

EE 2720, Spr. 06
HW# 4 Solutions.

Solutions of HW# 4

Problem 1: The left side of (T10) is:

$$X \cdot Y + X \cdot Y' \quad \text{This can be written as}$$

$$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\substack{\uparrow \\ \text{consensus} \\ \text{term}}} \quad \text{(according to (T11))}$$

$$= Y \cdot X + Y' \cdot X + X = Y \cdot X + Y' \cdot X + X \cdot 1 = X \cdot (1 + Y + Y') =$$

$$= X \cdot 1 = X. \quad \text{We now reached the right side of (T10) so the proof is completed.}$$

Problem 2:

$$(a) F = W \cdot X \cdot Y \cdot Z \cdot (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z + W \cdot X \cdot Y' \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z' +$$

$$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W' \cdot X \cdot Y \cdot Z +$$

$$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0$$

$$(b) F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$$

$$= A \cdot B \cdot 1 + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$$

$$= A \cdot B \cdot (1 + C' \cdot D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E =$$

$$= A \cdot B \cdot 1 + C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$$

$$(c) F = M \cdot N \cdot O + Q' \cdot P' \cdot N' + P \cdot R \cdot M + Q' \cdot O \cdot M \cdot P' + M \cdot R =$$

$$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R + P \cdot R \cdot M$$

$$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R + M \cdot R \cdot P =$$

$$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot 1 + M \cdot R \cdot P =$$

$$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot (1 + P) =$$

$$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot 1 =$$

$$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R =$$

$$= N \cdot (M \cdot O) + N' \cdot (Q' \cdot P') + \underbrace{(M \cdot O) \cdot (Q' \cdot P')}_{\substack{\uparrow \\ \text{consensus term and can} \\ \text{be eliminated (T11)}}} + M \cdot R =$$

Solutions of HW#4 cont.Problem 2(c) cont:

$$= N \cdot C \cdot 0 + N' \cdot (Q' \cdot P') + M \cdot R = \quad (\text{applying (T11)})$$

$$= N \cdot M \cdot 0 + N' \cdot Q' \cdot P' + M \cdot R$$

Problem 3: Apply (T8') first to get:

$$(A+B+C') \cdot (A'+B'+D) \cdot (A'+C+D') \cdot (A+C'+D) =$$

$$= (A+C'+B) \cdot (A+C'+D) \cdot (A'+B'+D) \cdot (A'+C+D') =$$

$$= (A+C'+B \cdot D) \cdot [A' + (B'+D) \cdot (C+D')] =$$

$$= (A+C'+B \cdot D) \cdot [A' + (D+B') \cdot (D'+C)]$$

Now apply theorem of eq. (1) on

$(D+B') \cdot (D'+C)$ to get

$$(A+C'+B \cdot D) \cdot (A' + D \cdot C + D' \cdot B') ; (\text{apply theorem of eq. (1) again})$$

$$= A \cdot (D \cdot C + D' \cdot B') + A' \cdot (C' + B \cdot D) ; (\text{apply (T8)})$$

$$= A \cdot D \cdot C + A \cdot D' \cdot B' + A' \cdot C' + A' \cdot B \cdot D$$

If we were to multiply out using only theorem (T8), we would generate $3 \times 3 \times 3 \times 3 = 81$ product terms and would have to eliminate 77 of them!!!; (too much trouble!). Here I only got 4 terms

Problem 4: Apply (T8) first to get:

$$W \cdot X \cdot Y' + W' \cdot X' \cdot Z + W \cdot Y' \cdot Z + W' \cdot Y \cdot Z' =$$

$$= W \cdot Y' \cdot X + W \cdot Y' \cdot Z + W' \cdot X' \cdot Z + W' \cdot Y \cdot Z' =$$

$$= W \cdot Y' \cdot (X+Z) + W' \cdot (X' \cdot Z + Y \cdot Z')$$

Apply now theorem of eq. (1) to get:

Problem 4 cont:

$$\begin{aligned}
 & W \cdot Y' \cdot (X+Z) + W' \cdot (X' \cdot Z + Y \cdot Z') = \\
 & = (W + X' \cdot Z + Y \cdot Z') \cdot [W' + Y' \cdot (X+Z)] \quad (\text{apply theorem of eq. (1) on } Z \cdot X' + Z' \cdot Y) \\
 & = (W + Z \cdot X' + Z' \cdot Y) \cdot [W' + Y' \cdot (X+Z)] \quad (\text{apply theorem of eq. (1) again}) \\
 & = [W + (Z+Y) \cdot (Z'+X')] \cdot [W' + Y' \cdot (X+Z)] \quad (\text{apply theorem of eq. (1) again}) \\
 & = [W + (Z+Y) \cdot (Z'+X')] \cdot [W' + Y' \cdot (X+Z)] \quad (\text{apply theorem (B')}) \\
 & = (W+Z+Y) \cdot (W+Z'+X') \cdot (W'+Y') \cdot (W'+X+Z)
 \end{aligned}$$

Problem 5: I will first provide the canonical sum and then the canonical product for each logic function.

