EE 2720, Spr. 06

HW #2 solutions

EE 2720
Homework #2 Solutions

Problem 1: The Dynamic Range is DR where DR is $DR = [-(2^{6-1}-1) \quad +(2^{6-1}-1)] = [-31 \; +31]$.

Problem 2: $+27_{10} = 011011_2$

Problem 3: $-20_{10} = 110100_2$

Problem 4: The first way is: $r=10$, $n=5$, so 10's-complement of $35865 = 10^5 - 35865 = 64135$.
For the second way we have $r=10$ so $r-1=9$. The digit 3 becomes $9-3=6$, the digit 5 becomes $9-5=4$, the digit 8 becomes $9-8=1$, the digit 6 becomes $9-6=3$ and the digit 5 becomes $9-5=4$. Thus
10's-complement of $35865 = 64134+1 = 64135$.

Problem 5: $DR = [-2^{8-1} \quad +(2^{8-1}-1)] = [-128 \; +127]$.

Problem 6: $11011011_2 = -2^7 \times 1 + 1\times 2^6 + 0\times 2^5 + 1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 = -128+64+16+8+2+1 = -128+91 = -37_{10}$

Problem 7: 11011010

↓ complement bits

$$\begin{array}{r} 00100101 \\ +\quad\quad\quad 1 \\ \hline 00100110 \end{array}$$

Problem 8:

$$\begin{array}{r} 1 \leftarrow \text{initial Cin of 1} \\ 101010 \\ +\ 111010 \quad \text{complementing bits of Y} \\ \hline 1\,\underbrace{100101} \end{array}$$

$\rightarrow X-Y=100101_2=-27_{10}$.

1 is overall carry out and must be ignored

Problem 9:

$$\begin{array}{r} 011001 \leftarrow \text{positive number} \\ +\ 011011 \leftarrow \text{positive number} \\ \hline 0\ 110100 \leftarrow \text{negative result; (wrong)}. \end{array}$$

0 is overall carry out and must be ignored

$\rightarrow$ sign bit $=1$. This means that we got a negative result; (observe that the obtained result is $110100_2=-12_{10}<0$). Here an overflow occured. Remember that the Dynamic Range (DR) of a 6-bit integer two's-complement system is $DR=[-2^{6-1} \;\; +(2^{6-1}-1)] = [-32 \;\; +31]$ and $X+Y=+25+27=+52>31$ which implies overflow.

Problem 10:

$$
\begin{array}{r}
101100 \\
+\ 110001 \\
\hline
1\ 011101
\end{array}
$$

101100 ← negative number
110001 ← negative number
011101 ← positive result; (wrong)

1 is overal carry out and must be ignored.

↳ sign bit = 0. This means that we got a positive result; (observe that the obtained result is $011101_2 = +29_{10} > 0$ which is wrong). Here an underflow occured. As we said in Problem 9 the Dynamic Range (DR) of a 6-bit integer two's-complement system is DR = [−32 +31] and $X+Y = (-20)+(-15) = -35 < -32$ which implies underflow.

Problem 11: The first way is: $r=10$, $n=5$, so 9's-complement of $85357 = 10^5 - 1 - 85357 = 14642$.
For the second way we have $r=10$ so $r-1=9$. The digit 8 becomes $9-8=1$, the digit 5 becomes $9-5=4$, the digit 3 becomes $9-3=6$, the digit 5 becomes $9-5=4$ and the digit 7 becomes $9-7=2$. Thus
9's-complement of 85357 = 14642.

Problem 12: $DR = [-(2^{7-1}-1) \quad +(2^{7-1}-1)] = [-63 \quad +63]$

Problem 13: $101110_2 = -(2^{6-1}-1)\times 1 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 = -(2^5-1)\times 1 + 8 + 4 + 2 = -31 + 14 = -17_{10}$.

Problem 14: 11011010

$\downarrow$ complement bits

00100101

Problem 15: The ones'-complement of Y is ones'-complement of (000101) = 111010. We now have

$$\begin{array}{r} 101010 \\ +\ 111010 \\ \hline 1\ 100100 \end{array} \qquad \begin{array}{r} 100100 \\ +\quad\ \ 1 \\ \hline 100101 \end{array} \text{ adding carry out}$$

1 is overal carry out of addition and must be added back to the result.

$\hookrightarrow X - Y = 100101_2 = -26_{10}$.

Problem 16:

$$\begin{array}{r} 011001 \\ +\ 011011 \\ \hline 110100 \end{array} \begin{array}{l} \leftarrow \text{positive number} \\ \leftarrow \text{positive number} \\ \leftarrow \text{negative result; (wrong)} \end{array}$$

$\hookrightarrow$ sign bit = 1. This means that we got a negative result; (observe that the obtained result $= 110100_2 = -11_{10} < 0$). Here an overflow occured. Remember that the Dynamic Range (DR) of a 6-bit integer ones'-complement system is $DR = [-(2^{6-1}-1)\ \ +(2^{6-1}-1)] =$

Problem 16 cont: $= [-31 \ \ +31]$ and $X+Y = +25+27 = +52 > 31$ which implies overflow.

