2720, Spring 06

Homework # 6 solutions

EE 2720
Homework # 6 Solutions

Problem 1: Let the inputs of the circuit be E_3, E_2, E_1, E_0 representing the Excess-3 number. Let the outputs of the circuit be B_3, B_2, B_1, B_0 representing the BCD number. Below I show the trut table.

E_3	E_2	E_1	E_0	B_3	B_2	B_1	B_0
0	0	0	0	d	d	d	d
0	0	0	1	d	d	d	d
0	0	1	0	d	d	d	d
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	1
1	1	0	1	d	d	d	d
1	1	1	0	d	d	d	d
1	1	1	1	d	d	d	d

} d means don't care

} d means don't care

From the above truth table, one can easily get the Karnaugh maps for the outputs B_3, B_2, B_1, B_0. These Karnaugh maps are shown on the next page.

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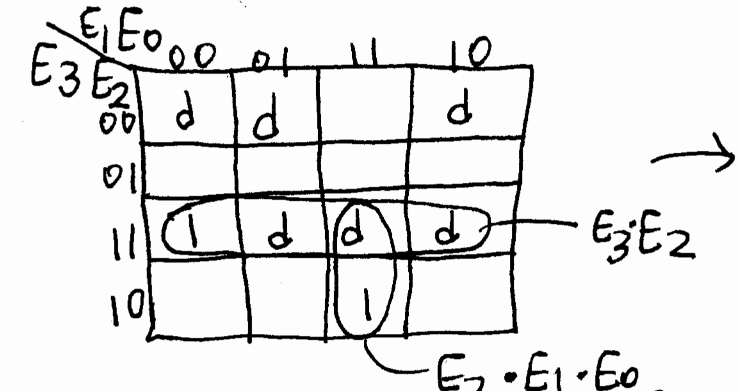
Problem 1 cont:

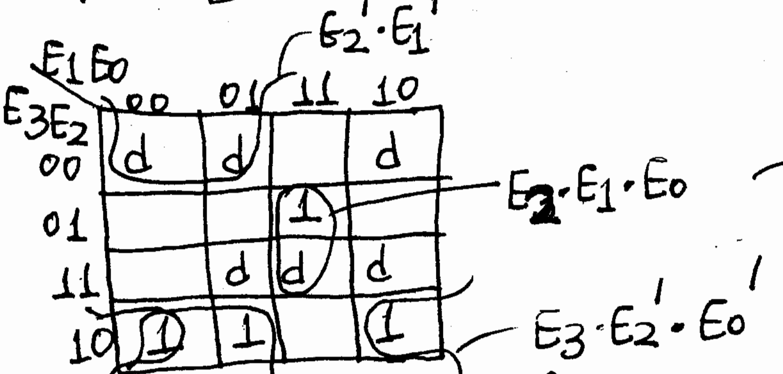


E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01				
11	1	d	d	d
10			1	

Fig. 1: Karnaugh map for output B_3.

From the Karnaugh map of fig. 1 we get the following expression for B_3:

$$B_3 = E_3 \cdot E_2 + E_3 \cdot E_1 \cdot E_0 \quad (1)$$

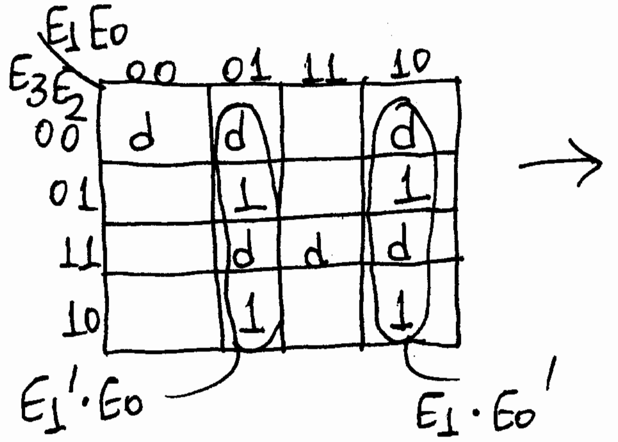


E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01			1	
11		d	d	d
10	1	1		1

Fig. 2: Karnaugh map for output B_2.

From the Karnaugh map of fig. 2 we get the following expression for B_2:

$$B_2 = E_2' \cdot E_1' + E_3 \cdot E_2' \cdot E_0' + E_2 \cdot E_1 \cdot E_0 \quad (2)$$



E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01		1		1
11		d	d	d
10		1		1

Fig. 3: Karnaugh map for output B_1

From the Karnaugh map of fig. 3 we get the following expression for B_1:

$$B_1 = E_1' \cdot E_0 + E_1 \cdot E_0' = E_1 \oplus E_0 \quad (3)$$

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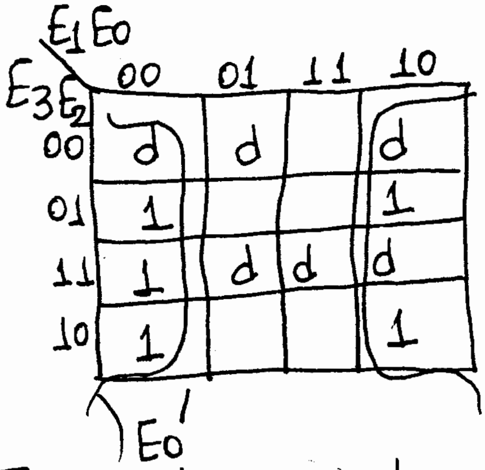
Problem 1 cont:



E_3E_2 \ E_1E_0	00	01	11	10
00	d	d		d
01	1			1
11	1	d	d	d
10	1			1

→ From the karnaugh map of fig. 4 we get the following expression for B_0:

$$B_0 = E_0' \quad (4).$$

Fig. 4: Karnaugh map for output B_0

From equations (1), (2), (3), (4) we get the following circuit shown in figure 5 below; (this is one possible realization).

