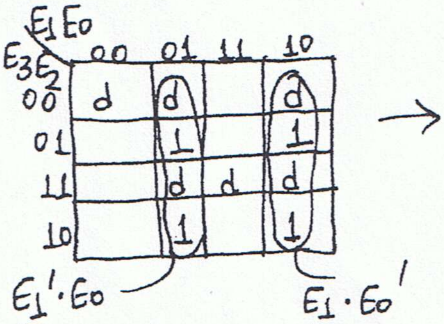


E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01			1	
11		d	d	d
10	1	1		1

Fig. 2: karnaugh map for output B_2.

→ From the karnaugh map of fig. 2 we get the following expression for B_2:

$$B_2 = E_2' \cdot E_1' + E_3 \cdot E_2' \cdot E_0' + E_2 \cdot E_1 \cdot E_0 \quad (2)$$



E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01		1		1
11		d	d	d
10		1		1

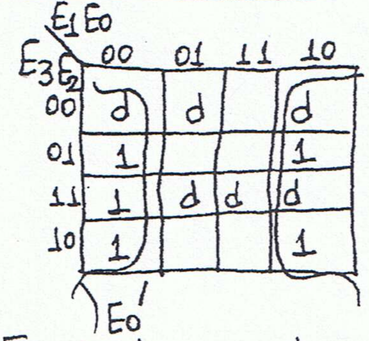
Fig. 3: Karnaugh map for output B_1

→ From the Karnaugh map of fig. 3 we get the following expression for B_1:

$$B_1 = E_1' \cdot E_0 + E_1 \cdot E_0' = E_1 \oplus E_0 \quad (3)$$

EE 2720 -
HW#4 Solutions cont.

Problem 1 cont:



E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01	1			1
11	1	d	d	d
10	1			1

From the Karnaugh map of fig. 4 we get the following expression for B_0:

$B_0 = E_0'$ (4).

Fig. 4: Karnaugh map for output B_0

From equations (1), (2), (3), (4) we get the following circuit shown in figure 5 below; (this is one possible realization).

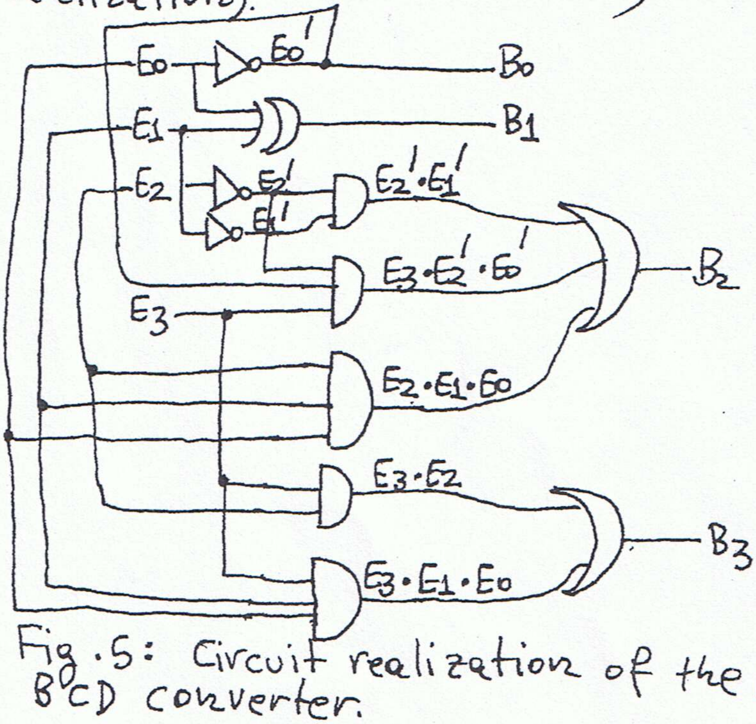


Fig. 5: Circuit realization of the Excess-3 to BCD converter.

EE 2720.

HW# 4 Solutions cont.

Problem 2: The inputs here are A, B, C and the outputs are X, Y, Z. Below I show the truth table.

