EE 2720, Fall 03

Solutions of HW# 4

Problem 1: The left side of (T10) is: $X \cdot Y + X \cdot Y'$. This can be written as

$$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\text{consensus term}} \quad \text{(according to (T11))}$$

$$= Y \cdot X + Y' \cdot X + X = Y \cdot X + Y' \cdot X + X \cdot 1 = X \cdot (1 + Y + Y') =$$

$= X \cdot 1 = X$. We now reached the right side of (T10) so the proof is completed.

Problem 2:

(a) $F = W \cdot X \cdot Y \cdot Z \cdot (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z + W \cdot X \cdot Y' \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z' + W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W' \cdot X \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0$

(b) $F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$

$= A \cdot B \cdot 1 + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$

$= A \cdot B \cdot (1 + C' \cdot D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E =$

$= A \cdot B \cdot 1 + C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$

(c) $F = M \cdot N \cdot O + Q' \cdot P' \cdot N' + P \cdot R \cdot M + Q' \cdot O \cdot M \cdot P' + M \cdot R =$

$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R + P \cdot R \cdot M$

$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R + M \cdot R \cdot P =$

$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot 1 + M \cdot R \cdot P =$

$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot (1 + P) =$

$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot 1 =$

$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R =$

$$= N \cdot (M \cdot O) + N' \cdot (Q' \cdot P') + \underbrace{(M \cdot O) \cdot (Q' \cdot P')}_{\text{consensus term and can be eliminated; (T11).}} + M \cdot R =$$

Problem 2(c) cont:

$= N\cdot(M\cdot O) + N'\cdot(Q'\cdot P') + M\cdot R =$ (applying (T11)).

$= N\cdot M\cdot O + N'\cdot Q'\cdot P' + M\cdot R$

Problem 3: Apply (T8') first to get:

$(A+B+C')\cdot(A'+B'+D)\cdot(A'+C+D')\cdot(A+C'+D) =$

$= (A+C'+B)\cdot(A+C'+D)\cdot(A'+B'+D)\cdot(A'+C+D') =$

$= (A+C'+B\cdot D)\cdot[A'+(B'+D)\cdot(C+D')] =$

$= (A+C'+B\cdot D)\cdot[A'+(D+B')\cdot(D'+C)]$.

Now apply theorem of eq. (1) on $(D+B')\cdot(D'+C)$ to get

$(A+C'+B\cdot D)\cdot(A'+D\cdot C+D'\cdot B')$; (apply theorem of eq. (1) again)

$= A\cdot(D\cdot C+D'\cdot B') + A'\cdot(C'+B\cdot D)$; (apply (T8))

$= A\cdot D\cdot C + A\cdot D'\cdot B' + A'\cdot C' + A'\cdot B\cdot D$

If we were to multiply out using only theorem (T8), we would generate $3\times3\times3\times3 = 81$ product terms and would have to eliminate 77 of them!!!; (too much trouble!). Here I only got 4 terms

Problem 4: Apply (T8) first to get:

$W\cdot X\cdot Y' + W'\cdot X'\cdot Z + W\cdot Y'\cdot Z + W'\cdot Y\cdot Z' =$

$= W\cdot Y'\cdot X + W\cdot Y'\cdot Z + W'\cdot X'\cdot Z + W'\cdot Y\cdot Z' =$

$= W\cdot Y'\cdot(X+Z) + W'\cdot(X'\cdot Z + Y\cdot Z')$

Apply now theorem of eq. (1) to get:

EE 2720, Fall 03
Solutions of HW# 4 cont.

Problem 4 cont:

$W\cdot Y'\cdot(X+Z)+W'\cdot(X'\cdot Z+Y\cdot Z')=$

$=(W+X'\cdot Z+Y\cdot Z')\cdot[W'+Y'\cdot(X+Z)]=$ (apply theorem of eq.(1) on $Z\cdot X'+Z'\cdot Y$)

$=(W+Z\cdot X'+Z'\cdot Y)\cdot[W'+Y'\cdot(X+Z)]$

$=[W+(Z+Y)\cdot(Z'+X')]\cdot[W'+Y'\cdot(X+Z)]$ (apply theorem of eq.(1) again)

$=W\cdot Y'\cdot(X+Z)+W'\cdot(Z+Y)\cdot(Z'+X')$ (apply theorem of eq.(1) again)

$=[W+(Z+Y)\cdot(Z'+X')]\cdot[W'+Y'\cdot(X+Z)]$ (apply theorem (T8'))

$=(W+Z+Y)\cdot(W+Z'+X')\cdot(W'+Y')\cdot(W'+X+Z)$

Problem 5: I will first provide the canonical sum and then the canonical product for each logic function.

