EE 2720, Fall 06

Homework # 3 solutions.

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Homework #3 sol.

Problem 1: All the theorems are one-variable theorems. Just view two cases: $\to X=0$, $\to X=1$ and then apply the axioms. Proofs are trivial. I did several in class

Problem 2: Prove th. (T7) with truth table

X Y Z	X+Y	Y+Z	(X+Y)+Z	X+(Y+Z)
0 0 0	0	0	0	0
0 0 1	0	1	1	1
0 1 0	1	1	1	1
0 1 1	1	1	1	1
1 0 0	1	0	1	1
1 0 1	1	1	1	1
1 1 0	1	1	1	1
1 1 1	1	1	1	1

Same identical columns

$\Rightarrow (X+Y)+Z = X+(Y+Z)$

$\Rightarrow$ proven!

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HW # 3 sol. cont.

Problem 3: (T10′) states:

$(X+Y)\cdot(X+Y') = X$

Proof:

$(X+Y)(X+Y') = X\cdot X + X\cdot Y' + Y\cdot X + Y\cdot Y'$

$= X + X\cdot Y' + X\cdot Y + 0 = X + X\cdot Y' + X\cdot Y$

$= X\cdot(1 + Y' + Y) = X\cdot 1 = X$

Problem 4: (T13′) states:

$(X_1+X_2+\dots+X_n)' = X_1'\cdot X_2'\cdot \dots \cdot X_n'$

Proof: I'll first prove (T13′) for n=2

or I'll prove that $(X_1+X_2)' = X_1'\cdot X_2'$ (1)

I'll prove (1) using a truth table

X_1 X_2	X_1+X_2	$(X_1+X_2)'$	X_1'	X_2'	$X_1'\cdot X_2'$
0 0	0	1	1	1	1
0 1	1	0	1	0	0
1 0	1	0	0	1	0
1 1	1	0	0	0	0

identical columns $\Rightarrow$

eq (1) true.

Assume now that theorem (T13') is true for $n=i$, or assume that

$$(X_1+X_2+\cdots X_i)' = X_1' \cdot X_2' \cdots X_i' \quad (2)$$

we need to prove that the theorem is also true for $n=i+1$ or we need to prove that

$$(X_1+X_2+\cdots+X_i+X_{i+1})' = X_1' \cdot X_2' \cdots X_i' \cdot X_{i+1}'$$

But $(X_1+X_2+\cdots+X_i+X_{i+1})' =$

$= [(X_1+X_2+\cdots+X_i)+X_{i+1}]'$

$= (X_1+X_2+\cdots+X_i)' \cdot X_{i+1}'$ according to $\boxed{(1)}$

$= X_1' \cdot X_2' \cdots X_i' \cdot X_{i+1}'$ according to (2)

The proof is now completed.

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Homework #3 sol. cont

Problem 5:

$$(X+Y)\cdot(X'+Z) = X\cdot X' + X\cdot Z + Y\cdot X' + Y\cdot Z$$

$$= 0 + X\cdot Z + X'\cdot Y + Y\cdot Z$$

$$= X\cdot Z + X'\cdot Y + \underbrace{Y\cdot Z}$$

consensus term and can be eliminated according to (T11)

$$= X\cdot Z + X'\cdot Y$$

It was an easy HW. wasn't it?

Alex