

EE 2720, Spring 07

Homework # 3 solutions

Homework #3 sol.

Problem 1: All the theorems are one-variable theorems. Just view two cases: $\begin{cases} \rightarrow x=0 \\ \rightarrow x=1 \end{cases}$ and they apply the axioms. Proofs are trivial. I did several in class

Problem 2: Prove th. (T7) with truth table

| $X Y Z$ | $X+Y$ | $Y+Z$ | $(X+Y)+Z$ | $X+(Y+Z)$ |
|---------|-------|-------|-----------|-----------|
| 0 0 0 | 0 | 0 | 0 | 0 |
| 0 0 1 | 0 | 1 | 1 | 1 |
| 0 1 0 | 1 | 1 | 1 | 1 |
| 0 1 1 | 1 | 1 | 1 | 1 |
| 1 0 0 | 1 | 0 | 1 | 1 |
| 1 0 1 | 1 | 1 | 1 | 1 |
| 1 1 0 | 1 | 1 | 1 | 1 |
| 1 1 1 | 1 | 1 | 1 | 1 |



Some identical columns
 $\Rightarrow (X+Y)+Z = X+(Y+Z)$
 \Rightarrow proved!

HW #3 sol. cont.

Problem 3: (T10') states:

$$(X+Y) \cdot (X+Y') = X$$

Proof:

$$\begin{aligned} (X+Y)(X+Y') &= X \cdot X + X \cdot Y' + Y \cdot X + Y \cdot Y' \\ &= X + X \cdot Y' + X \cdot Y + 0 = X + X \cdot Y' + X \cdot Y \\ &= X \cdot (1 + Y' + Y) = X \cdot 1 = X \end{aligned}$$

Problem 4: (T13') states:

$$(X_1 + X_2 + \dots + X_n)' = X_1' \cdot X_2' \cdot \dots \cdot X_n'$$

Proof: I'll first prove (T13') for $n=2$

or I'll prove that $(X_1 + X_2)' = X_1' \cdot X_2'$ (1)

I'll prove (1) using a truth table

| X_1 | X_2 | $X_1 + X_2$ | $(X_1 + X_2)'$ | X_1' | X_2' | $X_1' \cdot X_2'$ |
|-------|-------|-------------|----------------|--------|--------|-------------------|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |



identical columns \Rightarrow

eq(1) true.

Homework #3 sol. cont.

Assume now that theorem (T13') is true for $n=i$, or assume that

$$(X_1 + X_2 + \dots + X_i)' = X_1' \cdot X_2' \cdot \dots \cdot X_i' \quad (2)$$

we need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1 + X_2 + \dots + X_i + X_{i+1})' = X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}'$$

$$\text{But } (X_1 + X_2 + \dots + X_i + X_{i+1})' = \\ = [(X_1 + X_2 + \dots + X_i) + X_{i+1}]'$$

$$= (X_1 + X_2 + \dots + X_i)' \cdot X_{i+1}' \quad \text{according to } \boxed{(1)}$$

$$= X_1' \cdot X_2' \cdot \dots \cdot X_i' \cdot X_{i+1}' \quad \text{according to (2)}$$

The proof is now completed.

Homework #3 sol. contProblem 5:

$$(X+Y) \cdot (X'+Z) = X \cdot X' + X \cdot Z + Y \cdot X' + Y \cdot Z$$

$$= 0 + X \cdot Z + X' \cdot Y + Y \cdot Z$$

$$= X \cdot Z + X' \cdot Y + \underbrace{Y \cdot Z}$$

↑
consensus term
and can be eliminated
according to (T11)

$$= X \cdot Z + X' \cdot Y$$

It was an easy HW. wasn't it?

Alex