EE 2720, Fall 06

Solutions of HW #4

EE 2720

Solutions of HW# 4

Problem 1: The left side of (T10) is: $X \cdot Y + X \cdot Y'$. This can be written as

$$Y \cdot X + Y' \cdot X = Y \cdot X + Y' \cdot X + \underbrace{X \cdot X}_{\text{consensus term}} \quad \text{(according to (T11))}$$

$$= Y \cdot X + Y' \cdot X + X = Y \cdot X + Y' \cdot X + X \cdot 1 = X \cdot (1 + Y + Y') =$$

$= X \cdot 1 = X$. We now reached the right side of (CT10) so the proof is completed.

Problem 2:

(a) $F = W \cdot X \cdot Y \cdot Z \cdot (W \cdot X \cdot Y \cdot Z' + W \cdot X' \cdot Y \cdot Z + W' \cdot X \cdot Y \cdot Z$
$+ W \cdot X \cdot Y' \cdot Z) = W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y \cdot Z' +$
$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X' \cdot Y \cdot Z + W \cdot X \cdot Y \cdot Z \cdot W' \cdot X \cdot Y \cdot Z +$
$+ W \cdot X \cdot Y \cdot Z \cdot W \cdot X \cdot Y' \cdot Z = 0 + 0 + 0 + 0 = 0$

(b) $F = A \cdot B + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$
$= A \cdot B \cdot 1 + A \cdot B \cdot C' \cdot D + A \cdot B \cdot D \cdot E' + A \cdot B \cdot C' \cdot E + C' \cdot D \cdot E$
$= A \cdot B \cdot (1 + C' \cdot D + D \cdot E' + C' \cdot E) + C' \cdot D \cdot E =$
$= A \cdot B \cdot 1 + C' \cdot D \cdot E = A \cdot B + C' \cdot D \cdot E$

(c) $F = M \cdot N \cdot O + Q' \cdot P' \cdot N' + P \cdot R \cdot M + Q' \cdot O \cdot M \cdot P' + M \cdot R =$
$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R + P \cdot R \cdot M$
$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R + M \cdot R \cdot P =$
$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot 1 + M \cdot R \cdot P =$
$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot (1 + P) =$
$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R \cdot 1 =$
$= M \cdot N \cdot O + Q' \cdot P' \cdot N' + Q' \cdot O \cdot M \cdot P' + M \cdot R =$
$= N \cdot (M \cdot O) + N' \cdot (Q' \cdot P') + \underbrace{(M \cdot O) \cdot (Q' \cdot P')}_{\text{consensus term and can be eliminated; (T11).}} + M \cdot R =$

EE 2729

Solutions of HW#4 cont.

Problem 2(c) cont:

$= N\cdot(M\cdot O) + N'\cdot(Q'\cdot P') + M\cdot R =$ (applying (T11)).

$= N\cdot M\cdot O + N'\cdot Q'\cdot P' + M\cdot R$

Problem 3: Apply (T8') first to get:

$(A+B+C')\cdot(A'+B'+D)\cdot(A'+C+D')\cdot(A+C'+D) =$

$= (A+C'+B)\cdot(A+C'+D)\cdot(A'+B'+D)\cdot(A'+C+D') =$

$= (A+C'+B\cdot D)\cdot[A'+(B'+D)\cdot(C+D')] =$

$= (A+C'+B\cdot D)\cdot[A'+(D+B')\cdot(D'+C)]$.

Now apply theorem of eq. (1) on $(D+B')\cdot(D'+C)$ to get

$(A+C'+B\cdot D)\cdot(A'+D\cdot C+D'\cdot B')$; (apply theorem of eq. (1) again)

$= A\cdot(D\cdot C+D'\cdot B') + A'\cdot(C'+B\cdot D)$; (apply (T8))

$= A\cdot D\cdot C + A\cdot D'\cdot B' + A'\cdot C' + A'\cdot B\cdot D$

If we were to multiply out using only theorem (T8), we would generate $3\times3\times3\times3 = 81$ product terms and would have to eliminate 77 of them!!!; (too much trouble!). Here I only got 4 terms

Problem 4: Apply (T8) first to get:

$W\cdot X\cdot Y' + W'\cdot X'\cdot Z + W\cdot Y'\cdot Z + W'\cdot Y\cdot Z' =$

$= W\cdot Y'\cdot X + W\cdot Y'\cdot Z + W'\cdot X'\cdot Z + W'\cdot Y\cdot Z' =$

$= W\cdot Y'\cdot(X+Z) + W'\cdot(X'\cdot Z + Y\cdot Z')$

Apply now theorem of eq. (1) to get:

EE 2720

Solutions of HW# 4 cont.

