

EE 2720, Fall 06

Homework #6 solutions.

Problem 1: Let the inputs of the circuit be E_3, E_2, E_1, E_0 representing the Excess-3 number. Let the outputs of the circuit be B_3, B_2, B_1, B_0 representing the BCD number. Below I show the truth table.

E_3	E_2	E_1	E_0	B_3	B_2	B_1	B_0
0	0	0	0	d	d	d	d
0	0	0	1	d	d	d	d
0	0	1	0	d	d	d	d
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	1
1	1	0	1	d	d	d	d
1	1	1	0	d	d	d	d
1	1	1	1	d	d	d	d

} d means don't care

} d means don't care

From the above truth table, one can easily get the Karnaugh maps for the outputs B_3, B_2, B_1, B_0 . These Karnaugh maps are shown on the next page.

Problem 1 cont:

	$E_1 E_0$	00	01	11	10
$E_3 E_2$	00	d	d		d
	01				
	11	1	d	d	d
	10			1	

From the Karnaugh map of fig. 1 we get the following expression for B_3 :

$$B_3 = E_3 \cdot E_2 + E_3 \cdot E_1 \cdot E_0 \quad (1)$$

Fig. 1: Karnaugh map for output B_3 .

	$E_1 E_0$	00	01	11	10
$E_3 E_2$	00	d	d		d
	01			1	
	11		d	d	d
	10	1	1		1

From the Karnaugh map of fig. 2 we get the following expression for B_2 :

$$B_2 = E_2' \cdot E_1' + E_3 \cdot E_2' \cdot E_0' + E_2 \cdot E_1 \cdot E_0 \quad (2)$$

Fig. 2: Karnaugh map for output B_2 .

	$E_1 E_0$	00	01	11	10
$E_3 E_2$	00	d	d		d
	01		1		1
	11		d	d	d
	10		1		1

From the Karnaugh map of fig. 3 we get the following expression for B_1 :

$$B_1 = E_1' \cdot E_0 + E_1 \cdot E_0' = E_1 \oplus E_0 \quad (3)$$

Fig. 3: Karnaugh map for output B_1 .

EE 2720 -
 HW#6 Solutions cont.

(3)

Problem 1 cont:

$E_1 E_0$				
$E_3 E_2$	00	01	11	10
00	d	d		d
01	1			1
11	1	d	d	d
10	1			1

E_0'

From the Karnaugh map of Fig. 4 we get the following expression for B_0 :
 $B_0 = E_0'$ (4).

Fig. 4: Karnaugh map for output B_0

From equations (1), (2), (3), (4) we get the following circuit shown in figure 5 below; (this is one possible realization).

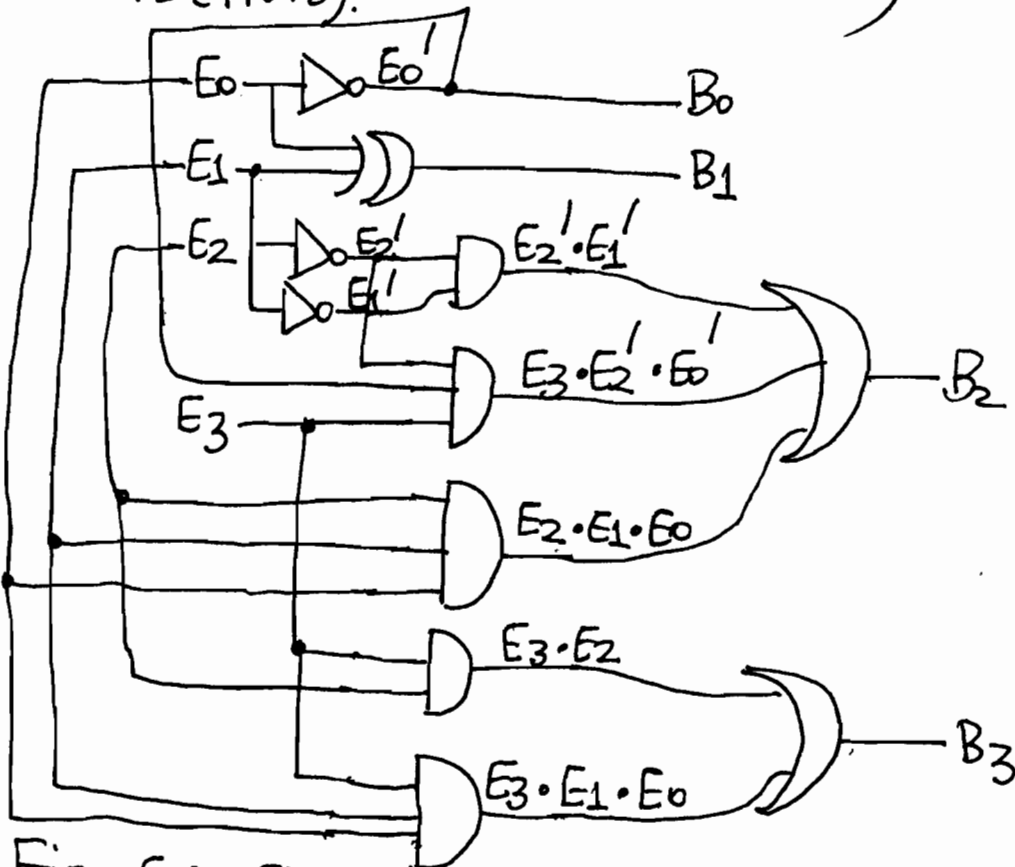


Fig. 5: Circuit realization of the Excess-3 to BCD converter.