(a). $F=\Sigma_{X,Y}(1,2)=X'\cdot Y+X\cdot Y'=\Pi_{X,Y}(0,3)=(X+Y)\cdot(X'+Y')$

(b) $F=\Pi_{A,B}(0,1,2)=$ minterm $3=A\cdot B=\Pi_{A,B}(0,1,2)=$

$=(A+B)\cdot(A+B')\cdot(A'+B)$

(c) $F=\Sigma_{A,B,C}(2,4,6,7)=A'\cdot B\cdot C'+A\cdot B'\cdot C'+A\cdot B\cdot C'+A\cdot B\cdot C$

$=\Pi_{A,B,C}(0,1,3,5)=(A+B+C)\cdot(A+B+C')\cdot(A+B'+C')\cdot$
$\cdot(A'+B+C')$

(d) $F=\Pi_{W,X,Y}(0,1,3,4,5)=\Sigma_{W,X,Y}(2,6,7)=$

$=W'\cdot X\cdot Y'+W\cdot X\cdot Y'+W\cdot X\cdot Y=\Pi_{W,X,Y}(0,1,3,4,5)$

$=(W+X+Y)\cdot(W+X+Y')\cdot(W+X'+Y')\cdot(W'+X+Y)\cdot(W'+X+Y')$

(e) $F=X+Y'\cdot Z'=X\cdot(\underbrace{Y+Y'}_{1})\cdot(\underbrace{Z+Z'}_{1})+Y'\cdot Z'\cdot(\underbrace{X+X'}_{1})=$

$=X\cdot(Y\cdot Z+Y\cdot Z'+Y'\cdot Z+Y'\cdot Z')+Y'\cdot Z'\cdot X+Y'\cdot Z'\cdot X'$

$=\underset{111}{X\cdot Y\cdot Z}+\underset{110}{X\cdot Y\cdot Z'}+\underset{101}{X\cdot Y'\cdot Z}+\underset{100}{X\cdot Y'\cdot Z'}+\cancel{\underset{100}{X\cdot Y'\cdot Z'}}+\underset{000}{X'\cdot Y'\cdot Z'}$

$=\Sigma_{X,Y,Z}(0,4,5,6,7)=\Pi_{X,Y,Z}(1,2,3)=(X+Y+Z')\cdot(X+Y'+Z)\cdot$
$\cdot(X+Y'+Z')$.

Problem 5 cont:

(f) $F = v' + (w' \cdot x)' = v' + w + x' = \text{maxterm } 5 = \Sigma_{v,w,x}(0,1,2,3,4,6,7).$

$= v' \cdot w' \cdot x' + v' \cdot w' \cdot x + v' \cdot w \cdot x' + v' \cdot w \cdot x + v \cdot w' \cdot x' + v \cdot w \cdot x' + v \cdot w \cdot x = \text{maxterm } 5 = v' + w + x'$

Problem 6: $F = (a+b) \cdot (a+c) \cdot (b+c) =$

$= (a + b + \underbrace{c \cdot c'}_{0}) \cdot (a + c + \underbrace{b \cdot b'}_{0}) \cdot (b + c + \underbrace{a \cdot a'}_{0})$

$= (a+b+c) \cdot (a+b+c') \cdot (\cancel{a+c+b}) \cdot (a+c+b') \cdot$

$\cdot (\cancel{b+c+a}) \cdot (b+c+a') =$

$= (a+b+c) \cdot (a+b+c') \cdot (a+b'+c) \cdot (a'+b+c)$

$= \prod_{a,b,c}(0,1,2,4).$

Note: Here I applied theorem (T8') several times.