

EE 2720, Spring 05
Homework # 3 solutions

EE 2720

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Solutions of HW#3

Problem 1:

Case $X=0$

• $0 \cdot 1 = 0$

Case $X=1$

• $1 \cdot 1 = 1$

Problem 2:

Case $X=0$

— $0 \cdot 0 = 0$

Case $X=1$

— $1 \cdot 0 = 0$

Problem 3:

Case $X=0$

— $0 + 0 = 0$

Case $X=1$

— $1 + 1 = 1$

Problem 4:

Case $X=0$

— $0 \cdot 0 = 0$

Case $X=1$

— $1 \cdot 1 = 1$

Problem 5:

Case $X=0$

— $(0')' = (1)' = 0$

Case $X=1$

— $(1')' = (0)' = 1$

Solutions of HW #3 cont.Problem 6:Case $X=0$

$$- 0 \cdot 0' = 0 \cdot 1 = 0$$

Case $X=1$

$$- 1 \cdot 1' = 1 \cdot 0 = 0$$

Problem 7: The truth table is shown below:

X	Y	Z	$X+Y$	$Y+Z$	$(X+Y)+Z$	$X+(Y+Z)$
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	0	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Looking at the two right most columns of the above truth table, we conclude that $(X+Y)+Z$ and $X+(Y+Z)$ are equal for all possible combinations of values of the variables X, Y, Z . Therefore theorem (T7) is valid and the proof is completed.

Problem 8:

$$\begin{aligned} (X+Y) \cdot (X+Y') &= X \cdot X + X \cdot Y' + Y \cdot X + Y \cdot Y' = X + X \cdot Y' + Y \cdot X + 0 \\ &= X + X \cdot Y' + X \cdot Y = X \cdot (1 + Y' + Y) = X \cdot 1 = X \end{aligned}$$

Problem 9:

The right side of (T11') is

$$(X+Y) \cdot (X'+Z) = X \cdot Z + X' \cdot Y \quad (1)$$

Solutions of HW# 3 cont.Problem 9 cont.:

The left side of (T11') is

$$\begin{aligned} (X+Y) \cdot (X'+Z) \cdot (Y+Z) &= (X \cdot Z + X' \cdot Y) \cdot (Y+Z) = \\ &= X \cdot Z \cdot Y + X \cdot Z \cdot Z + X' \cdot Y \cdot Y + X' \cdot Y \cdot Z = X \cdot Y \cdot Z + X \cdot Z + \\ &+ X' \cdot Y + X' \cdot Y \cdot Z = X \cdot Z + X' \cdot Y + X \cdot Y \cdot Z + X' \cdot Y \cdot Z = \\ &= X \cdot Z + X' \cdot Y + Y \cdot Z \cdot (X+X') = X \cdot Z + X' \cdot Y + Y \cdot Z \cdot 1 = \\ &= X \cdot Z + X' \cdot Y + Y \cdot Z = \end{aligned}$$

↑
consensus term
and can be
eliminated
according to (T11)

$$= X \cdot Z + X' \cdot Y \quad (2)$$

Because both right and left side reduced to the same expression (which is $X \cdot Z + X' \cdot Y$), theorem (T11') is valid.

Problem 10: I'll first prove that theorem (T13') is true for $n=2$ or I'll prove that $(X_1+X_2)' = X_1' \cdot X_2'$ (1). I'll prove eq. (1) using a truth table shown below:

X_1	X_2	X_1+X_2	$(X_1+X_2)'$	X_1'	X_2'	$X_1' \cdot X_2'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Looking at the 3rd and 6th column of the above truth table, we see that $(X_1+X_2)'$ and $X_1' \cdot X_2'$ are equal for all possible combinations of values

Solutions of HW#3 cont.

Problem 10 cont: of the variables X_1 and X_2 . Therefore eq. (1) is true and theorem (T13') holds true for $n=2$.

Assume now that theorem (T13') is true for $n=i$ or assume that

$$(X_1 + X_2 + \dots + X_i)' = X_1' \cdot X_2' \cdot \dots \cdot X_i' \quad (2)$$

We need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1 + X_2 + \dots + X_i + X_{i+1})' = X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}'$$

$$\text{But } (X_1 + X_2 + \dots + X_i + X_{i+1})' = [(X_1 + X_2 + \dots + X_i) + X_{i+1}]'$$

$$= (X_1 + X_2 + \dots + X_i)' \cdot X_{i+1}' \quad \text{according to (1)}$$

$$= X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}' \quad \text{according to (2)}$$

The proof is now completed and the theorem is true for all values of n .

Problem 11: $(X+Y) \cdot (X'+Z) = X \cdot X' + X \cdot Z + Y \cdot X' + Y \cdot Z =$

$$= 0 + X \cdot Z + X' \cdot Y + Y \cdot Z = X \cdot Z + X' \cdot Y + Y \cdot Z =$$

$$= X \cdot Z + X' \cdot Y.$$

↑
consensus term
and can be
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according to (T11)