HW#6 Solutions cont.

Problem 2: The inputs here are A, B, C and the outputs are X, Y, Z. Below I show the truth table.

A	B	C	X	Y	Z
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	1	0	0
0	1	1	1	0	1
1	0	0	0	1	0
1	0	1	0	1	1
1	1	0	1	0	0
1	1	1	1	0	1

From the truth table, one can easily get the Karnaugh maps for the outputs X, Y, Z. These Karnaugh maps are shown below:

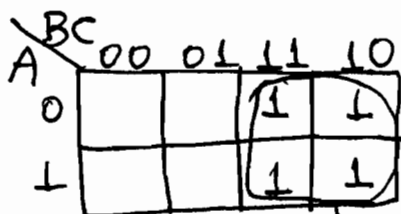


Fig. 1: Karnaugh map for output X

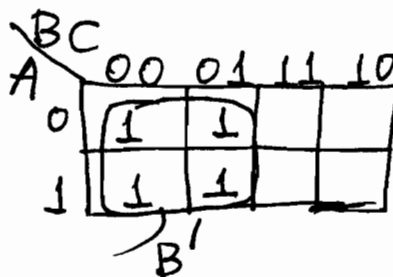


Fig. 2: Karnaugh map for output Y

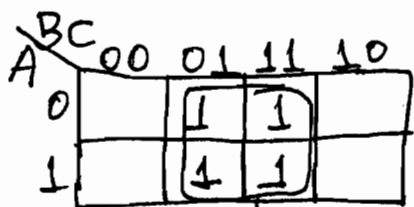


Fig. 3: Karnaugh map for output Z

From the Karnaugh maps of figures 1, 2, 3 we get the following expressions for X, Y, Z:

$$\begin{aligned} X &= B \quad (1) \\ Y &= B' \quad (2) \\ Z &= C \quad (3) \end{aligned}$$

From equations (1), (2), (3) above we get the following circuit:

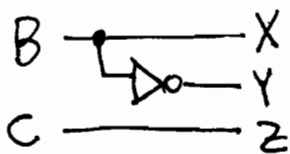


Fig. 4: Circuit realization for problem 2

Note: As seen, the circuit is very simple; it relies only on one inverter.

Problem 3: Below I show the truth table.

X	Y	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

→ From the truth table, one can easily get the karnaugh maps for the outputs S, Cout. These karnaugh maps are shown below:

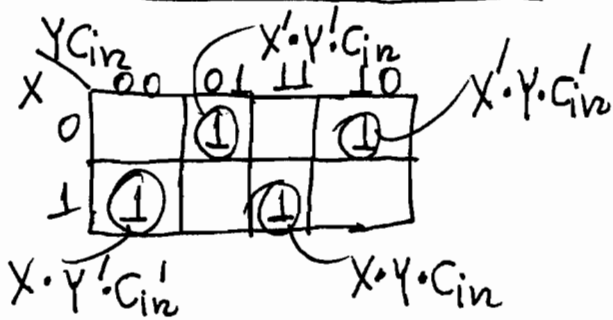


Fig. 1: Karnaugh map for output S

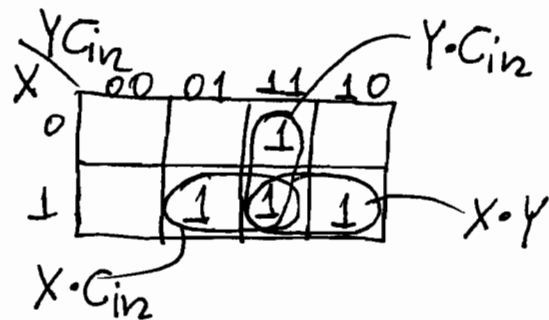


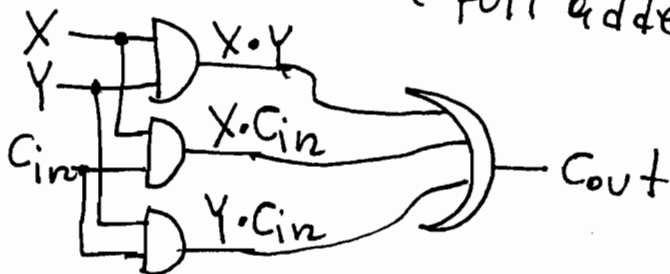
Fig. 2: Karnaugh map for output Cout.

