EE 2720, Fall 2011

Homework # 2 solutions

Note: Ignore solution of problem 5 since you didn't have to do problem 5.

EE 2720 Homework # 2 solutions

Problem 1:

```
          011001 ← positive number
       +) 011011 ← positive number
       ---------
         0110100 ← negative number
         ↑└→ sign bit=1 ⇒ we got a
overal ──┘   negative result. Here an
carry out    overflow occured.
to be ignored
```

Remember that the Dynamic range (DR) of a 6-bit two's-complement system is $DR = [-2^5 \quad (2^5-1)] = [-32 \quad +31]$ and $X+Y = +25+27 = +52 > 31$ which $\Rightarrow$ overflow.

Problem 2:

```
          101100 ← negative number
       +) 110001 ← negative   "
       ---------
         1011101 ← positive result
         └→ sign bit=0 ⇒ positive result
ignore ──┘  Here an underflow occured.
carry out
```

As said in prob. 1 the Dynamic Range DR is $[-32 \quad +31]$ and $X+Y = (-20)+(-15)$

$= -35 < -32) \Rightarrow$ underflow.

Problem 3: Here the two numbers X and Y are of different signs and we therefore have to perform (magnitude of X) − (magnitude of Y)

$= (10101) - (11111)$

NEXT PAGE →

= (10100) + (two's compl. of (11111))

$$
\begin{array}{r}
= \quad 10101 \\
+)\ 00001 \\
\hline
0\ 10\ 110
\end{array}
$$

↳ c = 0 ⇒ result < 0 or

(magnitude of X) − (magnitude of Y) < 0

or magnitude of X < magnitude of Y

Thus

- sign bit of X+Y = sign bit of Y = 1

  and magnitude of X+Y

  = two's compl. of (10110) = 01010

Therefore $X+Y = 101010_2 = -10_{10}$

Problem 4:

$$
\begin{array}{rl}
1100 & \text{multiplicand} \\
\times)\ 1111 & \text{multiplier} \\
\hline
1100 & \\
+)\ 1100 & \\
\hline
100100 & \\
+)\ 1100 & \\
\hline
1010100 & \\
+)\ 1100 & \\
\hline
10110100 & \rightarrow \text{product} = 180_{10}
\end{array}
$$

Problem 5:

```
          1001      multiplicand
        x)1010      multiplier
          0000
       +) 1001
        110010
      +)0000
       1110010
    +) 0111   <- shifted and negated multiplicand
   (1)0101010
```

ignore carry out

↳ product = $0101010_2 = +42_{10}$.

ignore this since you didn't have to do Pr. 5.

Problem 6:

$$\begin{array}{r} 8 \\ +\ 7 \\ \hline 15 \end{array}$$

```
     1000
  +) 0111
     1111   <- correction needed
  +) 0110
    10101
```

$\underbrace{1}_{1}\ \underbrace{0101}_{5}$ ← result = 15

Problem 7:

$$\begin{array}{r} 3 \\ +\ 4 \\ \hline 7 \end{array}$$

```
     0011
  +) 0100
     0111   <- no correction needed
```

$\underbrace{0111}_{7}$ ← result = 7.

Problem 8: All theorems are one-variable theorems. Just view two cases: $X=0$ and $X=1$ and apply the axioms. Proofs are trivial. See notes # 5 for similar proofs.

Problem 9:

X Y Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0 0 0	0	0	0	0
0 0 1	0	1	1	1
0 1 0	1	1	1	1
0 1 1	1	1	1	1
1 0 0	1	0	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

same identical columns

$\Rightarrow (X+Y)+Z = X+(Y+Z)$

and the theorem is proven.

Problem 10: (T10') states:

$$(X+Y)\cdot(X+Y')=X$$

Proof: $(X+Y)\cdot(X+Y') = X\cdot X + X\cdot Y' + Y\cdot X + Y\cdot Y'$

$= X + X\cdot Y' + X\cdot Y + 0 = X + X\cdot Y' + X\cdot Y =$

$X\cdot(1+Y'+Y) = X\cdot 1 = X$

Problem 11: (T13') states:

$(X_1+X_2+\dots+X_n)' = X_1'\cdot X_2'\cdot\dots\cdot X_n'$

Proof: I'll first prove (T13') for $n=2$

or I'll prove that $(X_1+X_2)' = X_1'\cdot X_2'$ (1)

I'll prove (1) using a truth table

$X_1 X_2$	X_1+X_2	$(X_1+X_2)'$	X_1'	X_2'	$X_1'\cdot X_2'$
00	0	1	1	1	1
01	1	0	1	0	0
10	1	0	0	1	0
11	1	0	0	0	0

identical column $\Rightarrow$

eq. (1) true.

Problem 11 cont. Assume now that theorem (T13') is true for $n=i$, or assume that

$$(X_1+X_2+\cdots+X_i)' = X_1'\cdot X_2'\cdot\cdots X_i' \quad (2)$$

we need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1+X_2+\cdots X_i+X_{i+1})' = X_1'\cdot X_2'\cdots X_i'\cdot X_{i+1}'$$

But $(X_1+X_2+\cdots X_i+X_{i+1})'$

$$= [(X_1+X_2+\cdots+X_i)+X_{i+1}]'$$

$$= (X_1+X_2+\cdots+X_i)'\cdot X_{i+1}' \quad \text{(according to (1))}$$

$$= X_1'\cdot X_2'\cdots X_i'\cdot X_{i+1}' \quad \text{(according to (2))}$$

The proof is now completed.

Problem 12: $(X+Y)\cdot(X'+Z) = X\cdot X' + X\cdot Z + Y\cdot X' + Y\cdot Z = 0 + X\cdot Z + X'\cdot Y + Y\cdot Z$

$= X\cdot Z + X'\cdot Y + Y\cdot Z$ → consensus term and can be eliminated according to (T11)

$= X\cdot Z + X'\cdot Y \Rightarrow$ proven.

Problem 13: The left side of (CT10) is

$X \cdot Y + X \cdot Y'$. This can be written as

$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}$ (according to (CT11))

consensus term

$= Y \cdot X + Y' \cdot X + X = X \cdot (Y + Y' + 1) = X \cdot 1 = X$. we

now reached the right side of (CT10) so the

proof is completed

Problem 14:

a) $F = W \cdot X \cdot Y \cdot Z \cdot (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z$

$+ W \cdot X \cdot Y' \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z'$

$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W' \cdot X \cdot Y \cdot Z$

$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0$.

e) $F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E$

$+ C' \cdot D \cdot E$

$= A \cdot B \cdot (1 + C' \cdot D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E = A \cdot B \cdot 1$

$+ C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$