ABC	XYZ
000	010
001	011
010	100
011	101
100	010
101	011
110	100
111	101

→ From the truth table, one can easily get the Karnaugh maps for the outputs X, Y, Z. These Karnaugh maps are shou below:

A \ BC	00	01	11	10
0			1	1
1			1	1

B

Fig. 1: Karnaugh map for output X

A \ BC	00	01	11	10
0	1	1		
1	1	1		

B'

Fig. 2: Karnaugh map for output Y

A \ BC	00	01	11	10
0		1	1	
1		1	1	

C

Fig. 3: Karnaugh map for output Z

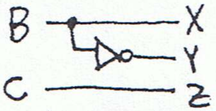
→ From the Karnaugh maps of figures 1, 2, 3 we get the following expressions for X, Y, Z:

$X = B$ (1)

$Y = B'$ (2)

$Z = C$ (3)

From equations (1), (2), (3) above we get the following circuit:



→ Note: As seen, the circuit is very simple; it relies only on one inverter

Fig. 4: Circuit realization for problem 2

Problem 3: Below I show the truth table.

X	Y	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

→ From the truth table, one can easily get the karnaugh maps for the outputs S, Cout. These karnaugh maps are shown below:

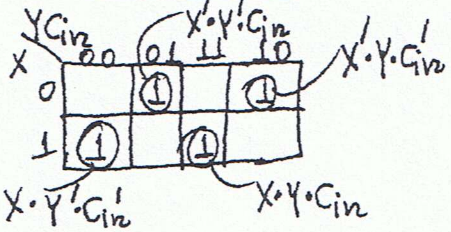


Fig. 1: Karnaugh map for output S

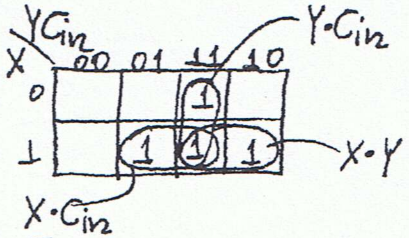


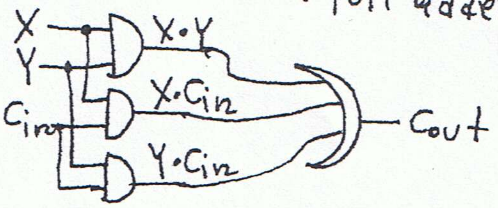
Fig. 2: karnaugh map for output Cout.

From the karnaugh maps of figures 1, 2 above, we get the following expressions for S and Cout:

$$S = X'\cdot Y'\cdot Cin + X'\cdot Y\cdot Cin' + X\cdot Y'\cdot Cin' + X\cdot Y\cdot Cin \quad (1)$$

$$Cout = X\cdot Y + X\cdot Cin + Y\cdot Cin \quad (2)$$

From equations (1), (2) above we get the following AND-OR realization for the full adder:



The rest of the figure showing the output S is on the next page.

EE 2720
HW#4 solutions cont.

Problem 3 cont:

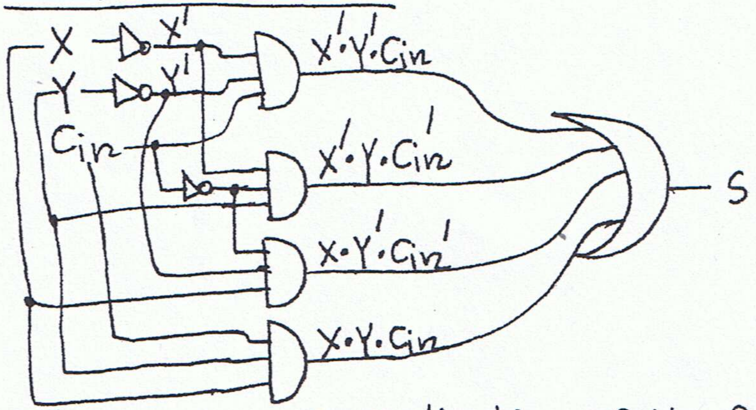


Fig. 3: AND-OR realization of the full adder.