From the karnaugh maps of figures 1, 2 above, we get the following expressions for S and Cout:

$$S = X' \cdot Y' \cdot C_{in} + X' \cdot Y \cdot C_{in}' + X \cdot Y' \cdot C_{in}' + X \cdot Y \cdot C_{in}$$

$$C_{out} = X \cdot Y + X \cdot C_{in} + Y \cdot C_{in} \quad (1)$$

From equations (1), (2) above we get the following AND-OR realization for the full adder:



The rest of the figure showing the output S is on the next page.

Problem 3 cont:

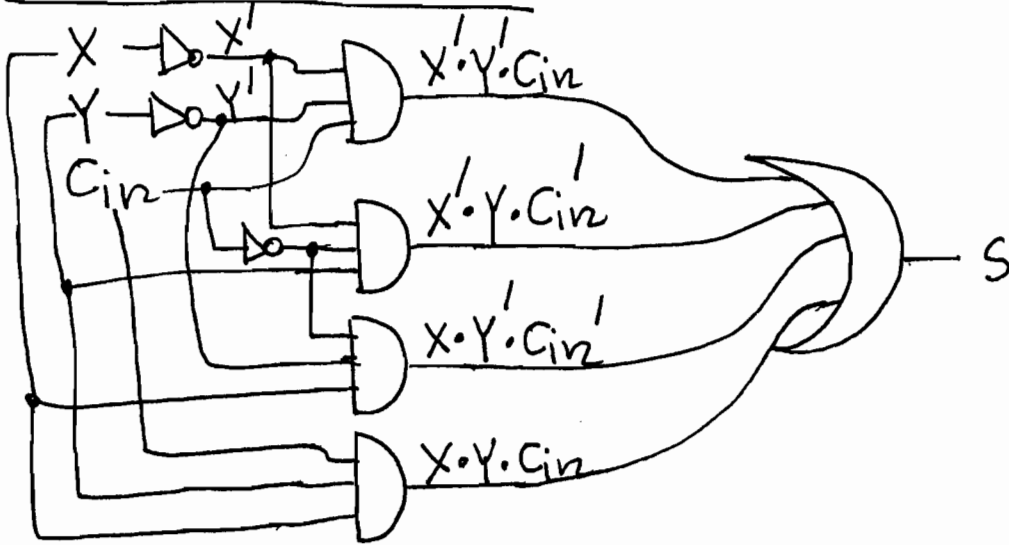


Fig. 3: AND-OR realization of the full adder.

Note: The expression of S provided in equation (1) cannot be simplified at all. It is a canonical sum actually: $S = \sum x_i y_i c_{in} (1, 2, 4, 7)$. As seen from the Karnaugh map of Fig. 1, we cannot combine any cells containing 1s (the same is true for cells containing 0s). Question: Can you provide another expression for S ?

Problem 4: Below I show the truth table

X	Y	S	Cout
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

→ From the truth table, one can easily get the Karnaugh maps for the outputs S , $Cout$ shown below:

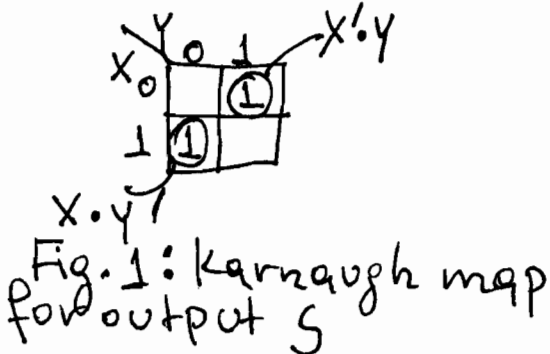


Fig. 1: Karnaugh map for output S

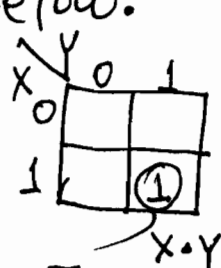


Fig. 2: Karnaugh map for output $Cout$

→ From the Karnaugh maps of figures 1, 2 we get the expressions:
 $S = X'Y + X.Y'$ (1)
 $Cout = X.Y$ (2)

Problem 4 cont: From equations (1), (2) of previous page we get the following AND-OR realization for the half adder:

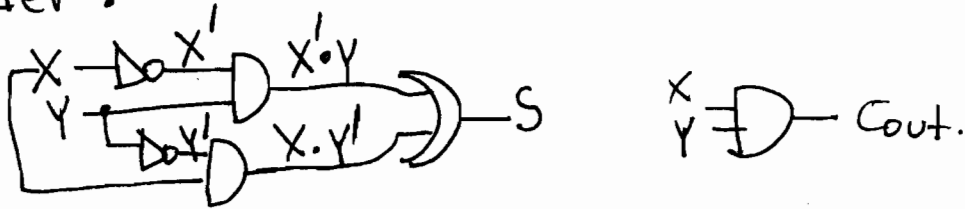


Fig. 3: AND-OR realization of the half adder.

