EE 2720, Fall 05

HW#3 Solutions.

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Solutions of HW#3

Problem 1:

Case $X=0$

- $0 \cdot 1 = 0$

Case $X=1$

- $1 \cdot 1 = 1$

Problem 2:

Case $X=0$

- $0 \cdot 0 = 0$

Case $X=1$

- $1 \cdot 0 = 0$

Problem 3:

Case $X=0$

- $0 + 0 = 0$

Case $X=1$

- $1 + 1 = 1$

Problem 4:

Case $X=0$

- $0 \cdot 0 = 0$

Case $X=1$

- $1 \cdot 1 = 1$

Problem 5:

Case $X=0$

- $(0')' = (1)' = 0$

Case $X=1$

- $(1')' = (0)' = 1$

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Solutions of HW#3 cont.

Problem 6:

Case $X=0$

— $0\cdot 0' = 0\cdot 1 = 0$

Case $X=1$

— $1\cdot 1' = 1\cdot 0 = 0$

Problem 7: The truth table is shown below:

X Y Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0 0 0	0	0	0	0
0 0 1	0	1	1	1
0 1 0	1	1	1	1
0 1 1	1	1	1	1
1 0 0	1	0	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

Looking at the two right most columns of the above truth table, we conclude that $(X+Y)+Z$ and $X+(Y+Z)$ are equal for all possible combinations of values of the variables X, Y, Z. Therefore theorem (T7) is valid and the proof is completed.

Problem 8:

$$(X+Y)\cdot(X+Y') = X\cdot X + X\cdot Y' + Y\cdot X + Y\cdot Y' = X + X\cdot Y' + Y\cdot X + 0$$
$$= X + X\cdot Y' + X\cdot Y = X\cdot(1+Y'+Y) = X\cdot 1 = X$$

Problem 9:

The right side of (T11') is

$(X+Y)\cdot(X'+Z) = X\cdot Z + X'\cdot Y$ (1)

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Problem 9 cont:

The left side of (T11') is

$(X+Y)\cdot(X'+Z)\cdot(Y+Z)=(X\cdot Z+X'\cdot Y)\cdot(Y+Z)=$
$=X\cdot Z\cdot Y+X\cdot Z\cdot Z+X'\cdot Y\cdot Y+X'\cdot Y\cdot Z=X\cdot Y\cdot Z+X\cdot Z+$
$+X'\cdot Y+X'\cdot Y\cdot Z=X\cdot Z+X'\cdot Y+X\cdot Y\cdot Z+X'\cdot Y\cdot Z=$
$=X\cdot Z+X'\cdot Y+Y\cdot Z\cdot(X+X')=X\cdot Z+X'\cdot Y+Y\cdot Z\cdot 1=$
$=X\cdot Z+X'\cdot Y+Y\cdot Z=$

consensus term and can be eliminated according to (T11)

$=X\cdot Z+X'\cdot Y$ (2)

Because both right and left side reduced to the same expression (which is $X\cdot Z+X'\cdot Y$), theorem (T11') is valid.

Problem 10:

I'll first prove that theorem (T13') is true for $n=2$ or I'll prove that $(X_1+X_2)'=X_1'\cdot X_2'$ (1). I'll prove eq. (1) using a truth table shown below:

X_1 X_2	X_1+X_2	$(X_1+X_2)'$	X_1'	X_2'	$X_1'\cdot X_2'$
0 0	0	1	1	1	1
0 1	1	0	1	0	0
1 0	1	0	0	1	0
1 1	1	0	0	0	0

Looking at the 3rd and 6th column of the above truth table, we see that $(X_1+X_2)'$ and $X_1'\cdot X_2'$ are equal for all possible combinations of values

Problem 10 cont: of the variables X_1 and X_2. Therefore eq. (1) is true and theorem (T13') holds true for $n=2$.

Assume now that theorem (T13') is true for $n=i$ or assume that

$(X_1+X_2+\cdots+X_i)' = X_1' \cdot X_2' \cdot \ldots X_i'$ (2)

We need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$(X_1+X_2+\cdots+X_i+X_{i+1})' = X_1' \cdot X_2' \cdot \ldots \cdot X_i' \cdot X_{i+1}'$

But $(X_1+X_2+\cdots X_i+X_{i+1})' = [(X_1+X_2+\cdots+X_i)+X_{i+1}]' =$

$= (X_1+X_2+\cdots+X_i)' \cdot X_{i+1}'$ according to (1)

$= X_1' \cdot X_2' \cdot \ldots \cdot X_i' \cdot X_{i+1}'$ according to (2)

The proof is now completed and the theorem is true for all values of n.

Problem 11: $(X+Y)\cdot(X'+Z) = X\cdot X' + X\cdot Z + Y\cdot X' + Y\cdot Z =$

$= 0 + X\cdot Z + X'\cdot Y + Y\cdot Z = X\cdot Z + X'\cdot Y + Y\cdot Z =$

consensus term and can be eliminated according to (T11)

$= X\cdot Z + X'\cdot Y.$