Note: The expession of S provided in equation (1) cannot be simplified at all. It is a canonical sum actually; ($S=\Sigma_{X,Y,C_{in}}(1,2,4,7)$). As seen from the Karnaugh map of fig. 1, we cannot combine any cells containing 1's (the same is true for cells containing 0's). Question: Can you provide another expression for S?

Problem 4:

Below I show the truth table

X	Y	S	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

→ From the truth table, one can easily get the Karnaugh maps for the outputs S, Cout shown below:

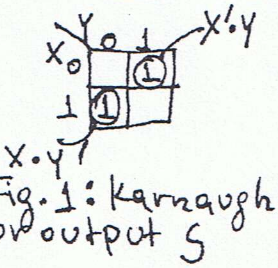


Fig. 1: Karnaugh map for output S

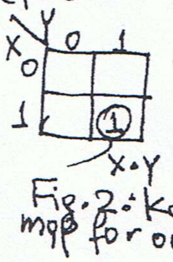


Fig. 2: Karnaugh map for output Cout

→ From the karnaugh maps of figures 1, 2 we get the expressions:

$S = X'\cdot Y + X\cdot Y'$ (1)

$C_{out} = X\cdot Y$ (2)

EE 2720

HW #4 Solutions cont.

Problem 4 cont: From equations (1), (2) of previous page, we get the following AND-OR realization for the half adder:

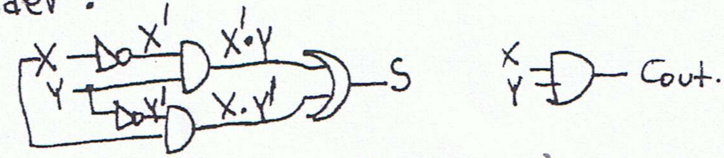


Fig. 3: AND-OR realization of the half adder.

Note: The expression of S provided by eq. (1) cannot be simplified at all. It is a canonical sum. As seen from the Karnaugh map of fig. 1, we cannot combine the two cells containing 1's; (they are not adjacent).

Note: This problem is so simple (trivial actually) that you don't need a truth table or Karnaugh map.

Problem 6:

wx \ yz	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

→ This function F can't be simplified at all. As seen from the Karnaugh map we can't combine any cells. So F is the given original expression which is a canonical sum, or F is

$F = \sum_{w,x,y,z}(1, 3, 4, 7, 8, 11, 13, 14)$. I hope you can now write an algebraic expression for F, but even if you give me this as an answer, it is ok.

Problem 7: The Karnaugh maps and the respective simplified sum-of-products expressions are shown on the next page.

Problem 7 cont:

(a),

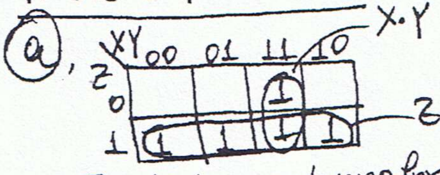


Fig. 1: karnaugh map for $F= \Sigma_{X,Y,Z}(1,3,5,6,7)$

From the karnaugh map of fig. 1 we get the following simplified sum-of-product expression for F:

$F= Z+X\cdot Y$ (1) ; (this is the minimal sum by the way)

(b).

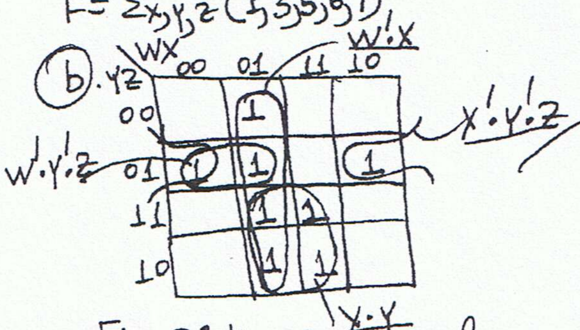


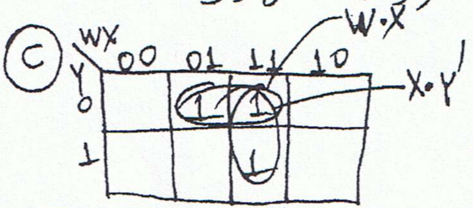
Fig. 2: karnaugh map for $F= \Sigma_{W,X,Y,Z}(1,4,5,6,7,9,14,15)$

From the karnaugh map of fig. 2 we get the following simplified sum-of-products expression for F.