Note: The expression of S provided by eq. (1) cannot be simplified at all. It is a canonical sum. As seen from the Karnaugh map of fig. 1, we cannot combine the two cells containing 1's; (they are not adjacent).

Note: This problem is so simple (trivial actually) that you don't need a truth table or Karnaugh map.

Problem 6:

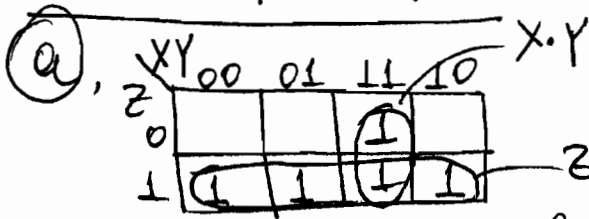
wx \ yz	00	01	11	10
00		1		1
01	1		1	
11		1		1
10	1		1	

→ This function F can't be simplified at all. As seen from the Karnaugh map we can't combine any cells. So F is the given original expression which is a canonical sum, or F is

$F = \sum_{w,x,y,z} (1, 3, 4, 7, 8, 11, 13, 14)$. I hope you can now write an algebraic expression for F , but even if you give me this as an answer, it is ok.

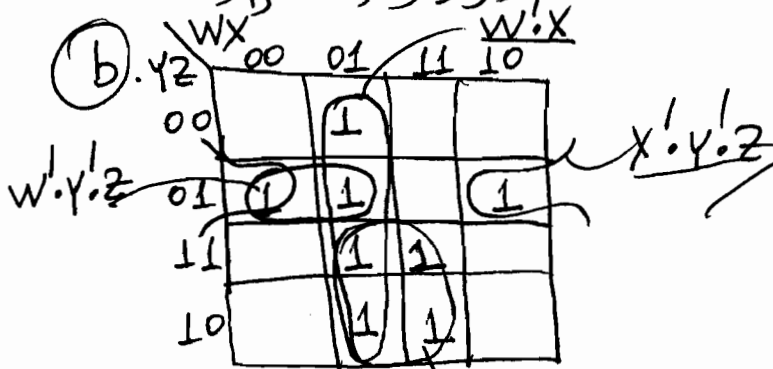
Problem 7: The Karnaugh maps and the respective simplified sum-of-products expressions are shown on the next page.

Problem 7 cont:



From the karnaugh map of Fig. 1 we get the following simplified sum-of-product expression for F:
 $F = Z + X \cdot Y$ (1); (this is the minimal sum by the way)

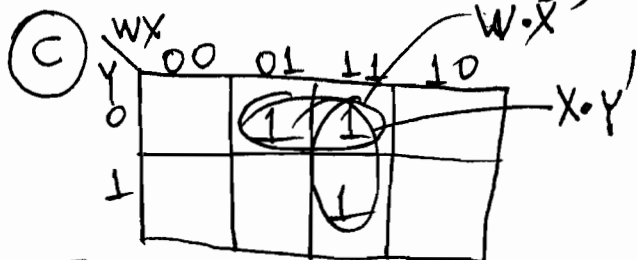
Fig. 1: karnaugh map for $F = \sum_{x,y,z} (1,3,5,6,7)$



From the karnaugh map of Fig. 2 we get the following simplified sum-of-product expression for F:
 $F = W' \cdot X + X' \cdot Y' \cdot Z + X \cdot Y + W' \cdot Y' \cdot Z$ (2)

Fig. 2: karnaugh map for $F = \sum_{w,x,y,z} (1,4,5,6,7,9,14,15)$

(this is not the minimal sum. Can you find the minimal sum?).



$F = \prod_{w,x,y} (0,1,3,4,5) = \sum_{w,x,y} (3,6,7)$

Fig. 3: karnaugh map for $F = \prod_{w,x,y} (0,1,3,4,5)$

From the karnaugh map of Fig. 3 we get the following simplified sum-of-product expression for F:
 $F = W \cdot X + X \cdot Y'$ (3); (this is the minimal sum by the way).

↳ Go to next page →

Problem 7 cont:

