

EE 2720, Spring 07

Homework #2 solutions

EE 2720
Homework #2 Solutions

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Problem 1: The Dynamic Range is DR where
 DR is $DR = [-(2^{6-1}-1) + (2^{6-1}-1)] = [-31+31]$.

Problem 2: $+27_{10} = 011011_2$

Problem 3: $-20_{10} = 110100_2$

Problem 4: The first way is: $r=10, n=5$, so
 10^5 -complement of 35865 = $10^5 - 35865 = 64135$.
 For the second way we have $r=10$ so $r-1=9$.
 The digit 3 becomes $9-3=6$, the digit 5 be-
 comes $9-5=4$, the digit 8 becomes $9-8=1$,
 the digit 6 becomes $9-6=3$ and the digit 5
 becomes $9-5=4$. Thus

10^5 -complement of 35865 = $64134+1 = 64135$.
Problem 5: $DR = [-(2^{8-1}-1) + (2^{8-1}-1)] = [-128+127]$

Problem 6: $11011011_2 = -2^7 \times 1 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 +$
 $+ 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = -128 + 64 + 16 + 8 + 2 + 1$
 $= -128 + 91 = -37_{10}$

Problem 7: 11011010
 \downarrow complement bits
 00100101
 $+ \quad \quad \quad 1$
 $\hline 00100110$

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Problem 8:

$$\begin{array}{r} 101010 \quad \uparrow \leftarrow \text{initial cin of 1} \\ + 111010 \quad \text{complementing bits of Y} \\ \hline 1100101 \end{array}$$

$\rightarrow X - Y = 100101_2 = -27_{10}$

1 is overall carry out and must be ignored

Problem 9:

$$\begin{array}{r} 011001 \quad \leftarrow \text{positive number} \\ + 011011 \quad \leftarrow \text{positive number} \\ \hline 0110100 \quad \leftarrow \text{negative result; (wrong)} \end{array}$$

0 is overall carry out and must be ignored

\rightarrow sign bit = 1. This means that we got a negative result; (observe that the obtained result is $110100_2 = -12_{10} < 0$). Here an overflow occurred. Remember that

the Dynamic Range (DR) of a 6-bit integer two's-complement system is $DR = [-2^{6-1}, +2^{6-1}] = [-32, +31]$ and $X + Y = +25 + 27 = +52 > 31$ which implies overflow.

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Problem 10:
$$\begin{array}{r} 101100 \leftarrow \text{negative number} \\ + 110001 \leftarrow \text{negative number} \\ \hline 1011101 \leftarrow \text{positive result; (wrong)} \end{array}$$

1 is overall carry out and must be ignored.

sign bit = 0. This means that we got a positive result; (observe that the obtained result is $011101_2 = +29_{10} > 0$ which is wrong). Here an underflow

occurred. As we said in Problem 9 the Dynamic Range (DR) of a 6-bit integer two's-complement system is $DR = [-32, +31]$ and $X+Y = (-20) + (-15) = -35 < -32$ which implies underflow.

Problem 11: The first way is: $r=10, n=5$, so 9^5 -complement of 85357 = $10^5 - 1 - 85357 = 14642$. For the second way we have $r=10$ so $r-1=9$. The digit 8 becomes $9-8=1$, the digit 5 becomes $9-5=4$, the digit 3 becomes $9-3=6$, the digit 5 becomes $9-5=4$ and the digit 7 becomes $9-7=2$. Thus

9^5 -complement of 85357 = 14642.

Problem 12: $DR = [-(2^{7-1}-1) \quad + (2^{7-1}-1)] = [-63 \quad +63]$

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Problem 13: $101110_2 = -(2^{6-1}-1) \times 1 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = -(2^5-1) \times 1 + 8 + 4 + 2 = -31 + 14 = -17_{10}$.

Problem 14: 11011010
 \downarrow complement bits
 00100101

Problem 15: The ones'-complement of Y is ones'-complement of $(000101) = 111010$. We now have

$$\begin{array}{r} 101010 \\ + 111010 \\ \hline 1100100 \end{array}$$

$$\begin{array}{r} 100100 \\ + \quad \quad 1 \text{ adding carry out} \\ \hline 100101 \end{array}$$

1 is overall carry out of addition and must be added back to the result.

$\rightarrow X - Y = 100101_2 = -26_{10}$.