$F=W'\cdot X+X'\cdot Y'\cdot Z+X\cdot Y+W'\cdot Y'\cdot Z$ (2)

; (this is not the minimal sum. Can you find the minimal sum?).

(c)



$F=\Pi_{W,X,Y}(0,1,3,4,5)=$
$=\Sigma_{W,X,Y}(2,6,7)$

Fig. 3: karnaugh map for $F=\Pi_{W,X,Y}(0,1,3,4,5)$

From the karnaugh map of fig. 3 we get the following simplified sum-of-products expression for F:

$F= W\cdot X+X\cdot Y'$ (3) ; (this is the minimal sum by the way).

↳ Go to next page →

EE 2730

HW # 4 Solutions cont.

Problem 7 cont:

ⓓ

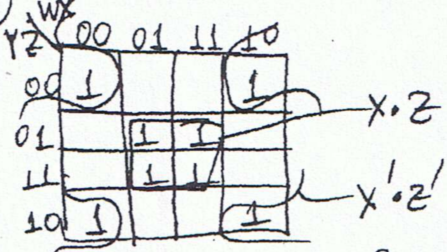
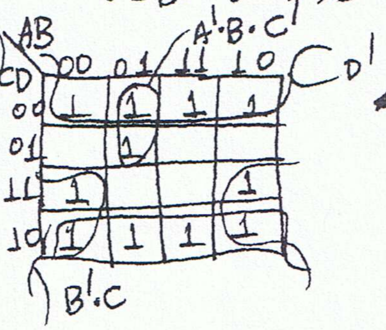


Fig. 4: karnaugh map for $F = \Sigma_{w,x,y,z}(0,3,5,7,8,10,13,15)$

From the karnaugh map of fig. 4, we get the following simplified sum-of-products expression for F:

$F = X \cdot Z + X' \cdot Z'$ (4); (this is the minimal sum by the way)

ⓔ



$F = \Pi_{A,B,C,D}(1,7,9,13,15) =$
$= \Sigma_{A,B,C,D}(0,2,3,4,5,6,8,10,11,12,$

Fig. 5: karnaugh map for $F = \Pi_{A,B,C,D}(1,7,9,13,15)$

From the karnaugh map of fig. 5, we get the following simplified sum-of-products expression for F:

$F = D' + B' \cdot C + A' \cdot B \cdot C'$ (5) (this is the minimal sum).

Problem 8:

ⓐ

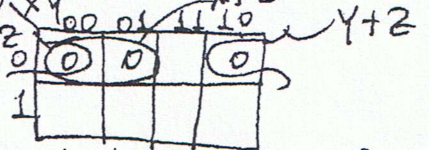


Fig. 1: karnaugh map for $F = \Sigma_{X,Y,Z}(1,3,5,6,7)$

From the karnaugh map of fig. 1 we get the following simplified product-of-sums expression for F

$F = (X+Z) \cdot (Y+Z)$ (1); (this is the minimal product by the way).

EE 2720
HW #4 Solutions cont.

Problem 8 cont:

ⓑ

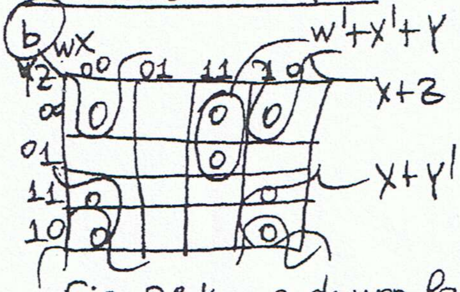


Fig. 2: Karnaugh map for $F=\Sigma_{w,x,y,z}(1,4,5,6,7,9,14,15)$

From the Karnaugh map of Fig. 2, we get the following simplified product-of-sums expression for F:

$F=(X+Z)\cdot(X+Y')\cdot(W'+X'+Y)$ (2);

(this is the minimal product by the way).