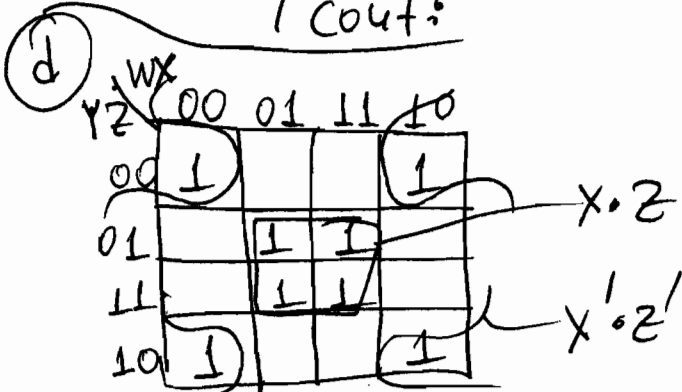
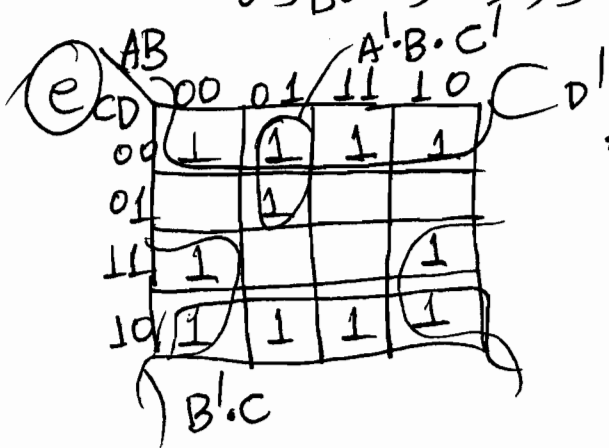


Fig. 4: karnaugh map for $F = \sum_{w,x,y,z} (0,3,5,7,8,10,13,15)$

From the karnaugh map of fig. 4, we get the following simplified sum-of-products expression for F:

$F = x \cdot z + x' \cdot z'$ (4); (this is the minimal sum by the way)



$F = \prod_{A,B,C,D} (1,7,9,13,15) = \sum_{A,B,C,D} (0,2,3,4,5,6,8,10,11,12,14)$

Fig. 5: karnaugh map for $F = \prod_{A,B,C,D} (1,7,9,13,15)$

From the karnaugh map of fig. 5, we get the following simplified sum-of-products expression for F:

$F = D + B' \cdot C + A' \cdot B \cdot C'$ (5); (this is the minimal sum).

Problem 8:

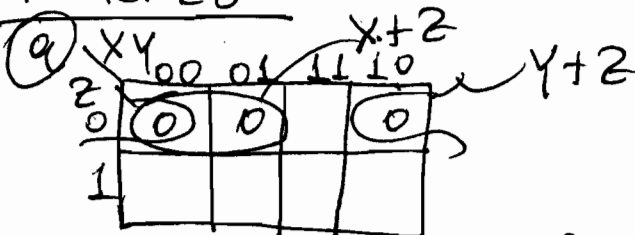


Fig. 1: karnaugh map for $F = \sum_{x,y,z} (1,3,5,6,7)$

From the karnaugh map of fig. 1, we get the following simplified product-of-sums expression for F:

$F = (x+z) \cdot (y+z)$ (1); (this is the minimal product by the way).

Problem 8 cont:

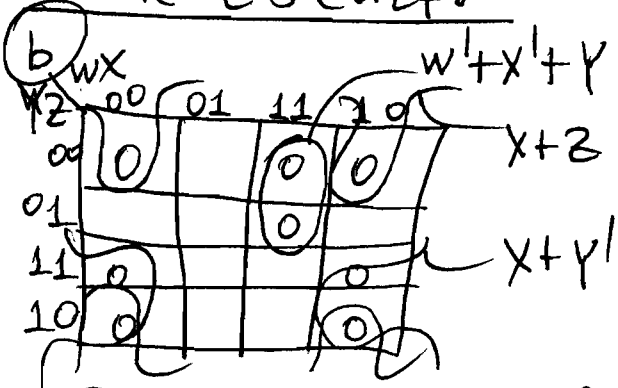


Fig. 2: Karnaugh map for $F = \sum_{w,x,y,z} (4, 5, 6, 7, 9, 14, 15)$

From the Karnaugh map of Fig. 2, we get the following simplified product-of-sums expression for F :
 $F = (x + z) \cdot (x + y') \cdot (w' + x' + y)$
 (this is the minimal product by the way).

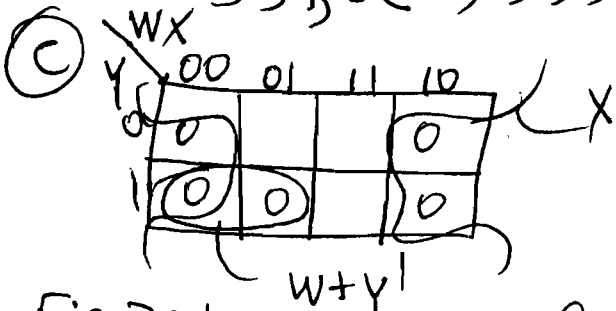


Fig. 3: Karnaugh map for $F = \prod_{w,x,y} (0, 1, 3, 4, 5)$

From the Karnaugh map of Fig. 3, we get the following simplified product-of-sums expression for F :
 $F = x \cdot (w + y')$
 (this is the minimal product)