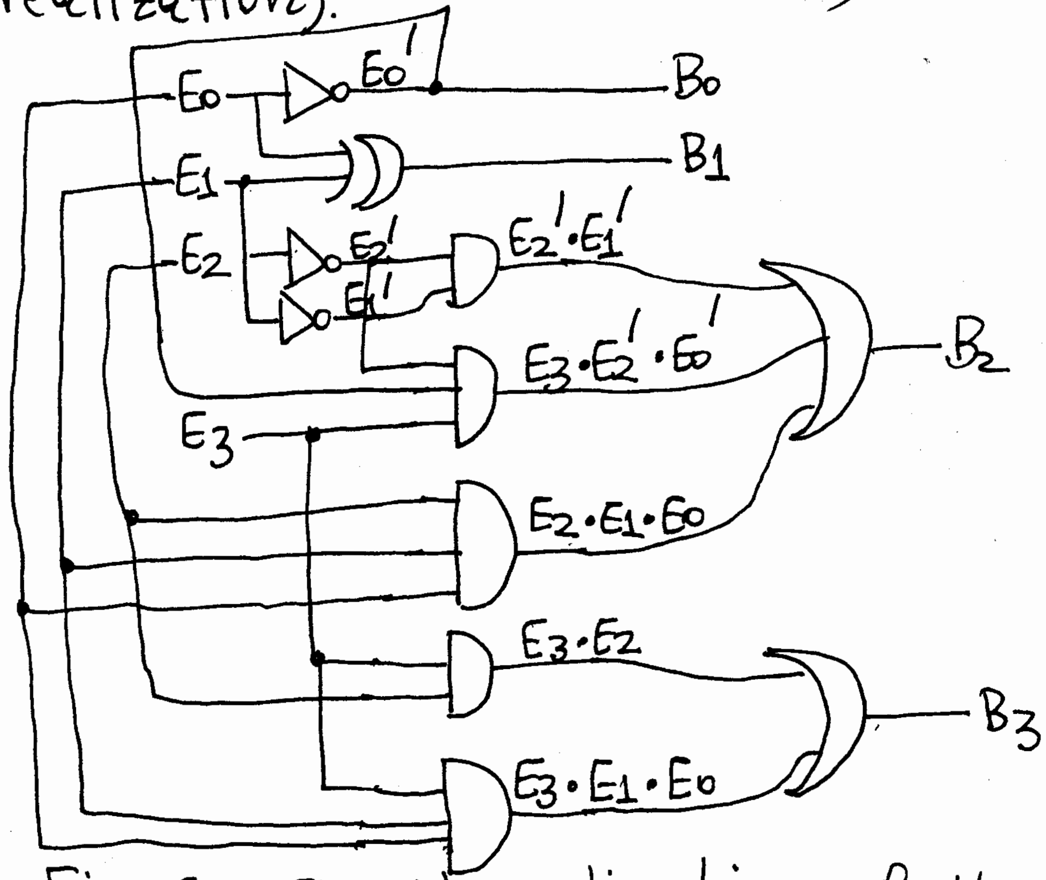


Fig. 5: Circuit realization of the Excess-3 to BCD converter.

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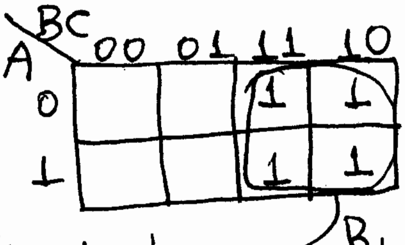
Problem 2: The inputs here are A, B, C and the outputs are X, Y, Z. Below I show the truth table.

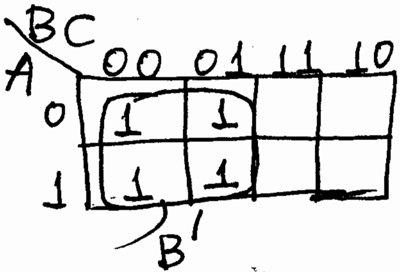
ABC	XYZ
000	010
001	011
010	100
011	101
100	010
101	011
110	100
111	101

→ From the truth table, one can easily get the Karnaugh maps for the outputs X, Y, Z. These Karnaugh maps are show below:



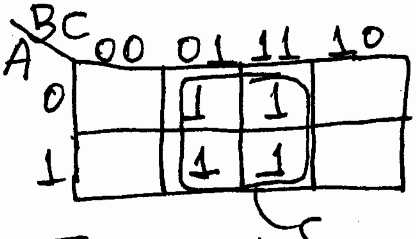
A \ BC	00	01	11	10
0			1	1
1			1	1

Fig. 1: Karnaugh map for output X



A \ BC	00	01	11	10
0	1	1		
1	1	1		

Fig. 2: Karnaugh map for output Y



A \ BC	00	01	11	10
0		1	1	
1		1	1	

Fig. 3: Karnaugh map for output Z

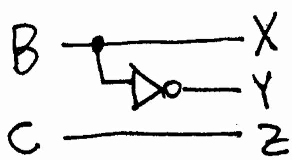
→ From the Karnaugh maps of figures 1, 2, 3 we get the following expressions for X, Y, Z:

$X = B$, (1)

$Y = B'$ (2)

$Z = C$ (3).