ⓒ

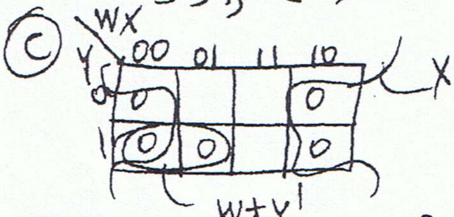


Fig. 3: Karnaugh map for $F=\Pi_{W,X,Y}(0,1,3,4,5)$

From the Karnaugh map of Fig. 3, we get the following simplified product-of-sums expression for F:

$F=X\cdot(W+Y')$ (3); (this is the minimal product)

ⓓ

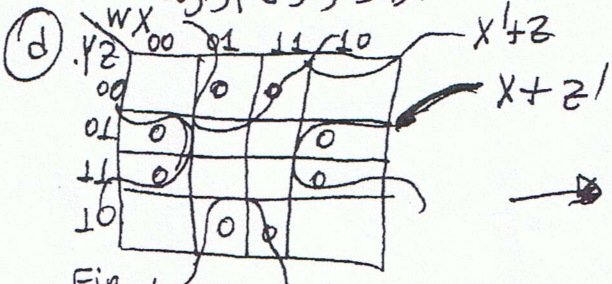


Fig. 4: Karnaugh map for $F=\Sigma_{w,x,y,z}(0,2,5,7,8,10,13,15)$

From the Karnaugh map of Fig. 4, we get the following simplified product-of-sums expression for F:

$F=(X'+Z)\cdot(X+Z')$ (4);

(this is the minimal product by the way).

EE 2720
HW # 4 Solutions cont.

Problem 8 cont:

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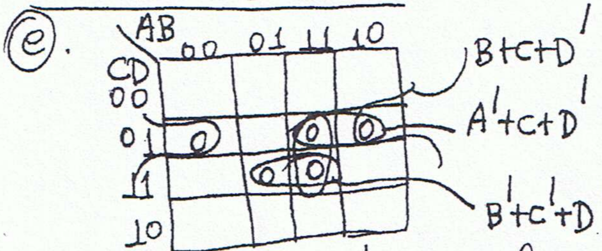

Fig. 5: karnaugh map for $F=\Pi_{A,B,C,D}(1,7,9,13,15)$

→ From the karnaugh map of fig. 5, we get the following simplified product-of-sums expression for F:
$F=(B+C+D')\cdot(B'+C'+D')\cdot(A'+C+D')$ (5); (this is a minimal product). You can also get another minimal product. Can you get it?

Problem 9:

ⓐ

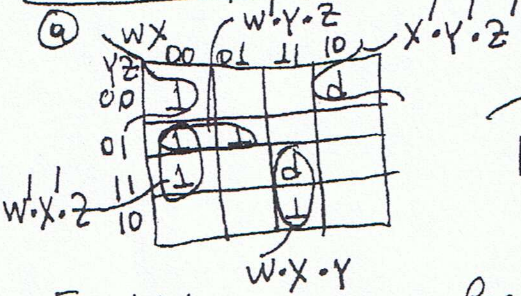

Fig. 1: karnaugh map for $F=\Sigma_{w,x,y,z}(0,1,3,5,14)+d(8,15)$

→ From the karnaugh map of fig. 1, we get the following simplified sum-of-products expression for F:
$F=w'\cdot y'\cdot z+w'\cdot x'\cdot z+$
$+w\cdot x\cdot y+x'\cdot y'\cdot z'$ (1);
(this is a minimal sum). You can also get one more minimal sum.

