EE7150 Theory and Application of Digital Signal Processing

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Quiz 1, Spring of 2005

Solution

An $N$-point real-valued discrete-time sequence $x[n]$ is given by $x[n] = \{x_0, x_1, \ldots, x_{N-1}\}$.

The corresponding DFT is

$$X[k] = [X_0, X_1, \ldots, X_{N-1}],$$

and the corresponding DCT is

$$X''[k] = [X''_0, X''_1, \ldots, X''_{N-1}].$$

The DFT of the zero-padded sequence $\overline{x}[n] = \{x_0, x_1, \ldots, x_{N-1}, 0, \ldots, 0\}$ is

$$\overline{X}[k] = [\overline{X}_0, \overline{X}_1, \ldots, \overline{X}_{2N-1}].$$

A $2N$-point discrete-time sequence $y[n]$ is given by

$$y[n] = \begin{cases} 
  x[n], & 0 \leq n \leq N - 1 \\
  x[2N - 1 - n], & N \leq n \leq 2N - 1.
\end{cases}$$

(a) Can we express DFT of $y[n]$ in terms of $X[k]$ or $\overline{X}[k]$? Write the expression. (50%)

(b) Determine the DCT of $y[n]$ in terms of $X''[k]$. (50%)
Answer:

(a) $DFT\{y[n]\} = \sum_{n=0}^{2N-1} y[n] e^{-j \frac{2\pi nk}{2N}} = \sum_{n=0}^{N-1} x[n] e^{-j \frac{\pi nk}{N}} + \sum_{n=N}^{2N-1} x[2N-1-n] e^{-j \frac{\pi nk}{N}}$

$$= \bar{X}[k] + \sum_{m=0}^{N-1} x[m] e^{-j \frac{\pi nk}{N}} - j \frac{2\pi Nk}{N} e^{-j \frac{\pi nk}{N}} = \bar{X}[k] + e^{j \frac{\pi nk}{N}} \sum_{m=0}^{N-1} x[m] e^{-j \frac{\pi nk}{N}}$$

$$= \bar{X}[k] + e^{j \frac{\pi nk}{N}} \bar{X}^*[k]$$

(b) $DCT\{y[n]\} = \sqrt{\frac{2}{2N}} C[k] \sum_{n=0}^{2N-1} y[n] \cos \left[ \frac{\pi k (2n+1)}{4N} \right]$