From equations (1), (2), (3) above we get the following circuit:



→ Note: As seen, the circuit is very simple; it relies only on one inverter

Fig. 4: Circuit realization for problem 2

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HW # 6 Solutions cont.

Problem 3: Below I show the truth table.

X	Y	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

→ From the truth table, one can easily get the karnaugh maps for the outputs S, Cout. These karnaugh maps are shown below:

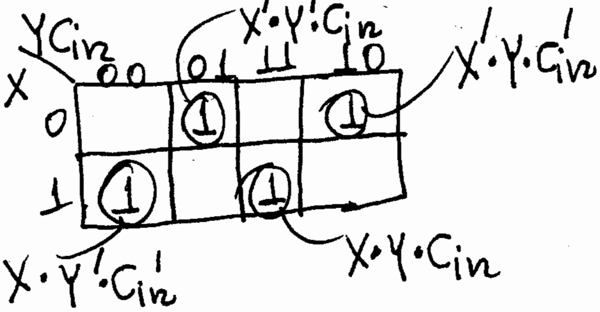


Fig. 1: karnaugh map for output S

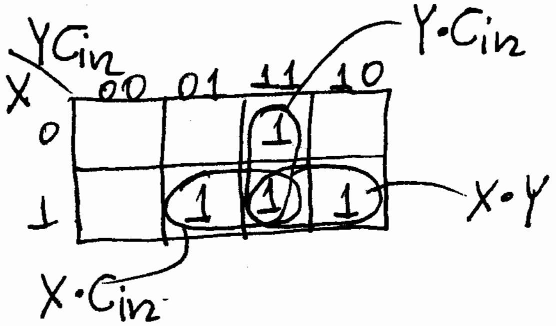


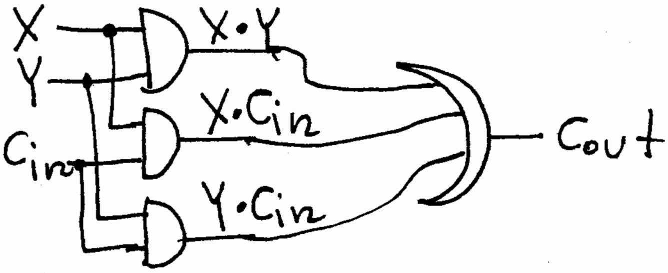
Fig. 2: karnaugh map for output Cout.

From the karnaugh maps of figures 1, 2 above, we get the following expressions for S and Cout:

$$S = X'\cdot Y'\cdot C_{in} + X'\cdot Y\cdot C_{in}' + X\cdot Y'\cdot C_{in}' + X\cdot Y\cdot C_{in} \quad (1)$$

$$C_{out} = X\cdot Y + X\cdot C_{in} + Y\cdot C_{in} \quad (2)$$

From equations (1), (2) above we get the following AND—O realization for the full adder:



The rest of the figure showing the output S is on the next page.

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Problem 3 cont:

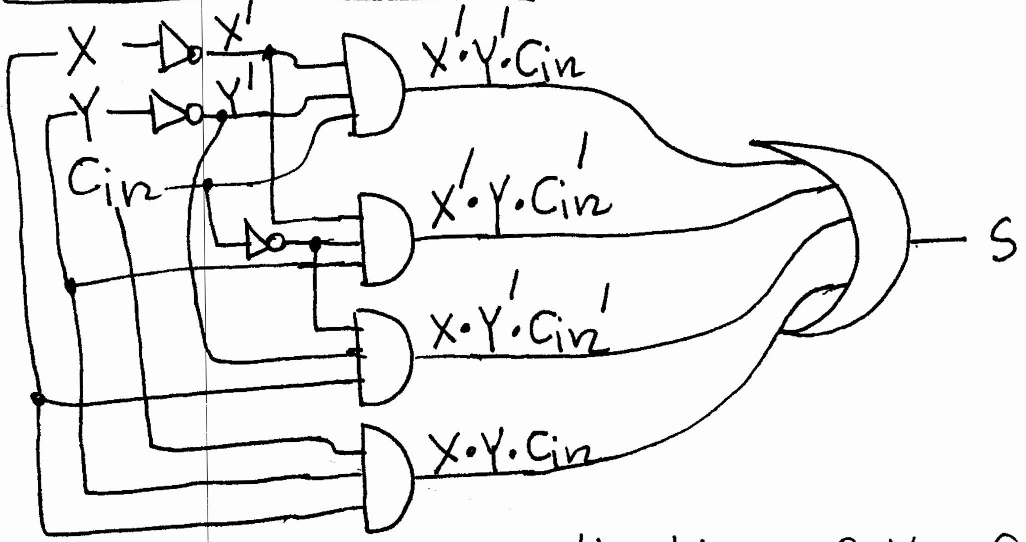

Fig. 3: AND-OR realization of the full adder.

Note: The expession of S provided in equation (1) cannot be simplified at all. It is a canonical sum actually. ($S=\Sigma_{X,Y,C_{in}}(1,2,4,7)$). As seen from the Karnaugh map of fig. 1, we cannot combine any cells containing 1's. (The same is true for cells containing 0's). Question: Can you provide another expression for S?