ⓑ

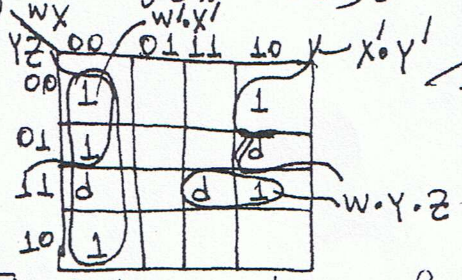

Fig. 2: karnaugh map for $F=\Sigma_{w,x,y,z}(0,1,2,8,11)+d(3,9,15)$.

→ From the karnaugh map of fig. 2 we get the following:
$F=w'\cdot x'+x\cdot y'+w\cdot y\cdot z$ (2).
(this is a minimal sum). You can get two more minimal sums. Can you get them?

EE 2720
HW #4 solutions cont.

Problem 9 cont:

ⓒ.

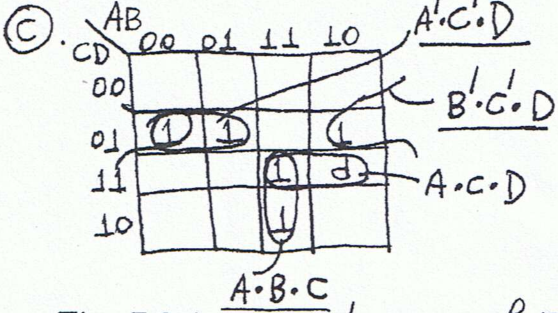

Fig. 3: Karnaugh map for $F=\Sigma_{A,B,C,D}(1,5,9,14,15)+d(11)$

→ From the Karnaugh map of fig. 3, we get the following:

$F = A\cdot B\cdot C + A'\cdot C'\cdot D + B'\cdot C'\cdot D + A\cdot C\cdot D$ (3);

(the above is <u>not</u> a minimal sum. Can you find a minimal sum?).

ⓓ.

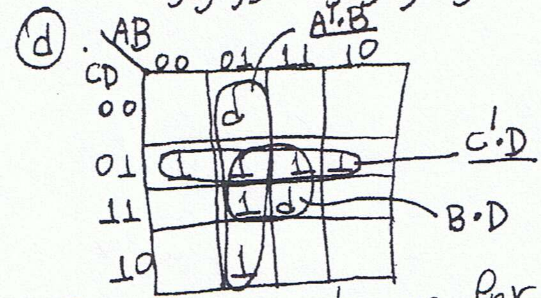

Fig. 4: Karnaugh map for $F=\Sigma_{A,B,C,D}(1,5,6,7,9,13)+d(4,15)$

→ From the Karnaugh map of fig. 4, we get the following:

$F = A'\cdot B + C'\cdot D + B\cdot D$ (4);

(the above is <u>not</u> a minimal sum. Can you find a minimal sum?).

Problem 10:

ⓐ.

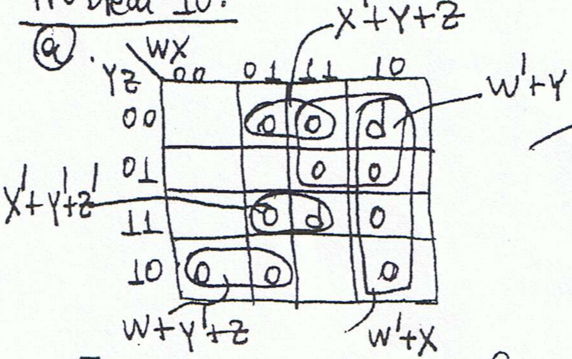

Fig. 1: Karnaugh map for $F=\Sigma_{W,X,Y,Z}(0,1,3,5,14)+d(8,15)$

→ From the Karnaugh map of fig. 1, we get the following:

$F=(X'+Y+Z)\cdot(W'+Y)\cdot(W'+X)\cdot(W+Y'+Z)\cdot(X'+Y'+Z')$ (1);

(the above is <u>not</u> a minimal product)

EE 2720

HW#4 Solutions cont.