Problem 17:

$$
\begin{array}{r}
101100 \leftarrow \text{negative number} \\
+\ 110001 \leftarrow \text{negative number} \\
\hline
1011101
\end{array}
$$

1 is overal carry out of addition and must be added back to the result

$$
\begin{array}{r}
011101 \\
+ \qquad\quad 1 \ \text{adding carry out} \\
\hline
011110 \leftarrow \text{positive result; (wrong).}
\end{array}
$$

↳ sign bit=0. This means that we got a positive result; (observe that the obtained result is $011110_2 = +30_{10} > 0$). Here an underflow occured. As we said in Problem 16, the Dynamic Range (DR) of a 6-bit integer ones'-complement system is $DR = [-31 \ \ +31]$ and $X+Y = (-19)+(-14) = -33 < -31$ which implies underflow.

Problem 18: Here the two numbers X and Y are of different signs and the addition X+Y needs to be performed. We thus have to perform

Problem 18 cont: the following subtraction:
(magnitude of X) − (magnitude of Y) =
= (10101) − (11111) =
= (10101) + (two's-complement of (11111)) =
= (10101) + (00001). We now have

$$\begin{array}{r} 10101 \\ 00001 \\ \hline 0\,10110 \end{array}$$

↳ since c = 0 it means result < 0 or
(magnitude of X) − (magnitude of Y) < 0,
or magnitude of X < magnitude of Y.

Therefore

- sign bit of result X+Y should be the sign bit of the number with the larger magnitude = sign bit of Y = 1

and

- magnitude of X+Y =
= two's-complement of (10110) = 01010

Therefore $X+Y = 101010_2 = -10_{10}$.

Problem 19:

$$
\begin{array}{r l}
1100 & \text{multiplicand} \\
\times\ 1111 & \text{multiplier} \\
\hline
1100 & \\
+\ 1100 & \\
\hline
100100 & \\
+\ 1100 & \\
\hline
1010100 & \\
+\ 1100 & \\
\hline
10110100 & \text{product} = 180_{10}
\end{array}
$$

Problem 20:

$$
\begin{array}{r l}
1001 & \text{multiplicand} \\
\times\ 1010 & \text{multiplier} \\
\hline
0000 & \\
+\ 1001 & \\
\hline
110010 & \\
+\ 0000 & \\
\hline
1110010 & \\
+\ 0111 & \leftarrow \text{shifted and negated multiplicand} \\
\hline
1\ \underbrace{0101010} &
\end{array}
$$

ignore this carry out

$\rightarrow$ product $= 0101010_2 = +42_{10}$.

Problem 21:

$$
\begin{array}{r}
8 \\
+\ 7 \\
\hline
15
\end{array}
\qquad
\begin{array}{r}
1000 \\
+\ 0111 \\
\hline
1111 \\
+\ 0110 \\
\hline
\underbrace{1}_{1}\,\underbrace{0101}_{5}
\end{array}
$$

1111 correction needed

1 5 ← result is 15

Problem 22:

$$
\begin{array}{r}
3 \\
+\ 4 \\
\hline
7
\end{array}
\qquad
\begin{array}{r}
0011 \\
+\ 0100 \\
\hline
\underbrace{0111}_{7}
\end{array}
$$

0111 ← no correction needed

7 ← result is 7.

Problem 23: Starting from the 3-bit code we get:

$$
\left.
\begin{array}{l}
\left.
\begin{array}{lcl}
\text{append } 0 & \rightarrow & 0000 \\
\quad '' \quad 0 & \rightarrow & 0001 \\
\quad '' \quad 0 & \rightarrow & 0011 \\
\quad '' \quad 0 & \rightarrow & 0010 \\
\quad '' \quad 0 & \rightarrow & 0110 \\
\quad '' \quad 0 & \rightarrow & 0111 \\
\quad '' \quad 0 & \rightarrow & 0101 \\
\quad '' \quad 0 & \rightarrow & 0100
\end{array}
\right\} \text{in order} \\
\left.
\begin{array}{lcl}
\quad '' \quad 1 & \rightarrow & 1100 \\
\quad '' \quad 1 & \rightarrow & 1101 \\
\quad '' \quad 1 & \rightarrow & 1111 \\
\quad '' \quad 1 & \rightarrow & 1110 \\
\quad '' \quad 1 & \rightarrow & 1010 \\
\quad '' \quad 1 & \rightarrow & 1011 \\
\quad '' \quad 1 & \rightarrow & 1001 \\
\quad '' \quad 1 & \rightarrow & 1000
\end{array}
\right\} \text{in reverse order}
\end{array}
\right\} \text{4-bit Gray code.}
$$