Problem 4:

Below I show the truth table

X	Y	S	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

→ From the truth table, one can easily get the Karnaugh maps for the outputs S, Cout shown below:

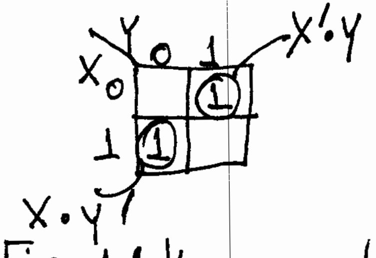

Fig. 1: Karnaugh map for output S

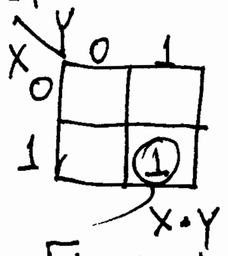

Fig. 2: Karnaugh map for output Cout

From the Karnaugh maps of figures 1, 2 we get the expressions:

$S = X'\cdot Y + X\cdot Y'$ (1)

$Cout = X\cdot Y$ (2)

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Problem 4 cont: From equations (1), (2) of previous page we get the following AND–OR realization for the half adder:

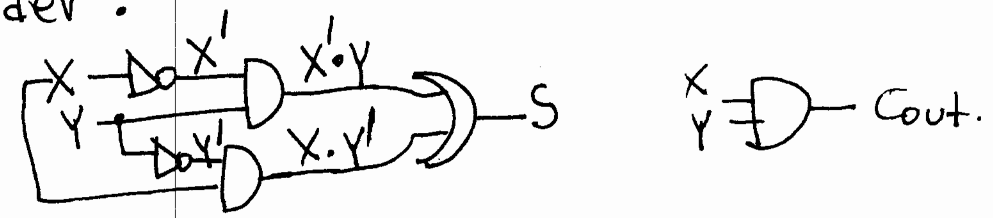

Fig. 3: AND–OR realization of the half adder.

Note: The expression of S provided by eq. (1) cannot be simplified at all. It is a canonical sum. As seen from the Karnaugh map of fig. 1, we cannot combine the two cells containg 1's; (they are not adjacent).
Note: This problem is so simple (trivial actually) that you don't need a truth table or Karnaugh map.

Problem 6:

WX \ YZ	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

→ This function F can't be simplified at all. As seen from the Karnaugh map we can't combine any cells. So F is the given original expression which is a canonical sum, or F is

$F = \sum_{w,x,y,z}(1, 3, 4, 7, 8, 11, 13, 14)$. I hope you can now write an algebraic expression for F, but even if you give me this as an answer, it is ok.

Problem 7: The Karnaugh maps and the respective simplified sum-of-products expressions are shown on the next page.

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HW#6 Solutions cont.

Problem 7 cont:

(a)

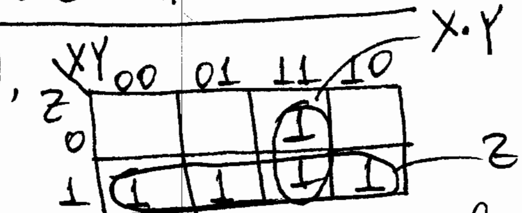


Fig. 1: karnaugh map for $F=\Sigma_{X,Y,Z}(1,3,5,6,7)$

From the karnaugh map of fig. 1 we get the following simplified sum-of-product expression for F:

$F = Z + X\cdot Y$ (1) ; (this is the minimal sum by the way)

(b)

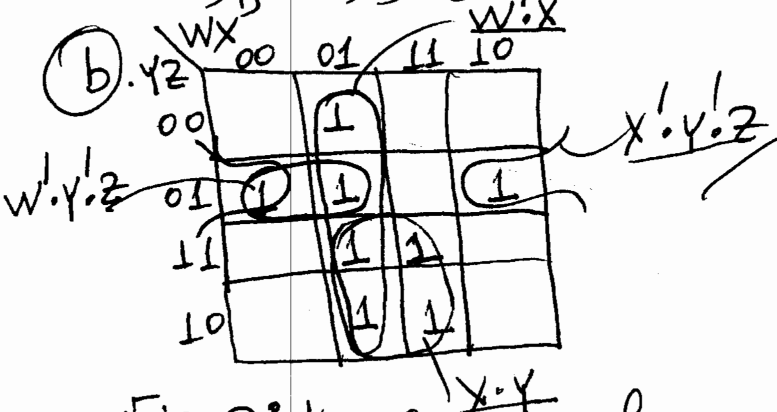


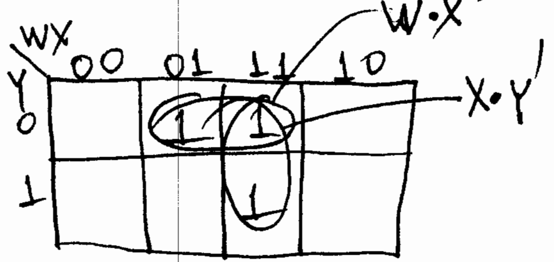
Fig. 2: karnaugh map for $F=\Sigma_{W,X,Y,Z}(1,4,5,6,7,9,14,15)$

From the karnaugh map of fig. 2 we get the following simplified sum-of-products expression for F.

$F = W'\cdot X + X'\cdot Y'\cdot Z + X\cdot Y + W'\cdot Y'\cdot Z$ (2)

; (this is not the minimal sum. Can you find the minimal sum?).

(c)



$F=\Pi_{W,X,Y}(0,1,3,4,5)=$
$=\Sigma_{W,X,Y}(2,6,7)$

Fig. 3: karnaugh map for $F=\Pi_{W,X,Y}(0,1,3,4,5)$

From the karnaugh map of fig. 3 we get the following simplified sum-of-products expression for F.

$F = W\cdot X + X\cdot Y'$ (3) ; (this is the minimal sum by the way).

