EE 2720, Spring 05

Solutions of Homework # 2

EE 2720, Solutions of HW # 2 ①

Problem 1: $DR = [-(2^6-1) \quad 2^6-1] = [-63 \quad 63]$

Problem 2: $+51_{10} = 0110011_2$

Problem 3: $-45_{10} = 1101101_2$

Problem 4: One way is the following. Here $r=10$ and $n=5$. So 10's-complement of 57932 $= 10^5 - 57932 = 100000 - 57932 = 42068$

For the other way we have $r=10$ so $r-1=9$.

The digit	5	becomes		$9-5=4$
"	"	7	"	$9-7=2$
"	"	9	"	$9-9=0$
"	"	3	"	$9-3=6$
"	"	2	"	$9-2=7$

So 10's complement of $57932 = 42067 + 1 = 42068$.

Problem 5: $DR = [-2^9 \quad 2^9-1] = [-512 \quad 511]$.

Problem 6: $101011011 = -2^8 + 2^6 + 2^4 + 2^3 + 2^1 + 2^0 =$
$= -256 + 64 + 16 + 8 + 2 + 1 = -165$

Problem 7: $X = 101010110$

↓ complementing bits

$$
\begin{array}{r}
010101001 \\
+)\quad\quad\quad 1 \\
\hline
010101010
\end{array}
$$

Problem 8:

$$
\begin{array}{r}
1 \leftarrow C_{in} \\
1011010 \\
+)\ 1111001 \\
\hline
1\,1010100
\end{array}
$$

ignore → ①

$\hookrightarrow X-Y = 1010100 = -44$

Problem 9:

$$
\begin{array}{rl}
0111001 & \leftarrow \text{positive number} \\
+)\ 0010010 & \leftarrow \quad " \qquad\qquad " \\
\hline
1001011 & \leftarrow \text{negetive result; (wrong).}
\end{array}
$$

Sign bit $=1 \Rightarrow$ we got a negative result $\Rightarrow$ overflow occured. See that $DR = [-2^6 \quad 2^6-1] = [-64 \quad 63]$ and $X+Y = 57+18 = 75 > 63$ which implies overflow

Problem 10:

$$
\begin{array}{rl}
1000111 & \leftarrow \text{negative number} \\
+)\ 1101110 & \leftarrow \quad " \qquad\qquad " \\
\hline
1\,0110101 & \leftarrow \text{positive result; (wrong)}
\end{array}
$$

ignore → ①

Sign bit $=0 \Rightarrow$ we got a positive result $\Rightarrow$ underflow occured. See that $DR = [-2^6 \quad 2^6-1] = [-64 \quad 63]$ and $X+Y = (-57)+(-18) = -75 < -64$ which implies underflow.

Problem 11: One way is the following. Here $r=10$ and $n=5$. So 9's-complement of 59725

$= 10^5-1-59725 = 100000-1-59725 = 40274$.

For the other way we have $r=10 \Rightarrow r-1=9$

The digit 5 becomes $9-5=4$
" " 9 " $9-9=0$
" " 7 " $9-7=2$
" " 2 " $9-2=7$
" " 5 " $9-5=4$

So 9's-complement of $59725 = 40274$.

Problem 12: $DR = [-(2^8-1) \quad 2^8-1] = [-255 \quad 255]$

Problem 13: $101011101_2 = -(2^8-1)+2^6+2^4+2^3+2^2+2^0 = -255+64+16+8+4+1 = -162$

Problem 14: $X = 101101101$

$\downarrow$ complementing bits

010010010

Problem 15:

$$\begin{array}{r} 1010010 \\ +)\ 1111001 \\ \hline ①1001011 \end{array} \qquad \begin{array}{r} 1001011 \\ +)\qquad\ \ 1 \\ \hline \underbrace{1001100} \end{array}$$

Carry out

$\hookrightarrow X-Y = 1001100 = -51$

Problem 16:

$$
\begin{array}{rl}
 & 0111001 \leftarrow \text{positive number} \\
+) & 0010010 \leftarrow \quad \text{"} \qquad \text{"} \\
\hline
 & 1001011 \leftarrow \text{negative result; (wrong)}
\end{array}
$$

sign bit $=1 \Rightarrow$ we got a negative result $\Rightarrow$ overflow occured. See that $DR = [-(2^6-1) \quad 2^6-1] = [-63 \quad 63]$ and $X+Y = 57+18 = 75 > 63$ which implies overflow.

Problem 17:

$$
\begin{array}{rl}
 & 1000111 \leftarrow \text{negative number} \\
+) & 1101110 \leftarrow \quad \text{"} \qquad \text{"} \\
\hline
 & 1\,0110101
\end{array}
$$

carry out

$$
\begin{array}{rl}
 & 0110101 \\
+) & 1 \\
\hline
 & 0110110 \leftarrow \text{positive result; (wrong)}
\end{array}
$$

sign bit $=0 \Rightarrow$ we got a positive result $\Rightarrow$ underflow occured. See that $DR = [-(2^6-1) \quad 2^6-1] = [-63 \quad 63]$ and $X+Y = (-56)+(-17) = -73 < -63$ which implies underflow.

