

EE 2720, Spring 07

HW # 1 solutions

Solutions of Homework #1

Problem 1:  $10011101.011_2 = 1 \times 2^7 + 0 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} + 1 \times 2^{-3}$   
 $= 128 + 0 + 0 + 16 + 8 + 4 + 0 + 1 + 0 + 0.25 + 0.125 =$   
 $= 157.375_{10}$

Problem 2:  $75.6_8 = 7 \times 8^1 + 5 \times 8^0 + 6 \times 8^{-1} = 56 + 5 +$   
 $0.75 = 61.75_{10}$

Problem 3:  $5AC.3_{16} = 5 \times 16^2 + 10 \times 16^1 + 12 \times 16^0 +$   
 $+ 3 \times 16^{-1} = 5 \times 256 + 10 \times 16 + 12 \times 1 + 0.1875 =$   
 $= 1280 + 160 + 12 + 0.1875 = 1452.1875_{10}$

Problem 4:  $423.1_5 = 4 \times 5^2 + 2 \times 5^1 + 3 \times 5^0 + 1 \times 5^{-1} =$   
 $= 4 \times 25 + 2 \times 5 + 3 \times 1 + 1 \times 0.2 = 100 + 10 + 3 + 0.2 =$   
 $113.2_{10}$

Problem 5: The length of the integer part is 10 bits which is not a multiple of 3 so we put two zeroes at its left to make its length 12 bits; (12 is multiple of 3). The length of the fractional part is 2 bits which is not a multiple of 3 so we put a zero at its right to make its length 3 bits. The number now becomes

$\underbrace{001}_{12} \underbrace{010}_{12} \underbrace{011}_{12} \underbrace{101}_{12} . \underbrace{110}_{12} \underbrace{0}_{12} = 1235.6_8$

Solutions of HW#1 cont.Problem 6:  $4567.75_{10} =$ 

$$= 100101110111.111101_2$$

Problem 7: The length of the integer part is 10 bits which is not a multiple of 4 so we put two zeroes at its left to make its length 12 bits; (12 is a multiple of 4). The length of the fractional part is 2 bits which is not a multiple of 4 so we put two zeroes at its right to make its length 4 bits. The number now becomes

$$\underbrace{0010} \underbrace{1001} \underbrace{1101} \underbrace{.1100}_2 = 29D.C_{16}$$

Problem 8:  $58E.A6_{16} =$ 

$$010110001110.10100110_2$$

Problem 9: The procedure is very simple. Separate the bits of the binary number into groups of two bits and replace each group with the corresponding radix-4 digit; (the radix-4 digits are 0, 1, 2, 3). Put zero~~s~~ at the left of the integer part of the binary number and at the right of the fractional part of the binary number to make the length of both integer and fractional part multiple of 2.

Solutions of HW#1 contProblem 9 cont:

Here the length of the integer part is 7 bits which is not a multiple of 2 so we put a zero at its left to make its length 8 bits; (8 is a multiple of 2). The length of the fractional part is 3 bits which is not a multiple of 2 so we put a zero at its right to make its length 4 bits; (4 is a multiple of 2). The number now becomes

$$\underbrace{01}_{\text{8 bits}} \underbrace{01}_{\text{8 bits}} \underbrace{11}_{\text{8 bits}} \underbrace{01}_{\text{8 bits}} \cdot \underbrace{11}_{\text{8 bits}} \underbrace{10}_{\text{8 bits}}_2 = 1131.324$$

Problem 10:

Integer part of 173

	Quotient	Remainder
173/2	86	1 LSB
86/2	43	0
43/2	21	1
21/2	10	1
10/2	5	0
5/2	2	1
2/2	1	0
1/2	0	1 MSB

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Problem 10 cont:

Fractional part of 0.625

	Fractional part	Integer part
$0.625 \times 2 = 1.25$	0.25	1 MSB
$0.25 \times 2 = 0.5$	0.5	0
$0.5 \times 2 = 1.0$	0.0	1 LSB

So  $173.625_{10} = 10101101.101_2$

Problem 11:

	Fractional part	Integer part
$0.7 \times 2 = 1.4$	0.4	1 MSB
$0.4 \times 2 = 0.8$	0.8	0
$0.8 \times 2 = 1.6$	0.6	1
$0.6 \times 2 = 1.2$	0.2	1
$0.2 \times 2 = 0.4$	0.4	0
$0.4 \times 2 = 0.8$	0.8	0
$0.8 \times 2 = 1.6$	0.6	1
$0.6 \times 2 = 1.2$	0.2	1
$0.2 \times 2 = 0.4$	0.4	0

Process does not terminate. What you get is

$0.7_{10} = 0.1 \underbrace{0110} \underbrace{0110} \underbrace{0110} \dots_2$ . It is a binary fraction with infinite number of bits.