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Problem 7 cont:

(d)

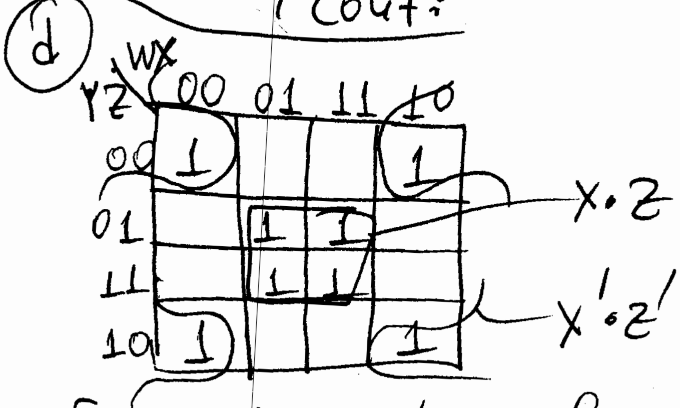

Fig. 4: karnaugh map for
$F=\Sigma_{W,X,Y,Z}(0,3,5,7,8,10,13,15)$

From the karnaugh map of fig. 4, we get the following simplified sum-of-products expression for F:

$F = X\cdot Z + X'\cdot Z'$ (4); (this is the minimal sum by the way)

(e)

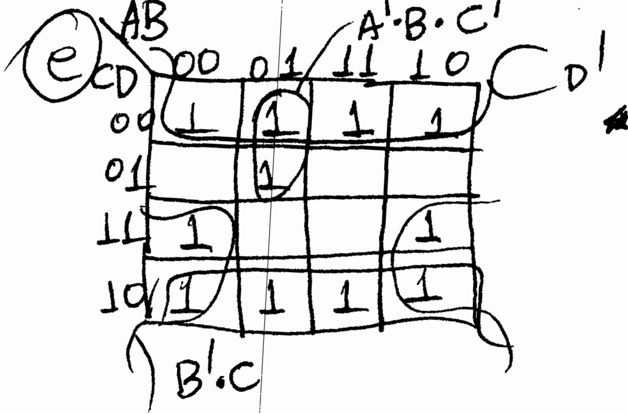

Fig. 5: karnaugh map for
$F=\Pi_{A,B,C,D}(1,7,9,13,15)$

$F=\Pi_{A,B,C,D}(1,7,9,13,15)=$
$=\Sigma_{A,B,C,D}(0,2,3,4,5,6,8,10,11,12,14)$

From the karnaugh map of fig. 5, we get the following simplified sum-of-products expression for F:

$F = D' + B'\cdot C + A'\cdot B\cdot C'$ (5); (this is the minimal sum).

Problem 8:

(a)

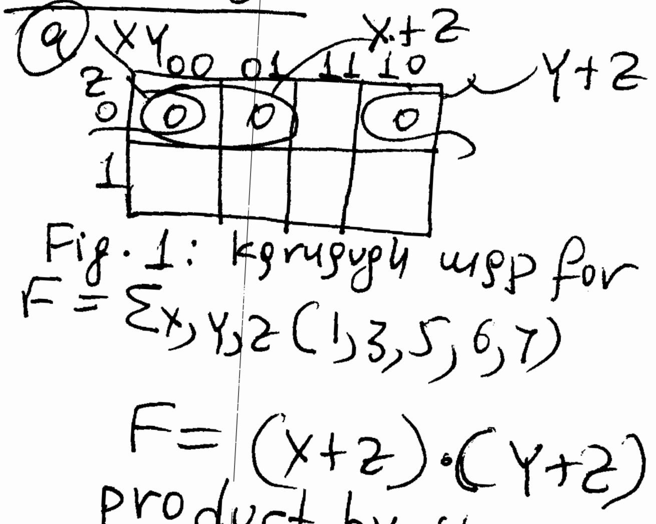

Fig. 1: karnaugh map for
$F=\Sigma_{X,Y,Z}(1,3,5,6,7)$

From the karnaugh map of fig. 1 we get the following simplified product-of-sums expression for F:

$F=(X+Z)\cdot(Y+Z)$ (1); (this is the minimal product by the way).

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HW #6 Solutions cont.

Problem 8 cont:

(b)

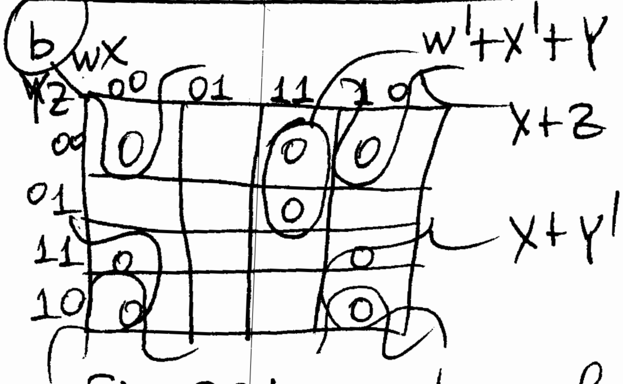

Fig. 2: karnaugh map for $F = \Sigma_{w,x,y,z}(1,4,5,6,7,9,14,15)$

From the karnaugh map of fig. 2, we get the following simplified product-of-sums expression for F:

$$F = (x+z)\cdot(x+y')\cdot(w'+x'+y) \quad (2);$$

(this is the minimal product by the way).