Problem 4 cont:

$W\cdot Y'\cdot(X+Z)+W'\cdot(X'\cdot Z+Y\cdot Z')=$

$=(W+X'\cdot Z+Y\cdot Z')\cdot[W'+Y'\cdot(X+Z)]=$ (apply theorem of eq. (1) on $Z\cdot X'+Z'\cdot Y$)

$=(W+Z\cdot X'+Z'\cdot Y)\cdot[W'+Y'\cdot(X+Z)]$ (apply theorem of eq. (1) again)

$=[W+(Z+Y)\cdot(Z'+X')]\cdot[W'+Y'\cdot(X+Z)]$ (apply theorem of eq. (1) again)

$=W\cdot Y'\cdot(X+Z)+W'\cdot(Z+Y)\cdot(Z'+X')$ (apply theorem (T8'))

$=[W+(Z+Y)\cdot(Z'+X')]\cdot[W'+Y'\cdot(X+Z)]$

$=(W+Z+Y)\cdot(W+Z'+X')\cdot(W'+Y')\cdot(W'+X+Z)$

Problem 5: I will first provide the canonical sum and then the canonical product for each logic function.

(a). $F=\Sigma_{X,Y}(1,2)=X'\cdot Y+X\cdot Y'=\Pi_{X,Y}(0,3)=(X+Y)\cdot(X'+Y')$

(b) $F=\Pi_{A,B}(0,1,2)=$ minterm $3=A\cdot B=\Pi_{A,B}(0,1,2)=$

$=(A+B)\cdot(A+B')\cdot(A'+B)$

(c) $F=\Sigma_{A,B,C}(2,4,6,7)=A'\cdot B\cdot C'+A\cdot B'\cdot C'+A\cdot B\cdot C'+A\cdot B\cdot C$

$=\Pi_{A,B,C}(0,1,3,5)=(A+B+C)\cdot(A+B+C')\cdot(A+B'+C')\cdot$

$\cdot(A'+B+C')$

↓sorry wrong.

~~[illegible]~~ ← Sorry Alex!!

(e) $F=X+Y'\cdot Z'=X\cdot(Y+Y')\cdot(Z+Z')+Y'\cdot Z'\cdot(X+X')=$

$=X\cdot(Y\cdot Z+Y\cdot Z'+Y'\cdot Z+Y'\cdot Z')+Y'\cdot Z'\cdot X+Y'\cdot Z'\cdot X'$

$=X\cdot Y\cdot Z+X\cdot Y\cdot Z'+X\cdot Y'\cdot Z+X\cdot Y'\cdot Z'+\cancel{X\cdot Y'\cdot Z'}+X'\cdot Y'\cdot Z'$

111 110 101 100 100 000

$=\Sigma_{X,Y,Z}(0,4,5,6,7)=\Pi_{X,Y,Z}(1,2,3)=(X+Y+Z')\cdot(X+Y'+Z)\cdot$

$\cdot(X+Y'+Z')$.

(d) $F = \prod_{M,N,P}(0,1,3,6,7) = \sum_{M,N,P}(2,4,5)$

$= M'\cdot N\cdot P' + M\cdot N'\cdot P' + M\cdot N'\cdot P$

$= \prod_{M,N,P}(0,1,3,6,7) =$

$(M+N+P)\cdot(M+N+P')\cdot(M+N'+P')\cdot(M'+N'+P)\cdot(M'+N'+P')$

(e) $F = A'\cdot B + B'\cdot C + A$

$= A'\cdot B\cdot\underbrace{(C+C')}_{1} + B'\cdot C\underbrace{(A+A')}_{1} + A\cdot\underbrace{(B+B')}_{1}\cdot\underbrace{(C+C')}_{1}$

$= A'\cdot B\cdot C + A'\cdot B\cdot C' + A\cdot B'\cdot C + A'\cdot B'\cdot C$

$+ A\cdot(B\cdot C + B\cdot C' + B'\cdot C + B'\cdot C') =$

$= A'\cdot B\cdot C + A'\cdot B\cdot C' + \cancel{A\cdot B'\cdot C} + A'\cdot B'\cdot C$

$+ A\cdot B\cdot C + A\cdot B\cdot C' + A\cdot B'\cdot C + A\cdot B'\cdot C'$

$= \sum_{A,B,C}(1,2,3,4,5,6,7) =$ maxterm $0 = A+B+C$

Problem 6: $(a+b)\cdot(a+c)\cdot(b+c)$

$$= (a+b+\underbrace{c\cdot c'}_{0})\cdot(a+c+\underbrace{b\cdot b'}_{0})\cdot(b+c+\underbrace{a\cdot a'}_{0})$$

$$= \cancel{(a+b+c)}\cdot(a+b+c')\cdot(a+b+c)\cdot(a+b'+c)$$

$$\cdot\cancel{(a+b+c)}\cdot(a'+b+c)$$

$$= \prod_{a,b,c}(0, 1, 2, 4).$$