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Problem 12:

Integer part of 785

	Quotient	Remainder
785/8	98	1 LSD
98/8	12	2
12/8	1	4
1/8	0	1 MSD

Fractional part of 0.125

	Fractional part	Integer part
$0.125 \times 8 = 1.0$	0.0	1 MSD

So  $785.125_{10} = 1421.1_8$

Double check if you want.  $1421.1_8 = 1 \times 8^3 + 4 \times 8^2 + 2 \times 8^1 + 1 \times 8^0 + 1 \times 8^{-1} = 1 \times 512 + 4 \times 64 + 2 \times 8 + 1 \times 1 + \frac{1}{8} = 785.125_{10}$

Problem 13:

Integer part of 1895

	Quotient	Remainder
1895/16	118	7 LSD
118/16	7	6
7/16	0	7 MSD

Fractional part of 0.625

	Fractional part	Integer part
$0.625 \times 16 = 10.00$	0.0	10 MSD

So  $1895.625_{10} = 767.A_{16}$

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Problem 13 cont:

Double check if you want.  $767.A_{16} = 7 \times 16^2 + 6 \times 16^1 + 7 \times 16^0 + 10 \times 16^{-1} = 7 \times 256 + 6 \times 16 + 7 \times 1 + \frac{10}{16} = 1895.625_{10}$

Problem 14: I first convert  $231.3_4$  into a decimal number.  $231.3_4 = 2 \times 4^2 + 3 \times 4^1 + 1 \times 4^0 + 3 \times 4^{-1} = 2 \times 16 + 3 \times 4 + 1 \times 1 + \frac{3}{4} = 32 + 12 + 1 + 0.75 = 45.75_{10}$   
 I now convert  $45.75_{10}$  into a radix-7 number

Integer part or 45

	Quotient	Remainder
45/7	6	3 LSD
6/7	0	6 MSD

Fractional part or 0.75

	Fractional part	Integer part
$0.75 \times 7 = 5.25$	0.25	5 MSD
$0.25 \times 7 = 1.75$	0.75	1
$0.75 \times 7 = 5.25$	0.25	5
$0.25 \times 7 = 1.75$	0.75	1

The same fractional digits keep repeated over and over again. The process does not terminate. What you get is

$231.3_4 = 63.515151 \dots_7$ . The fractional part of the radix-7 number has infinite number of digits.

Solutions of HW#1 cont

Problem 15:  $DR = [0 \ 2^{10} - 1] = [0 \ 1024 - 1] =$   
 $= [0 \ 1023]$

Problem 16: The Dynamic Range (DR) of a 6-bit binary unsigned system is  $DR = [0 \ 2^6 - 1] = [0 \ 63]$ . The addition follows

$$\begin{array}{r}
 000100 \text{ carry} \\
 101011 \\
 + 010010 \\
 \hline
 0111101 \text{ sum} \\
 \uparrow
 \end{array}$$

Here overall carry out is  $c=0$ . So overflow did not occur. The correct result is  $X+Y = 111101_2 = 61_{10}$ . This result is within the Dynamic Range (DR); (Remember  $DR = [0 \ 63]$ ). Just see that  $61 < 63$  which means no overflow.

Problem 17: As we said in problem 16  $DR = [0 \ 63]$ . The addition follows

$$\begin{array}{r}
 111110 \text{ carry} \\
 101111 \\
 + 010111 \\
 \hline
 1000110 \text{ sum} \\
 \uparrow
 \end{array}$$

Here overall carry out is  $c=1$ . So overflow occurred. The obtained result is  $X+Y = 1000110_2 = 70_{10}$  which is outside the Dynamic Range. Just see that  $70 > 63$  which implies overflow.