(c)

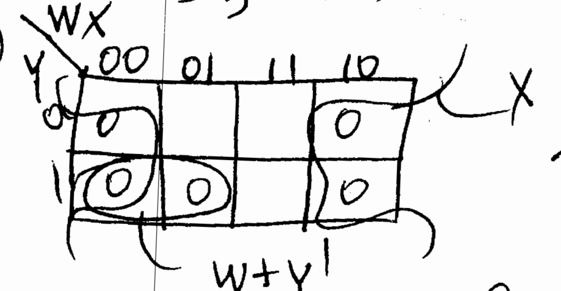

Fig. 3: karnaugh map for $F = \Pi_{w,x,y}(0,1,3,4,5)$

From the karnaugh map of fig. 3, we get the following simplified product-of-sums expression for F:

$F = x\cdot(w+y')$ (3); (this is the minimal product)

(d)

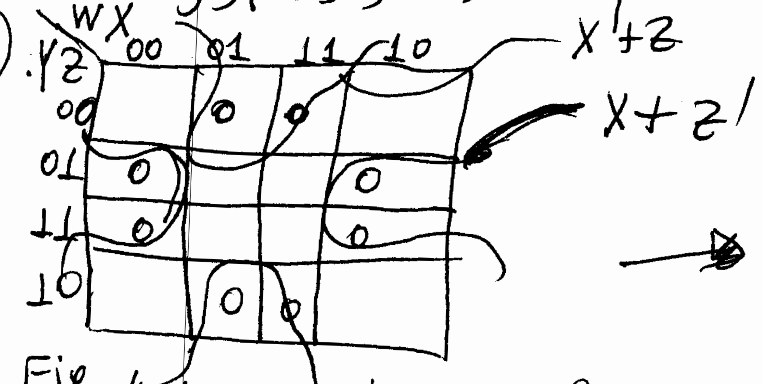

Fig. 4: karnaugh map for $F = \Sigma_{w,x,y,z}(0,2,5,7,8,10,13,15)$

From the karnaugh map of fig. 4, we get the following simplified product-of-sums expression for F:

$F = (x'+z)\cdot(x+z')$ (4); (this is the minimal product by the way).

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Problem 8 cont:

ⓔ.

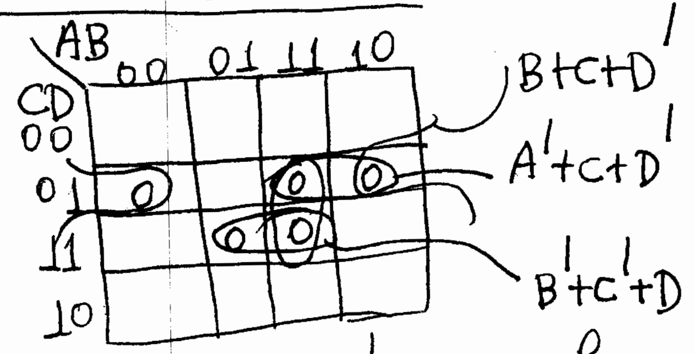

Fig. 5: Karnaugh map for $F=\prod_{A,B,C,D}(1,7,9,13,15)$

From the karnaugh map of fig. 5, we get the following simplified product-of-sums expression for F: $F=(B+C+D')\cdot(B'+C'+D')\cdot(A'+C+D')$ (5); (this is a minimal product). You can also get another minimal product. Can you get it?

Problem 9:

ⓐ

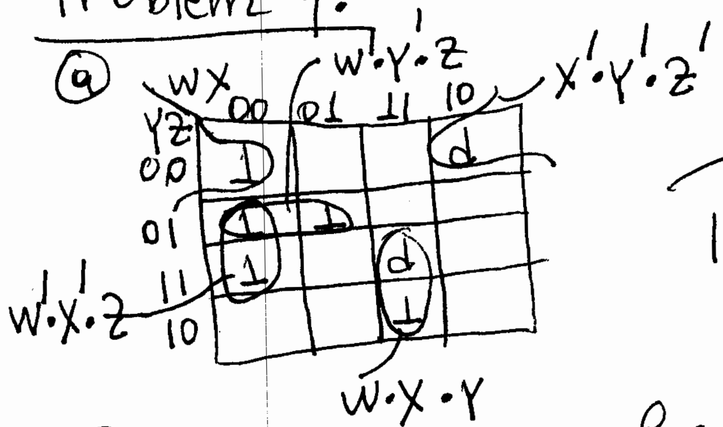

Fig. 1: karnaugh map for $F=\sum_{W,X,Y,Z}(0,1,3,5,14)+d(8,15)$

From the karnaugh map of fig. 1, we get the following simplified sum-of-products expression for F:

$$F=W'\cdot Y'\cdot Z+W'\cdot X'\cdot Z+$$
$$+W\cdot X\cdot Y+X'\cdot Y'\cdot Z' \quad (1);$$

(this is a minimal sum). You can also get one more minimal sum.

ⓑ

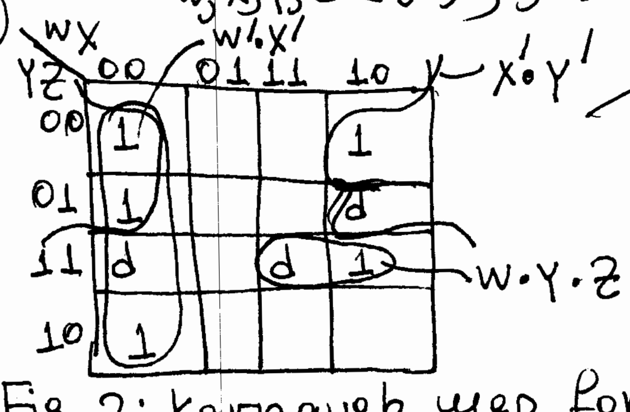

Fig. 2: karnaugh map for $F=\sum_{W,X,Y,Z}(0,1,2,8,11)+d(3,9,15)$.

