EE 2720, Fall 05

HW# 2 Solutions

EE 2720
Homework #2 Solutions

Problem 1: The Dynamic Range is DR where DR is $DR = [-(2^{6-1}-1) \quad +(2^{6-1}-1)] = [-31 \; +31]$.

Problem 2: $+27_{10} = 011011_2$

Problem 3: $-20_{10} = 110100_2$

Problem 4: The first way is: $r=10$, $n=5$, so 10's-complement of $35865 = 10^5 - 35865 = 64135$.
For the second way we have $r=10$ so $r-1=9$. The digit 3 becomes $9-3=6$, the digit 5 becomes $9-5=4$, the digit 8 becomes $9-8=1$, the digit 6 becomes $9-6=3$ and the digit 5 becomes $9-5=4$. Thus
10's-complement of $35865 = 64134 + 1 = 64135$.

Problem 5: $DR = [-2^{8-1} \quad +(2^{8-1}-1)] = [-128 \; +127]$.

Problem 6: $11011011_2 = -2^7 \times 1 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = -128 + 64 + 16 + 8 + 2 + 1 = -128 + 91 = -37_{10}$

Problem 7: 11011010

$\downarrow$ complement bits

```
  00100101
+        1
----------
  00100110
```

Problem 8:

$$
\begin{array}{rl}
 & 1 \leftarrow \text{initial } C_{in} \text{ of } 1 \\
 & 101010 \\
+ & 111010 \quad \text{complementing bits of } Y \\
\hline
1 & \underbrace{100101}
\end{array}
$$

$\rightarrow X - Y = 100101_2 = -27_{10}$.

1 is overall carry out and must be ignored

Problem 9:

$$
\begin{array}{rl}
 & 011001 \leftarrow \text{positive number} \\
+ & 011011 \leftarrow \text{positive number} \\
\hline
0 & 110100 \leftarrow \text{negative result; (wrong).}
\end{array}
$$

0 is overall carry out and must be ignored

$\rightarrow$ sign bit = 1. This means that we got a negative result; (observe that the obtained result is $110100_2 = -12_{10} < 0$). Here an overflow occured. Remember that the Dynamic Range (DR) of a 6-bit integer two's-complement system is $DR = [-2^{6-1} \;\; +(2^{6-1}-1)] = [-32 \;\; +31]$ and $X+Y = +25+27 = +52 > 31$ which implies overflow.

Problem 10:

$$\begin{array}{r} 101100 \leftarrow \text{negative number} \\ +\ 110001 \leftarrow \text{negative number} \\ \hline 1\ 011101 \leftarrow \text{positive result; (wrong)} \end{array}$$

1 is overal carry out and must be ignored.

→ sign bit = 0. This means that we got a positive result; (observe that the obtained result is $011101_2 = +29_{10} > 0$ which is wrong). Here an underflow occured. As we said in Problem 9 the Dynamic Range (DR) of a 6-bit integer two's-complement system is DR = [−32 +31] and $X+Y = (-20)+(-15) = -35 < -32$ which implies underflow.

Problem 11: The first way is: $r=10$, $n=5$, so 9's-complement of $85357 = 10^5 - 1 - 85357 = 14642$.

For the second way we have $r=10$ so $r-1=9$. The digit 8 becomes $9-8=1$, the digit 5 becomes $9-5=4$, the digit 3 becomes $9-3=6$, the digit 5 becomes $9-5=4$ and the digit 7 becomes $9-7=2$. Thus

9's-complement of 85357 = 14642.

Problem 12: $DR = [-(2^{7-1}-1) \quad +(2^{7-1}-1)] = [-63 \quad +63]$

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Problem 13: $101110_2 = -(2^{6-1}-1)\times 1 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 1\times 2^1 + 0\times 2^0 = -(2^5-1)\times 1 + 8 + 4 + 2 = -31 + 14 = -17_{10}$.