Problem 18: Here X and Y are of different signs and X+Y needs to be performed. We thus perform $\longrightarrow$ Next page $\longrightarrow$

Problem 18 cont.:

(magnitude of X) − (magnitude of Y) =

$10100 - 11101 = 10100 + 2\text{'s compl. of } (11101)$

$= 10100 + 00011 =$

$$
\begin{array}{r}
10100 \\
+)\ 00011 \\
\hline
010111
\end{array}
$$

↳ carry out = 0 ⇒ (mag. of X) − (mag. of Y) < 0

⇒ mag. of X < mag. of Y.

Therefore sign bit of X + Y = sign bit of Y = 1.

Magnitude of X + Y = two's compl. of (10111)

= 01001

So the result is $X + Y = 101001_2 =$

$= -9_{10}$.

EE 2720, Sol. of HW#2 cont.

Problem 19:

$$
\begin{array}{rr}
 & 10101 \\
\times) & 10111 \\
\hline
 & 10101 \\
+) & 10101 \\
\hline
 & 111111 \\
+) & 10101 \\
\hline
 & 10010011 \\
+) & 00000 \\
\hline
 & 10010011 \\
+) & 10101 \\
\hline
 & \underbrace{111100011}
\end{array}
$$

$\hookrightarrow$ product $= 111100011 = 483 =$
$= 21 \times 23$

Problem 20:

$$
\begin{array}{rr}
 & 1001 \\
\times) & 1010 \\
\hline
 & 00000 \\
+) & 1001 \\
\hline
 & 110010 \\
+) & 0000 \\
\hline
 & 1110010 \\
+) & 0111 \\
\hline
 & 1\,\underbrace{0101010}
\end{array}
$$

$\longleftarrow$ shifted and negated multiplicand.

ignore

$\hookrightarrow$ product $= 0101010 = 42 = (-7) \times (-6)$

Problem 21:

$$
\begin{array}{rl}
 & 9 = 1001 \\
+) & 8 = 1000 \\
\hline
 & 10001 \leftarrow \text{correction needed} \\
+) & 0110 \\
\hline
 & \underbrace{1}_{1}\,\underbrace{0111}_{7} \Rightarrow \text{result} = 17
\end{array}
$$

Problem 22:

$$
\begin{array}{rl}
 & 4 = 0100 \\
+) & 5 = 0101 \\
\hline
 & \underbrace{1001}_{9} \leftarrow \text{no correction needed}
\end{array}
$$

$9 \Rightarrow$ result is 9.

Problem 23: The 3-bit Gray code is

$$
\left.
\begin{array}{c}
000 \\
001 \\
011 \\
010 \\
110 \\
111 \\
101 \\
100
\end{array}
\right\} \leftarrow \text{3-bit Gray code}
$$

For the 4-bit Gray code we have

↳ Next page ⟶

Problem 23 cont:

$$
\left.\begin{array}{l}
\left.\begin{array}{lll}
\text{append } 0 & \rightarrow & 0000 \\
\text{"} \quad 0 & \rightarrow & 0001 \\
\text{"} \quad 0 & \rightarrow & 0011 \\
\text{"} \quad 0 & \rightarrow & 0010 \\
\text{"} \quad 0 & \rightarrow & 0110 \\
\text{"} \quad 0 & \rightarrow & 0111 \\
\text{"} \quad 0 & \rightarrow & 0101 \\
\text{"} \quad 0 & \rightarrow & 0100
\end{array}\right\} \text{in order} \\
\left.\begin{array}{lll}
\text{"} \quad 1 & \rightarrow & 1100 \\
\text{"} \quad 1 & \rightarrow & 1101 \\
\text{"} \quad 1 & \rightarrow & 1111 \\
\text{"} \quad 1 & \rightarrow & 1110 \\
\text{"} \quad 1 & \rightarrow & 1010 \\
\text{"} \quad 1 & \rightarrow & 1011 \\
\text{"} \quad 1 & \rightarrow & 1001 \\
\text{"} \quad 1 & \rightarrow & 1000
\end{array}\right\} \text{in reverse order}
\end{array}\right\} \text{4-bit Gray code}
$$

The 5-bit Gray code is shown on the next page

EE 2720, Sol. of HW # 2 cont.

append 0 → 00000		append 1 → 11000	
" 0 → 00001		" 1 → 11001	
" 0 → 00011		" 1 → 11011	
" 0 → 00010		" 1 → 11010	
" 0 → 00110		" 1 → 11110	
" 0 → 00111		" 1 → 11111	
" 0 → 00101		" 1 → 11101	
" 0 → 00100		" 1 → 11100	
" 0 → 01100		" 1 → 10100	
" 0 → 01101		" 1 → 10101	
" 0 → 01111		" 1 → 10111	
" 0 → 01110		" 1 → 10110	
" 0 → 01010		" 1 → 10010	
" 0 → 01011		" 1 → 10011	
" 0 → 01001		" 1 → 10001	
" 0 → 01000		" 1 → 10000	

in order

in reverse order

5-bit Gray code