From the karnaugh map of fig. 2 we get the following: $F=W'\cdot X'+X'\cdot Y'+W\cdot Y\cdot Z$ (2). (this is a minimal sum). You can get two more minimal sums. Can you get them?

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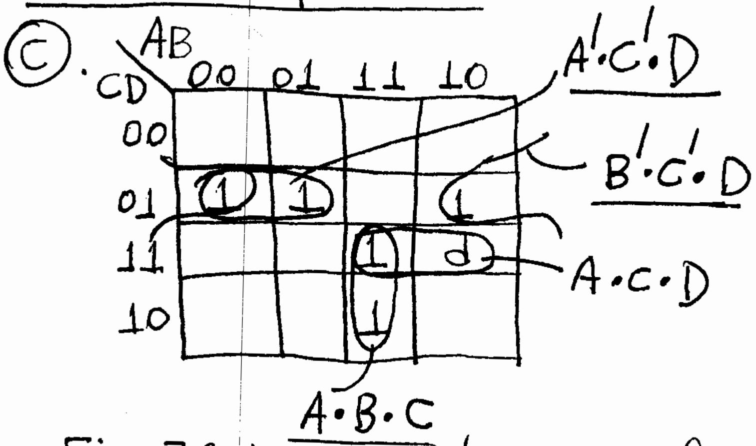
Problem 9 cont:

ⓒ.



CD \ AB	00	01	11	10
00				
01	1	1		1
11			1	d
10			1	

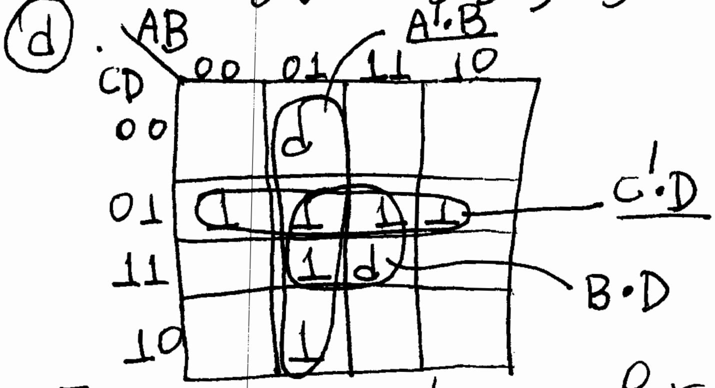
Fig. 3: Karnaugh map for $F = \sum_{A,B,C,D}(1,5,9,14,15) + d(11)$

From the Karnaugh map of fig. 3, we get the following:

$F = A \cdot B \cdot C + A' \cdot C' \cdot D + + B' \cdot C' \cdot D + A \cdot C \cdot D$ (3);

(the above is not a minimal sum. Can you find a minimal sum?)

ⓓ.



CD \ AB	00	01	11	10
00		d		
01	1	1	1	1
11		1	d	
10		1		

Fig. 4: Karnaugh map for $F = \sum_{A,B,C,D}(1,5,6,7,9,13) + d(4,15)$

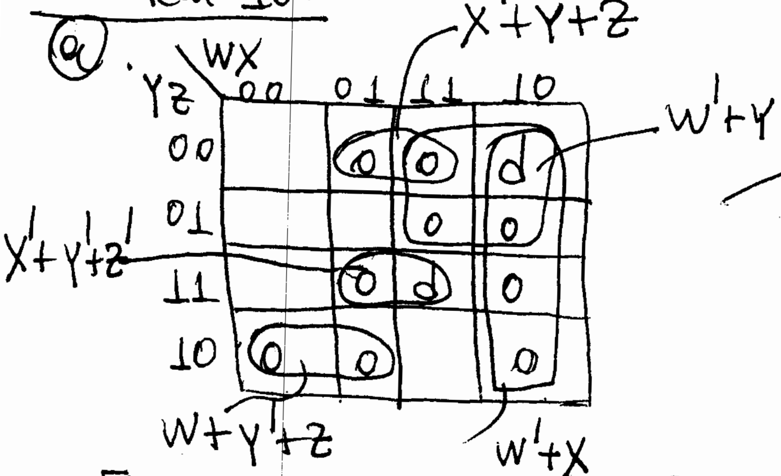
From the Karnaugh map of fig. 4, we get the following:

$F = A' \cdot B + C' \cdot D + B \cdot D$ (4).

(the above is not a minimal sum. Can you find a minimal sum?).

Problem 10:

ⓐ.



YZ \ WX	00	01	11	10
00		0	0	d
01			0	0
11		0	d	0
10	0	0		0

Fig. 1: Karnaugh map for $F = \sum_{W,X,Y,Z}(0,1,3,5,14) + d(8,15)$

From the Karnaugh map of fig. 1, we get the following:

$F = (X' + Y + Z) \cdot (W' + Y) \cdot (W' + X) \cdot (W + Y' + Z) \cdot (X' + Y' + Z')$ (1);

(the above is not a minimal product)

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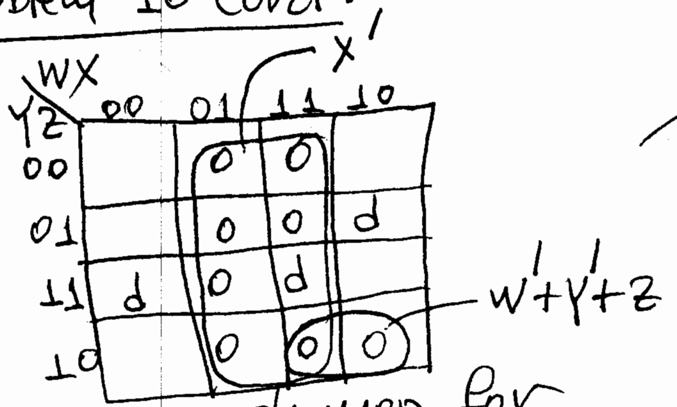
Problem 10 cont:

(b).