Problem 16: $011001 \leftarrow$ positive number
 $+ 011011 \leftarrow$ positive number
 $\hline 110100 \leftarrow$ negative result; (wrong)

\rightarrow sign bit = 1. This means that we got a negative result; (observe that the obtained result = $110100_2 = -11_{10} < 0$). Here an overflow occurred. Remember that the Dynamic Range (DR) of a 6-bit integer ones'-complement system is $DR = [-(2^{6-1}-1) + (2^{6-1}-1)] =$

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Problem 16 cont: $= [-31 \ +31]$ and $X+Y = +25+27 = +52 > 31$ which implies overflow.

Problem 17: $101100 \leftarrow$ negative number
 $+ 110001 \leftarrow$ negative number
 $\hline 1011101$

↑
1 is overall carry out of addition and must be added back to the result

011101
 $+ \quad \quad \quad 1$ adding carry out
 $\hline 011110 \leftarrow$ positive result; (wrong).

↳ sign bit = 0. This means that we got a positive result; (observe that the obtained result is $011110_2 = +30_{10} > 0$). Here an underflow occurred. As we said in Problem 16, the Dynamic Range (DR) of a 6-bit integer ones'-complement system is $DR = [-31 \ +31]$ and $X+Y = (-19) + (-14) = -33 < -31$ which implies underflow.

Problem 18: Here the two numbers X and Y are of different signs and the addition $X+Y$ needs to be performed. We thus have to perform

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Problem 18 cont: the following subtraction:

$$\begin{aligned} & (\text{magnitude of } X) - (\text{magnitude of } Y) = \\ & = (10101) - (11111) = \\ & = (10101) + (\text{two's-complement of } (11111)) = \\ & = (10101) + (00001). \text{ We now have} \end{aligned}$$

$$\begin{array}{r} 10101 \\ 00001 \\ \hline 010110 \end{array}$$

↳ since $c=0$ it means $\text{result} < 0$ or

$$(\text{magnitude of } X) - (\text{magnitude of } Y) < 0$$

or $\text{magnitude of } X < \text{magnitude of } Y$.

Therefore

- sign bit of result $X+Y$ should be the sign bit of the number with the larger magnitude = sign bit of $Y = 1$

and

- magnitude of $X+Y =$
 $= \text{two's-complement of } (10110) = 01010$

Therefore $X+Y = 101010_2 = -10_{10}$.

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Problem 19: 1100 multiplicand
x 1111 multiplier

$$\begin{array}{r} 1100 \\ + 1100 \\ \hline 100100 \\ + 1100 \\ \hline 1010100 \\ + 1100 \\ \hline 10110100 \end{array} \text{ product} = 180_{10}$$

Problem 20: 1001 multiplicand
x 1010 multiplier

$$\begin{array}{r} 0000 \\ + 1001 \\ \hline 110010 \\ + 0000 \\ \hline 1110010 \\ + 0111 \\ \hline 10101010 \end{array}$$

← shifted and negated multiplicand

ignore this carry out

↳ product = 0101010₂ = +42₁₀.

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Problem 21:

$$\begin{array}{r}
 8 \\
 + 7 \\
 \hline
 15
 \end{array}
 \qquad
 \begin{array}{r}
 1000 \\
 + 0111 \\
 \hline
 1111 \\
 + 0110 \\
 \hline
 10101
 \end{array}$$

correction needed

$$\begin{array}{r}
 \underbrace{1}_{1} \quad \underbrace{0101}_{5} \\
 \leftarrow \text{result is 15}
 \end{array}$$

Problem 22:

$$\begin{array}{r}
 3 \\
 + 4 \\
 \hline
 7
 \end{array}
 \qquad
 \begin{array}{r}
 0011 \\
 + 0100 \\
 \hline
 0111
 \end{array}$$

← no correction needed

7 ← result is 7.

Problem 23: Starting from the 3-bit code we get:

append 0	→	0000	} in order	} 4-bit Gray code.	
"	0	→			0001
"	0	→			0011
"	0	→			0010
"	0	→			0110
"	0	→			0111
"	0	→			0101
"	0	→	0100		} in reverse order
"	1	→	1100		
"	1	→	1101		
"	1	→	1111		
"	1	→	1110		
"	1	→	1010		
"	1	→	1011		
"	1	→	1001	}	
"	1	→	1000		