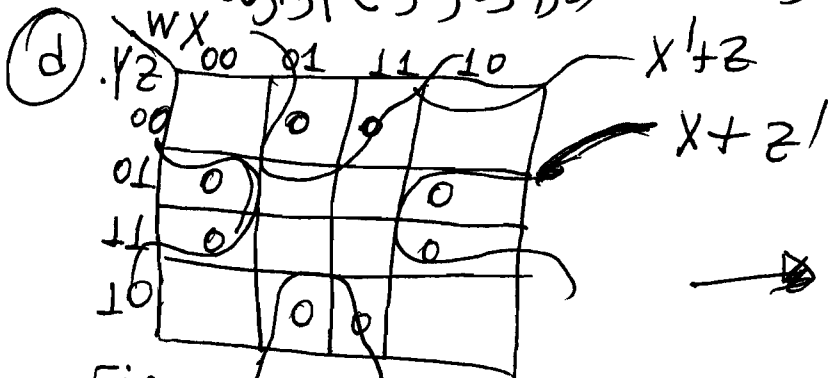


Fig. 4: Karnaugh map for $F = \sum_{w,x,y,z} (0, 2, 5, 7, 8, 10, 13, 15)$

From the Karnaugh map of Fig. 4, we get the following simplified product-of-sums expression for F :
 $F = (x' + z) \cdot (x + z')$
 (this is the minimal product by the way).

Problem 8 cont:

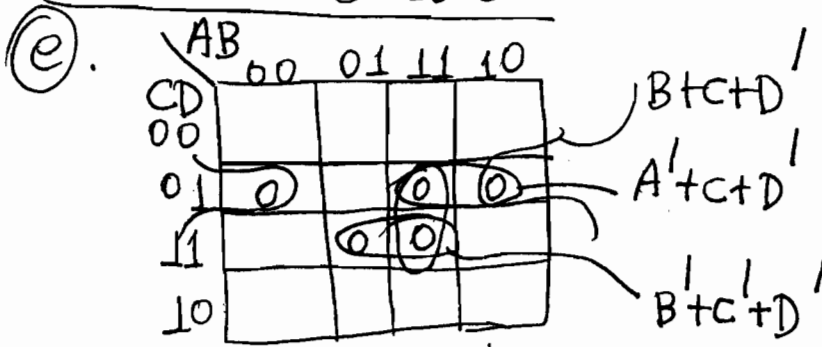


Fig. 5: Karnaugh map for $F = \prod_{A,B,C,D} (5, 7, 9, 13, 15)$

From the Karnaugh map of fig. 5, we get the following simplified product-of-sums expression for F: $F = (BtctD') \cdot (B'tc'tD') \cdot (A'tctD')$ (5); (this is a minimal product). You can also get another minimal product. Can you get it?

Problem 9:

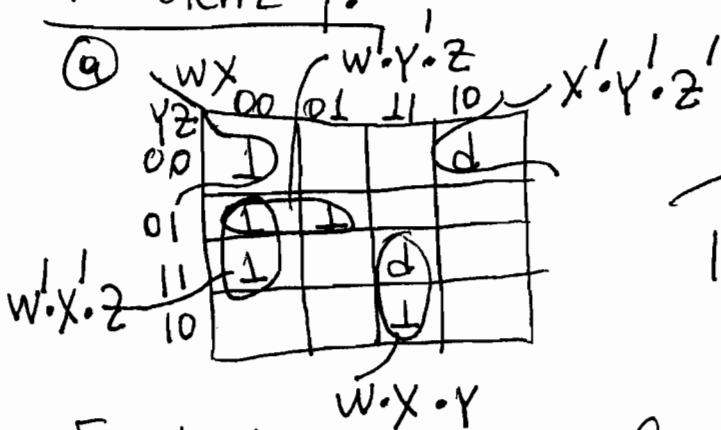


Fig. 1: Karnaugh map for $F = \sum_{w,x,y,z} (0, 1, 3, 5, 14) + d(8, 15)$

From the Karnaugh map of fig. 1, we get the following simplified sum-of-products expression for F: $F = w'y'z + w'x'z + w'x'y + x'y'z'$ (1); (this is a minimal sum). You can also get one more minimal sum.

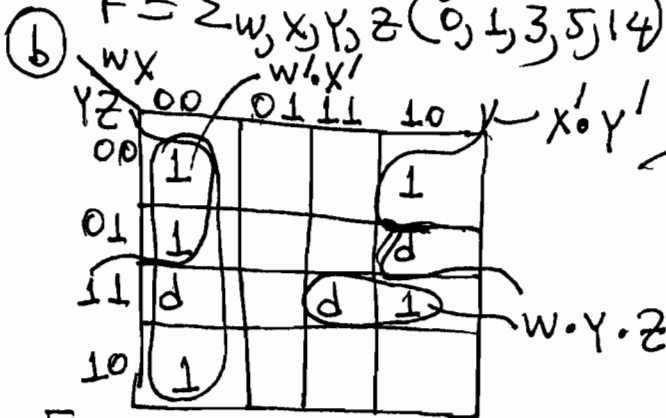


Fig. 2: Karnaugh map for $F = \sum_{w,x,y,z} (0, 1, 2, 8, 14) + d(3, 9, 15)$

From the Karnaugh map of fig. 2 we get the following: $F = w'x' + x'y' + w \cdot y \cdot z$ (2). (this is a minimal sum). You can get two more minimal sums. Can you get them?

