EE 2720, Fall 03
Solutions of HW#3

Problem 1:

Case $X=0$

• $0\cdot1=0$

Case $X=1$

• $1\cdot1=1$

Problem 2:

Case $X=0$

— $0\cdot0=0$

Case $X=1$

— $1\cdot0=0$

Problem 3:

Case $X=0$

— $0+0=0$

Case $X=1$

— $1+1=1$

Problem 4:

Case $X=0$

— $0\cdot0=0$

Case $X=1$

— $1\cdot1=1$

Problem 5:

Case $X=0$

— $(0')'=(1)'=0$

Case $X=1$

— $(1')'=(0)'=1$

Problem 6:

Case $X=0$

— $0 \cdot 0' = 0 \cdot 1 = 0$

Case $X=1$

— $1 \cdot 1' = 1 \cdot 0 = 0$

Problem 7:

The truth table is shown below:

X Y Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0 0 0	0	0	0	0
0 0 1	0	1	1	1
0 1 0	1	1	1	1
0 1 1	1	1	1	1
1 0 0	1	0	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

Looking at the two right most columns of the above truth table, we conclude that $(X+Y)+Z$ and $X+(Y+Z)$ are equal for all possible combinations of values of the variables X, Y, Z. Therefore theorem (T7) is valid and the proof is completed.

Problem 8:

$$(X+Y)\cdot(X+Y') = X\cdot X + X\cdot Y' + Y\cdot X + Y\cdot Y' = X + X\cdot Y' + Y\cdot X + 0$$
$$= X + X\cdot Y' + X\cdot Y = X\cdot(1+Y'+Y) = X\cdot 1 = X$$

Problem 9:

The right side of (T11') is

$$(X+Y)\cdot(X'+Z) = X\cdot Z + X'\cdot Y \quad (1)$$

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Solutions of HW# 3 cont.

Problem 9 cont:

The left side of (T11') is

$$(X+Y)\cdot(X'+Z)\cdot(Y+Z)=(X\cdot Z+X'\cdot Y)\cdot(Y+Z)=$$
$$= X\cdot Z\cdot Y + X\cdot Z\cdot Z + X'\cdot Y\cdot Y + X'\cdot Y\cdot Z = X\cdot Y\cdot Z + X\cdot Z +$$
$$+ X'\cdot Y + X'\cdot Y\cdot Z = X\cdot Z + X'\cdot Y + X\cdot Y\cdot Z + X'\cdot Y\cdot Z =$$
$$= X\cdot Z + X'\cdot Y + Y\cdot Z\cdot(X+X') = X\cdot Z + X'\cdot Y + Y\cdot Z\cdot 1 =$$
$$= X\cdot Z + X'\cdot Y + Y\cdot Z =$$

consensus term and can be eliminated according to (T11)

$$= X\cdot Z + X'\cdot Y \quad (2)$$

Because both right and left side reduced to the same expression (which is $X\cdot Z + X'\cdot Y$), theorem (T11') is valid.

Problem 10: I'll first prove that theorem (T13') is true for $n=2$ or I'll prove that $(X_1+X_2)'=X_1'\cdot X_2'$ (1). I'll prove eq. (1) using a truth table shown below:

$X_1\ X_2$	X_1+X_2	$(X_1+X_2)'$	X_1'	X_2'	$X_1'\cdot X_2'$
0 0	0	1	1	1	1
0 1	1	0	1	0	0
1 0	1	0	0	1	0
1 1	1	0	0	0	0

Looking at the 3rd and 6th column of the above truth table, we see that $(X_1+X_2)'$ and $X_1'\cdot X_2'$ are equal for all possible combinations of values

Problem 10 cont: of the variables X_1 and X_2. Therefore eq. (1) is true and theorem (T13') holds true for $n=2$.

Assume now that theorem (T13') is true for $n=i$ or assume that

$$(X_1+X_2+\cdots+X_i)' = X_1'\cdot X_2'\cdot\ldots X_i' \quad (2)$$

We need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1+X_2+\cdots+X_i+X_{i+1})' = X_1'\cdot X_2'\cdot\ldots\cdot X_i'\cdot X_{i+1}'$$

But $(X_1+X_2+\cdots X_i+X_{i+1})' = [(X_1+X_2+\cdots+X_i)+X_{i+1}]' =$

$= (X_1+X_2+\cdots+X_i)'\cdot X_{i+1}'$ according to (1)

$= X_1'\cdot X_2'\cdot\ldots\cdot X_i'\cdot X_{i+1}'$ according to (2)

The proof is now completed and the theorem is true for all values of n.

Problem 11: $(X+Y)\cdot(X'+Z) = X\cdot X'+X\cdot Z+Y\cdot X'+Y\cdot Z =$

$= 0+X\cdot Z+X'\cdot Y+Y\cdot Z = X\cdot Z+X'\cdot Y+\underbrace{Y\cdot Z}_{\text{consensus term and can be eliminated according to (T11)}} =$

$= X\cdot Z+X'\cdot Y.$