Problem 10 cont:

ⓑ

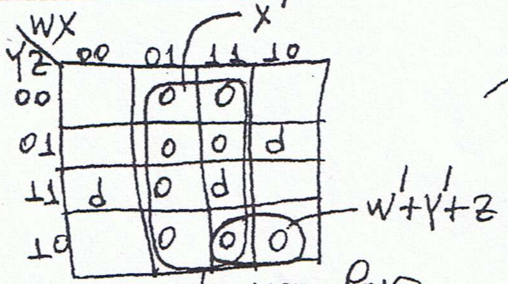

Fig. 2: karnaugh map for $F = \Sigma_{W,X,Y,Z}(0,1,2,8,11) + d(3,9,15)$

From the karnaugh map of fig. 2, we get the following

$F = X' \cdot (W'+Y'+Z)$ (2); (th

expression is a minimal product).

ⓒ

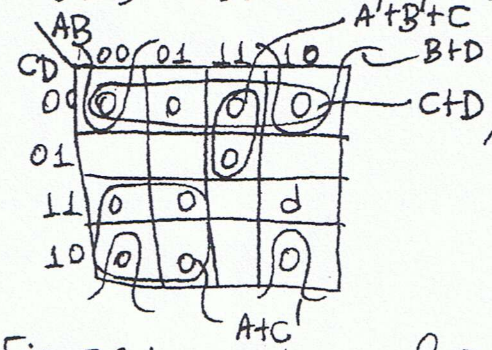

Fig. 3: karnaugh map for $F = \Sigma_{A,B,C,D}(1,5,9,14,15) + d(11)$

From the karnaugh map of fig. 3, we get the following:

$F = (A'+B'+C) \cdot (B+D) \cdot (C+D) \cdot (A+C')$ (3); (this is a minimal product).

ⓓ

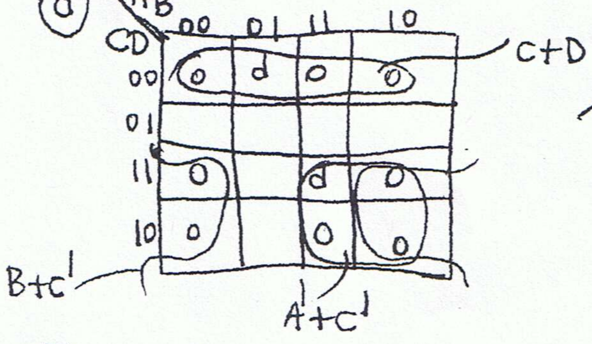

Fig. 4: karnaugh map for $F = \Sigma_{A,B,C,D}(1,5,6,7,9,13) + d(4,15)$

From the karnaugh map of fig. 4, we get the following:

$F = (C+D) \cdot (A'+C') \cdot (B+C')$ (4

; (the above expression is a minimal product).

You can get two more minimal products. Can you get them?

EE 2720

HW # 4 Solutions cont.

(14).

Problem 11:

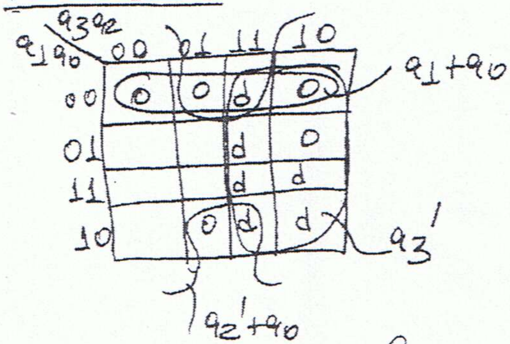

Fig. 1: Karnaugh map for the prime BCD-digit detector showing 0's and d's.

→ From the Karnaugh map of Fig. 1 we get the following:

$F = a_3' \cdot (a_2' + a_0) \cdot (a_1 + a$

(1); (the above i

a minimal product

Note: This HW was a little bit time consuming but I think it was lots of fun!! So I hope you enjoyed it!! [redacted]