Problem 9 cont:

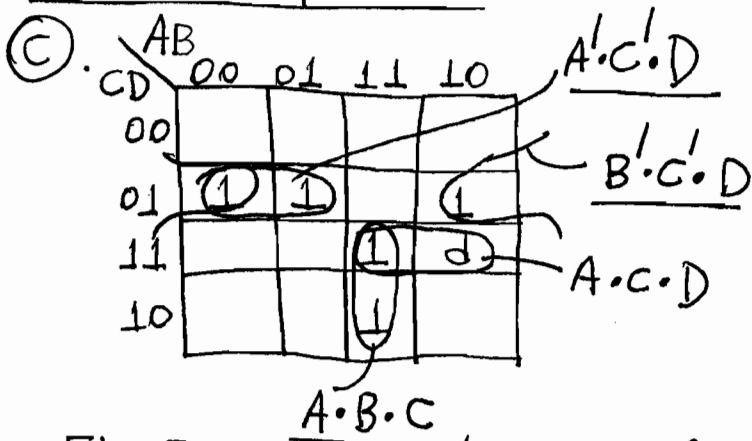


Fig. 3: Karnaugh map for $F = \sum_{A,B,C,D} (1, 5, 9, 14, 15) + d(11)$

From the Karnaugh map of fig. 3, we get the following:
 $F = A \cdot B \cdot C + A \cdot C \cdot D + B \cdot C \cdot D + A \cdot C \cdot D$ (3);
 (the above is not a minimal sum. Can you find a minimal sum?)

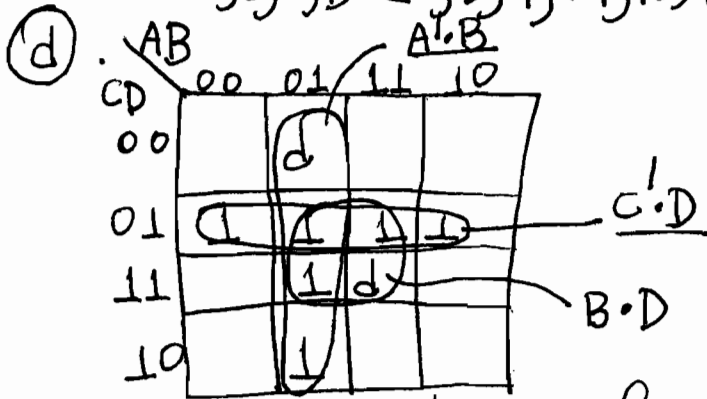


Fig. 4: Karnaugh map for $F = \sum_{A,B,C,D} (1, 5, 6, 7, 9, 13) + d(4, 15)$

From the Karnaugh map of fig. 4, we get the following:
 $F = A \cdot B + C \cdot D + B \cdot D$ (4);
 (the above is not a minimal sum. Can you find a minimal sum?)

Problem 10:

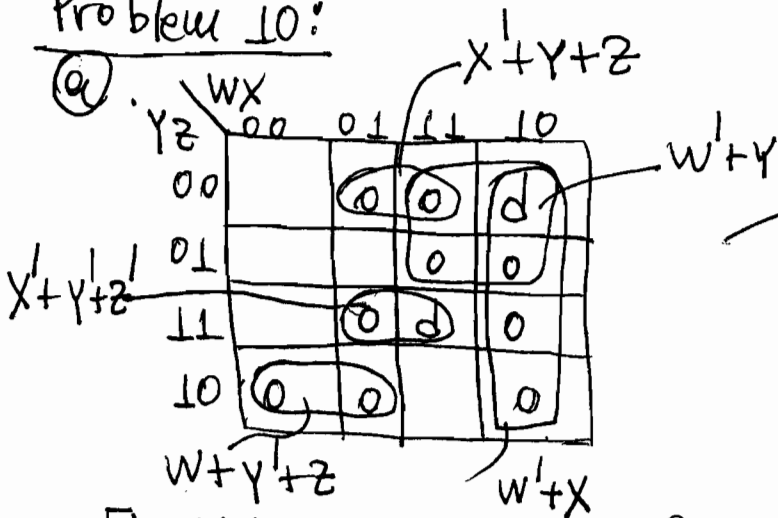
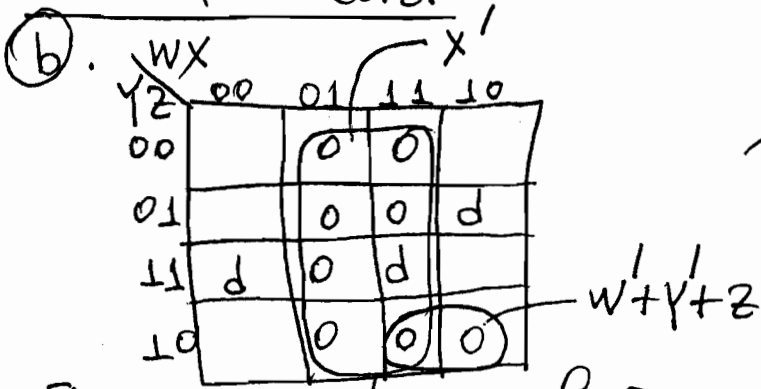


