EE 2720, fall 05

HW #4 Solutions; <u>Corrected</u>

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Solutions of HW# 4

Problem 1: The left side of (CT10) is: $X\cdot Y + X\cdot Y'$. This can be written as

$$Y\cdot X + Y'\cdot X = Y\cdot X + Y'\cdot X + \underbrace{X\cdot X}_{\text{consensus term}} \quad \text{(according to (T11))}$$

$$= Y\cdot X + Y'\cdot X + X = Y\cdot X + Y'\cdot X + X\cdot 1 = X\cdot(1+Y+Y') =$$

$= X\cdot 1 = X$. We now reached the right side of (CT10) so the proof is completed.

Problem 2:

(a) $F = W\cdot X\cdot Y\cdot Z\cdot(W\cdot X\cdot Y\cdot Z' + W\cdot X'\cdot Y\cdot Z + W'\cdot X\cdot Y\cdot Z + W\cdot X\cdot Y'\cdot Z) = W\cdot X\cdot Y\cdot Z\cdot W\cdot X\cdot Y\cdot Z' + W\cdot X\cdot Y\cdot Z\cdot W\cdot X'\cdot Y\cdot Z + W\cdot X\cdot Y\cdot Z\cdot W'\cdot X\cdot Y\cdot Z + W\cdot X\cdot Y\cdot Z\cdot W\cdot X\cdot Y'\cdot Z = 0+0+0+0 = 0$

(b) $F = A\cdot B + A\cdot B\cdot C'\cdot D + A\cdot B\cdot D\cdot E' + A\cdot B\cdot C'\cdot E + C'\cdot D\cdot E$

$= A\cdot B\cdot 1 + A\cdot B\cdot C'\cdot D + A\cdot B\cdot D\cdot E' + A\cdot B\cdot C'\cdot E + C'\cdot D\cdot E$

$= A\cdot B\cdot(1 + C'\cdot D + D\cdot E' + C'\cdot E) + C'\cdot D\cdot E =$

$= A\cdot B\cdot 1 + C'\cdot D\cdot E = A\cdot B + C'\cdot D\cdot E$

(c) $F = M\cdot N\cdot O + Q'\cdot P'\cdot N' + P\cdot R\cdot M + Q'\cdot O\cdot M\cdot P' + M\cdot R =$

$= M\cdot N\cdot O + Q'\cdot P'\cdot N' + Q'\cdot O\cdot M\cdot P' + M\cdot R + P\cdot R\cdot M$

$= M\cdot N\cdot O + Q'\cdot P'\cdot N' + Q'\cdot O\cdot M\cdot P' + M\cdot R + M\cdot R\cdot P =$

$= M\cdot N\cdot O + Q'\cdot P'\cdot N' + Q'\cdot O\cdot M\cdot P' + M\cdot R\cdot 1 + M\cdot R\cdot P =$

$= M\cdot N\cdot O + Q'\cdot P'\cdot N' + Q'\cdot O\cdot M\cdot P' + M\cdot R\cdot(1+P) =$

$= M\cdot N\cdot O + Q'\cdot P'\cdot N' + Q'\cdot O\cdot M\cdot P' + M\cdot R\cdot 1 =$

$= M\cdot N\cdot O + Q'\cdot P'\cdot N' + Q'\cdot O\cdot M\cdot P' + M\cdot R =$

$$= N\cdot(M\cdot O) + N'\cdot(Q'\cdot P') + \underbrace{(M\cdot O)\cdot(Q'\cdot P')}_{\text{consensus term and can be eliminated; (T11).}} + M\cdot R =$$

Solutions of HW#4 cont.

Problem 2(c) cont:

$= N\cdot(M\cdot O) + N'\cdot(Q'\cdot P') + M\cdot R =$ (applying (T11)).

$= N\cdot M\cdot O + N'\cdot Q'\cdot P' + M\cdot R$

Problem 3: Apply (T8') first to get:

$(A+B+C')\cdot(A'+B'+D)\cdot(A'+C+D')\cdot(A+C'+D) =$

$= (A+C'+B)\cdot(A+C'+D)\cdot(A'+B'+D)\cdot(A'+C+D') =$

$= (A+C'+B\cdot D)\cdot[A'+(B'+D)\cdot(C+D')] =$

$= (A+C'+B\cdot D)\cdot[A'+(D+B')\cdot(D'+C)]$.

Now apply theorem of eq. (1) on $(D+B')\cdot(D'+C)$ to get

$(A+C'+B\cdot D)\cdot(A'+D\cdot C+D'\cdot B')$; (apply theorem of eq. (1) again)

$= A\cdot(D\cdot C+D'\cdot B') + A'\cdot(C'+B\cdot D)$; (apply (T8))

$= A\cdot D\cdot C + A\cdot D'\cdot B' + A'\cdot C' + A'\cdot B\cdot D$

If we were to multiply out using only theorem (T8), we would generate $3\times3\times3\times3 = 81$ product terms and would have to eliminate 77 of them!!! ; (too much trouble!). Here I only got 4 terms

Problem 4: Apply (T8) first to get:

$W\cdot X\cdot Y' + W'\cdot X'\cdot Z + W\cdot Y'\cdot Z + W'\cdot Y\cdot Z' =$

$= W\cdot Y'\cdot X + W\cdot Y'\cdot Z + W'\cdot X'\cdot Z + W'\cdot Y\cdot Z' =$