(a) $F = \sum_{x,y} \gamma(1,2) = X' \cdot Y + X \cdot Y' = \prod_{x,y} \gamma(0,3) = (X+Y) \cdot (X'+Y')$

(b) $F = \prod_{A,B} \gamma(0,1,2) = \text{minterm } 3 = A \cdot B = \prod_{A,B} \gamma(0,1,2) = (A+B) \cdot (A+B') \cdot (A'+B)$

(c) $F = \sum_{A,B,C} \gamma(2,4,6,7) = A' \cdot B \cdot C' + A \cdot B' \cdot C' + A \cdot B \cdot C' + A \cdot B \cdot C$
 $= \prod_{A,B,C} \gamma(0,1,3,5) = (A+B+C) \cdot (A+B+C') \cdot (A+B'+C') \cdot (A'+B+C')$

↓ sorry wrap.

~~(a) $F = X' \cdot Y + X \cdot Y' = (X+Y) \cdot (X'+Y')$~~
~~(b) $F = A \cdot B = (A+B) \cdot (A+B') \cdot (A'+B)$~~
~~(c) $F = X' \cdot Y + X \cdot Y' = (X+Y) \cdot (X'+Y')$~~

← sorry Alex!!

(e) $F = X + Y' \cdot Z' = X \cdot (Y+Y') \cdot (Z+Z') + Y' \cdot Z' \cdot (X+X') =$
 $= X \cdot (Y \cdot Z + Y \cdot Z' + Y' \cdot Z + Y' \cdot Z') + Y' \cdot Z' \cdot X + Y' \cdot Z' \cdot X'$
 $= X \cdot Y \cdot Z + X \cdot Y \cdot Z' + X \cdot Y' \cdot Z + X \cdot Y' \cdot Z' + X' \cdot Y' \cdot Z' + X' \cdot Y' \cdot Z'$
 $= \sum_{x,y,z} \gamma(0,4,5,6,7) = \prod_{x,y,z} \gamma(1,2,3) = (X+Y+Z') \cdot (X+Y'+Z) \cdot (X+Y'+Z')$

(d) → Sorry that (e) is before (d). Friendly; Alex!!

$$F = \sum_{M,N,P} (0, 1, 3, 6, 7) = \sum_{M,N,P} (2, 4, 5) = M' \cdot N \cdot P' + M \cdot N' \cdot P$$

$$= \sum_{M,N,P} (0, 1, 3, 6, 7) = (M+N+P) \cdot (M+N+P) \cdot (M+N+P) \cdot (M+N+P)$$

(f) $F = A' \cdot B + B' \cdot C + A =$

$$A' \cdot B \cdot (C+C') + B' \cdot C \cdot (A+A') + A \cdot (B+B') \cdot (C+C') =$$

$$A' \cdot B \cdot C + A' \cdot B \cdot C' + A' \cdot B \cdot C' + A' \cdot B \cdot C + A \cdot B' \cdot C + A \cdot B' \cdot C' + A \cdot B \cdot C + A \cdot B \cdot C' + A \cdot B' \cdot C + A \cdot B' \cdot C'$$

↓ don't delete this

$$= \sum_{A,B,C} (1, 2, 3, 4, 6, 7) = \sum_{A,B,C} (0, 5)$$

(A+B+C) = max term 0 = A+B+C

Problem 6: (a+b) \cdot (a+c) \cdot (b+c)

$$= (a+b+c \cdot c') \cdot (a+c+b \cdot b') \cdot (b+c+a \cdot a') =$$

$$= (a+b+c) \cdot (a+b+c') \cdot (a+b+c) \cdot (a+b'+c) \cdot (a+b+c) \cdot (a'+b+c) =$$

$$(a+b+c) \cdot (a'+b+c) = \prod_{a,b,c} (0, 1, 2, 4)$$

Here I applied theorem T8 several times. Not very difficult after all!