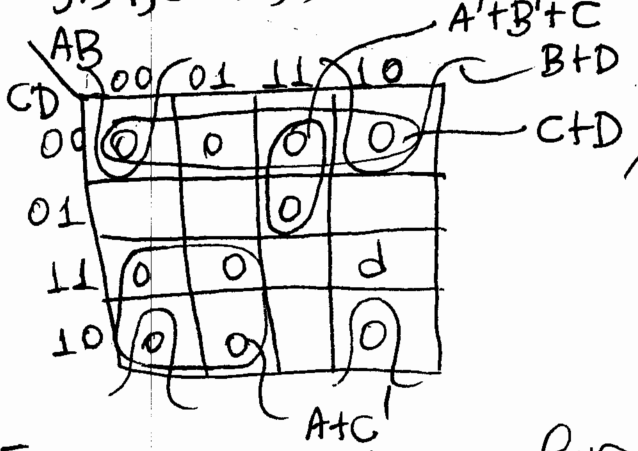


YZ \ WX	00	01	11	10
00		0	0	
01		0	0	d
11	d	0	d	
10		0	0	0

Fig. 2: karnaugh map for
$F=\Sigma_{W,X,Y,Z}(0,1,2,8,11)+d(3,9,15)$

From the karnaugh map of fig. 2, we get the following:
$F=X'\cdot(W'+Y'+Z)$ (2); (this expression is a minimal product).

(c)

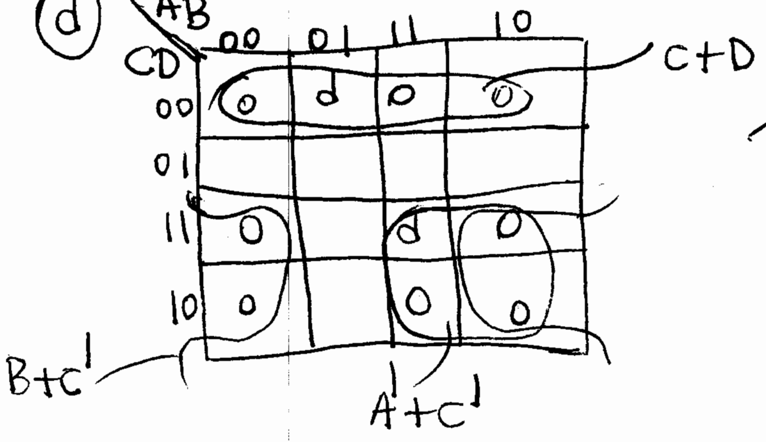


CD \ AB	00	01	11	10
00	0	0	0	0
01			0	
11	0	0		d
10	0	0		0

Fig. 3: karnaugh map for
$F=\Sigma_{A,B,C,D}(1,5,9,14,15)+d(11)$

From the karnaugh map of fig. 3, we get the following:
$F=(A'+B'+C)\cdot(B+D)\cdot(C+D)\cdot(A+C')$ (3); (this is a minimal product).

(d)



CD \ AB	00	01	11	10
00	0	d	0	0
01				
11	0		d	0
10	0		0	0

Fig. 4: karnaugh map for
$F=\Sigma_{A,B,C,D}(1,5,6,7,9,13)+d(4,15)$

From the karnaugh map of fig. 4, we get the following:
$F=(C+D)\cdot(A'+C')\cdot(B+C')$ (4); (the above expression is a minimal product).
You can get two more minimal products. Can you get them?

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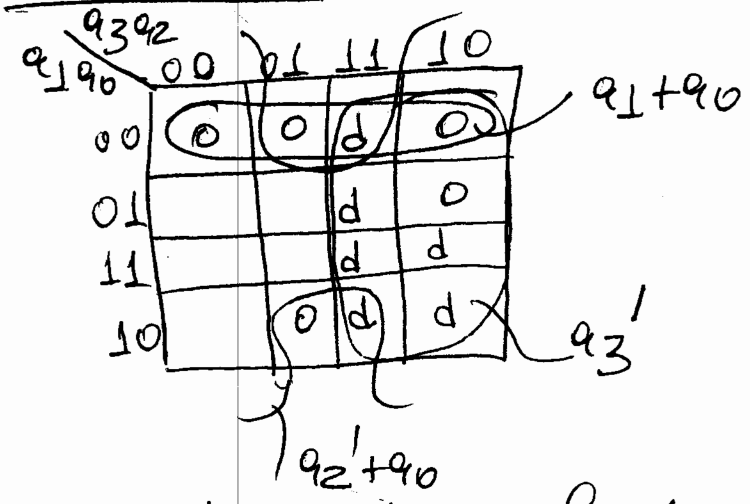
⑭.

HW#6 solutions cont.

Problem 11:



From the Karnaugh map of Fig. 1 we get the following:

$F = q_3' \cdot (q_2'+q_0) \cdot (q_1+q_0$

(1); (the above is a minimal product)

Fig. 1: Karnaugh map for the prime BCD-digit detector showing 0's and d's.

Note: This HW was a little bit time consuming but I think it was lots of fun!! So I hope you enjoyed it!! The next HW, (if we have one) is not going to be that time consuming. We will definitely have one more HW, but I might not grade it. However, like always, I will post the solutions of it.