$= W\cdot Y'\cdot(X+Z) + W'\cdot(X'\cdot Z + Y\cdot Z')$

Apply now theorem of eq. (1) to get:

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Problem 4 cont:

$W\cdot Y'\cdot(X+Z)+W'\cdot(X'\cdot Z+Y\cdot Z')=$

$=(W+X'\cdot Z+Y\cdot Z')\cdot[W'+Y'\cdot(X+Z)]=$ (apply theorem of eq.(1) on $Z\cdot X'+Z'\cdot Y$)

$=(W+Z\cdot X'+Z'\cdot Y)\cdot[W'+Y'\cdot(X+Z)]$ (apply theorem of eq.(1) again)

$=[W+(Z+Y)\cdot(Z'+X')]\cdot[W'+Y'\cdot(X+Z)]$ (apply theorem of eq.(1) again)

$=W\cdot Y'\cdot(X+Z)+W'\cdot(Z+Y)\cdot(Z'+X')$ (apply theorem (T8'))

$=[W+(Z+Y)\cdot(Z'+X')]\cdot[W'+Y'\cdot(X+Z)]$

$=(W+Z+Y)\cdot(W+Z'+X')\cdot(W'+Y')\cdot(W'+X+Z)$

Problem 5: I will first provide the canonical sum and then the canonical product for each logic function.

(a). $F=\Sigma_{X,Y}(1,2)=X'\cdot Y+X\cdot Y'=\Pi_{X,Y}(0,3)=(X+Y)\cdot(X'+Y')$

(b) $F=\Pi_{A,B}(0,1,2)=$ minterm 3 $=A\cdot B=\Pi_{A,B}(0,1,2)=$

$=(A+B)\cdot(A+B')\cdot(A'+B)$

(c) $F=\Sigma_{A,B,C}(2,4,6,7)=A'\cdot B\cdot C'+A\cdot B'\cdot C'+A\cdot B\cdot C'+A\cdot B\cdot C$

$=\Pi_{A,B,C}(0,1,3,5)=(A+B+C)\cdot(A+B+C')\cdot(A+B'+C')\cdot$
$\cdot(A'+B+C')$

sorry wrong.

~~[illegible]~~ ← Sorry Alex!!

(e) $F=X+Y'\cdot Z'=X\cdot(Y+Y')\cdot(Z+Z')+Y'\cdot Z'\cdot(X+X')=$

$=X\cdot(Y\cdot Z+Y\cdot Z'+Y'\cdot Z+Y'\cdot Z')+Y'\cdot Z'\cdot X+Y'\cdot Z'\cdot X'$

$=X\cdot Y\cdot Z+X\cdot Y\cdot Z'+X\cdot Y'\cdot Z+X\cdot Y'\cdot Z'+\cancel{X\cdot Y'\cdot Z'}+X'\cdot Y'\cdot Z'$

1 1 1   1 1 0   1 0 1   1 0 0   1 0 0   0 0 0

$=\Sigma_{X,Y,Z}(0,4,5,6,7)=\Pi_{X,Y,Z}(1,2,3)=(X+Y+Z')\cdot(X+Y'+Z)\cdot$
$\cdot(X+Y'+Z')$

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(d) → Sorry that (e) is before (d). Friendly; Alex!!

$F = \prod_{M,N,P}(0,1,3,6,7) = \sum_{M,N,P}(2,4,5) =$

$M' \cdot N \cdot P' + M \cdot N' \cdot P' + M \cdot N' \cdot P$

$= \prod_{M,N,P}(0,1,3,6,7) =$

$(M+N+P) \cdot (M+N+P') \cdot (M+N'+P') \cdot (M'+N'+P) \cdot (M'+N'+P')$

(f) $F = A' \cdot B + B' \cdot C + A =$

$A' \cdot B \cdot \underbrace{(C+C')}_{1} + B' \cdot C \cdot \underbrace{(A+A')}_{1} + A \cdot \underbrace{(B+B')}_{1} \cdot \underbrace{(C+C')}_{1} =$

$A' \cdot B \cdot C + A' \cdot B \cdot C' + A \cdot B' \cdot C + A' \cdot B' \cdot C +$

$A \cdot (B \cdot C + B \cdot C' + B' \cdot C + B' \cdot C') =$ don't delete this

$= A' \cdot B \cdot C + A' \cdot B \cdot C' + \cancel{A \cdot B' \cdot C} + A' \cdot B' \cdot C$

$+ A \cdot B \cdot C + A \cdot B \cdot C' + \cancel{A \cdot B' \cdot C} + A \cdot B' \cdot C'$

$= \sum_{A,B,C}(1,2,3,4,5,6,7) =$ = maxterm 0 = $A+B+C$

Problem 6: $(a+b) \cdot (a+c) \cdot (b+c)$

$= (a+b+\underbrace{c \cdot c'}_{0}) \cdot (a+c+\underbrace{b \cdot b'}_{0}) \cdot (b+c+\underbrace{a \cdot a'}_{0}) =$

$= (\cancel{a+b}+c) \cdot (a+b+c') \cdot (a+b+c) \cdot (a+b'+c) \cdot$

$(\cancel{a+b}+c) \cdot (a'+b+c) = (a+b+c) \cdot (a+b+c') \cdot$

$(a+b'+c) \cdot (a'+b+c) = \prod_{a,b,c}(0,1,2,4).$

Here I applied theorem T8' several times.
Not very difficult after all!