Problem 14:

$$\begin{array}{c} 11011010 \\ \downarrow \text{complement bits} \\ 00100101 \end{array}$$

Problem 15: The ones'-complement of Y is ones'-complement of (000101) = 111010. We now have

$$\begin{array}{r} 101010 \\ +\ 111010 \\ \hline 1\ 100100 \end{array} \qquad \begin{array}{r} 100100 \\ +\ \ \ \ \ 1 \\ \hline 100101 \end{array} \text{ adding carry out}$$

1 is overal carry out of addition and must be added back to the result.

$\hookrightarrow X - Y = 100101_2 = -26_{10}$.

Problem 16:

$$\begin{array}{r} 011001 \\ +\ 011011 \\ \hline 110100 \end{array} \begin{array}{l} \leftarrow \text{positive number} \\ \leftarrow \text{positive number} \\ \leftarrow \text{negative result; (wrong)} \end{array}$$

$\hookrightarrow$ sign bit = 1. This means that we got a negative result; (observe that the obtained result $= 110100_2 = -11_{10} < 0$). Here an overflow occured. Remember that the Dynamic Range (DR) of a 6-bit integer ones'-complement system is $DR = [-(2^{6-1}-1)\ \ +(2^{6-1}-1)] =$

Problem 16 cont: $=[-31 \;\; +31]$ and $X+Y = +25+27=+52>31$ which implies overflow.

Problem 17:

$$\begin{array}{r} 101100 \leftarrow \text{negative number} \\ +\,110001 \leftarrow \text{negative number} \\ \hline 1011101 \end{array}$$

1 is overal carry out of addition and must be added back to the result

$$\begin{array}{r} 011101 \\ +\qquad 1 \text{ adding carry out} \\ \hline 011110 \leftarrow \text{positive result; (wrong).} \end{array}$$

↳ sign bit=0. This means that we got a positive result; (observe that the obtained result is $011110_2 = +30_{10} > 0$). Here an underflow occured. As we said in Problem 16, the Dynamic Range (DR) of a 6-bit integer ones'-complement system is $DR=[-31 \;\; +31]$ and $X+Y=(-19)+(-14)=-33<-31$ which implies underflow.

Problem 18: Here the two numbers X and Y are of different signs and the addition X+Y needs to be performed. We thus have to perform

Problem 18 cont: the following subtraction:
(magnitude of X) − (magnitude of Y) =
= (10101) − (11111) =
= (10101) + (two's-complement of (11111)) =
= (10101) + (00001). We now have

$$\begin{array}{r} 10101 \\ 00001 \\ \hline 0\,10110 \end{array}$$

↳ since c=0 it means result < 0 or
(magnitude of X) − (magnitude of Y) < 0

or magnitude of X < magnitude of Y.

Therefore

- sign bit of result X+Y should be the sign bit of the number with the larger magnitude = sign bit of Y = 1

and

- magnitude of X+Y =

= two's-complement of (10110) = 01010

Therefore $X+Y = 101010_2 = -10_{10}$.

Problem 19:

```
         1100     multiplicand
       x 1111     multiplier
       ------
         1100
      + 1100
      -------
       100100
    + 1100
    --------
     1010100
   + 1100
   ---------
    10110100      product = 180₁₀
```

Problem 20:

```
         1001     multiplicand
       x 1010     multiplier
       ------
         0000
      + 1001
      -------
       110010
    + 0000
    --------
     1110010
   + 0111         <- shifted and negated multiplicand
   ---------
   1 0101010
```

ignore this carry out: 1

$\underbrace{0101010}$ → product $= 0101010_2 = +42_{10}$.

$$\begin{array}{r} +4 \\ \hline 7 \end{array} \qquad \begin{array}{c} \hline \underbrace{0111}_{7} \end{array}$$

0111 ← no correction needed

7 ← result is 7.

<u>Problem 23:</u> Starting from the 3-bitcode we get:

append 0 → 0000
" 0 → 0001
" 0 → 0011
" 0 → 0010
" 0 → 0110
" 0 → 0111
" 0 → 0101
" 0 → 0100

in order

" 1 → 1100
" 1 → 1101
" 1 → 1111
" 1 → 1110
" 1 → 1010
" 1 → 1011
" 1 → 1001
" 1 → 1000

in reverse order

4-bit Gray code.