Fig. 1: Karnaugh map for $F = \sum_{W,X,Y,Z} (0, 1, 3, 5, 14) + d(8, 15)$

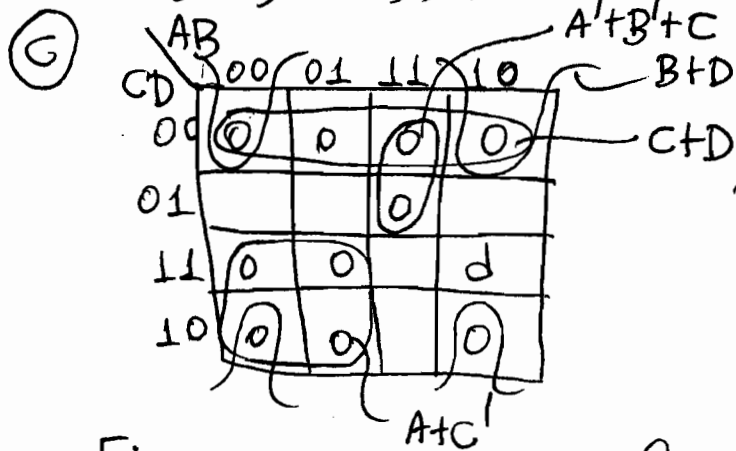
From the Karnaugh map of fig. 1, we get the following:
 $F = (X' + Y + Z) \cdot (W' + Y) \cdot (W' + X) \cdot (W + Y' + Z) \cdot (X' + Y' + Z')$ (1);
 (the above is not a minimal product)

Problem 10 cont:



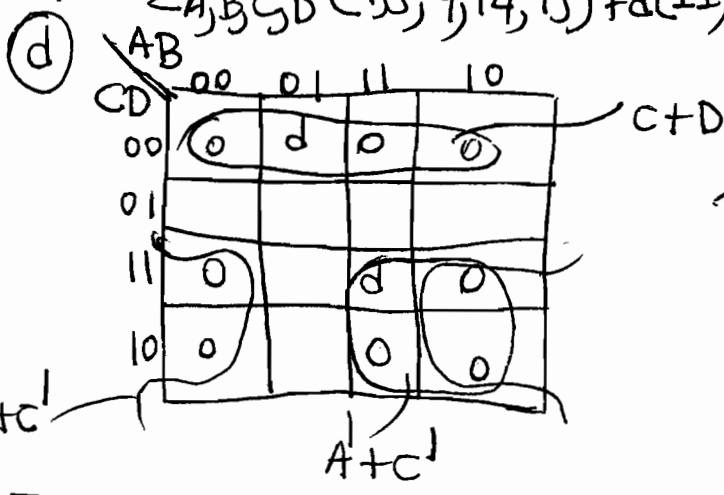
From the karnaugh map of fig. 2, we get the following:
 $F = x' \cdot (w' + y' + z) \cdot z$; (this expression is a minimal product).

Fig. 2: karnaugh map for $F = \sum_{w,x,y,z} (0,1,2,8,11) + d(3,9,15)$



From the karnaugh map of fig. 3, we get the following:
 $F = (A'+B'+C) \cdot (B+D) \cdot (C+D)$.
 (this is a minimal product).

Fig. 3: karnaugh map for $F = \sum_{A,B,C,D} (1,5,9,14,15) + d(11)$



From the karnaugh map of fig. 4, we get the following:
 $F = (C+D) \cdot (A'+C') \cdot (B+c')$.
 (the above expression is a minimal product).
 You can get two more minimal products. Can you get them?

Fig. 4: karnaugh map for $F = \sum_{A,B,C,D} (1,5,6,7,9,13) + d(4,15)$

Problem 11:

$a_3 a_2$	00	01	11	10
$a_1 a_0$	00	0	d	0
01			d	0
11			d	d
10		0	d	d

$a_1 + a_0$ (circled in the top row)
 a_3' (circled in the bottom row)
 $a_2' + a_0$ (circled in the bottom-left cell)

From the Karnaugh map of Fig. 1 we get the following:

$$F = a_3' \cdot (a_2' + a_0) \cdot (a_1 + a_0)$$

(1); (the above a minimal product)

Fig. 1: Karnaugh map for the prime BCD-digit detector showing 0's and d's.

Note: This HW was a little bit time consuming but I think it was lots of fun!! So I hope you enjoyed it!! The next HW, (if we have one) is not going to be first time consuming. We will definitely have one more HW, but I might not grade it. However, like always, I will post the solutions of it.