1. (a) \( f(t) \) is plotted below for three periods:

(b) Fundamental period, \( T_0 = 4\pi \)

(c) We have

\[
C_n = \frac{1}{T_0} \int_{-T_0}^{T_0} f(t)e^{-jn\omega_0 t} \, dt
\]

where \( T_0 = 4\pi \) and \( \omega_0 = \frac{2\pi}{T_0} = \frac{1}{2} \). This yields

\[
C_n = \frac{1}{4\pi} \int_{0}^{2\pi} \sin(t)e^{-jn\omega_0 t} \, dt = \frac{1}{4\pi} \int_{0}^{2\pi} e^{jt} - e^{-jt} - e^{-jt}e^{jt} \, dt
\]

\[
= \frac{1 - e^{jn\pi}}{\pi(4 - n^2)} = \begin{cases} \frac{2}{\pi(4 - n^2)} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}
\]

(d) Call the function \( f(t) \) shifted to the left by \( \pi \) as \( g(t) \), i.e., \( g(t) = f(t + \pi) \). Now, \( f(t) = \sum_{-\infty}^{\infty} C_n e^{jn\omega_0 t} \). Therefore,

\[
g(t) = f(t + \pi) = \sum_{-\infty}^{\infty} C_n e^{jn\omega_0(t+\pi)} = \sum_{-\infty}^{\infty} C_n e^{jn\omega_0 t} e^{jn\omega_0 \pi} = \sum_{-\infty}^{\infty} C_n e^{jn\omega_0 t}
\]

where \( C_n^g = C_n e^{jn\omega_0 \pi} = C_n e^{j\frac{n\pi}{2}} \) is the Fourier series coefficient of \( g(t) \). Thus if we compute the Fourier series coefficients \( C_n^g \) for the function \( g(t) \), then we can get the Fourier series coefficients \( C_n \) for the function \( f(t) \) from the relation

\[
C_n = C_n^g e^{-j\frac{n\pi}{2}} \quad (1)
\]
Now \( g(t) \) is a periodic function with period \( 4\pi \). Therefore

\[
C_n^g = \frac{1}{4\pi} \int_{-\pi}^{\pi} g(t) e^{-jn\omega_0 t} \, dt = \frac{1}{4\pi} \int_{-\pi}^{\pi} e^{jt} - e^{-jt} e^{-jn\frac{\pi}{2}} \, dt
\]

\[
= \frac{1}{j(1 - \frac{n}{2})} e^{j(1 - \frac{n}{2})t} \big|_{-\pi}^{\pi} + \frac{1}{j(1 + \frac{n}{2})} e^{j(1 + \frac{n}{2})t} \big|_{-\pi}^{\pi}
\]

\[
= \frac{2j}{\pi(n^2 - 4)} \sin \frac{n\pi}{2}
\]

OR

\[
C_n^g = \begin{cases} 
\frac{2j}{\pi(n^2 - 4)} & n = \ldots -7, -3, 1, 5, 9, \ldots \\
\frac{2j}{\pi(n^2 - 4)} & n = \ldots -9, -5, -1, 3, 7, \ldots \\
0 & \text{otherwise. (i.e., } n \text{ even)}
\end{cases}
\]

Now from (1) and (2) we get \( C_n \) which is the same as before.

2. (a)

\[
\int_{-\infty}^{\infty} \delta(t - 2) \sin(t) \, dt = \int_{-2}^{2} \delta(t - 2) \sin(t) \, dt = \sin(2) = 0.9092
\]

(b)

\[
\int_{-\infty}^{\infty} \delta(t + 2) e^{-t} \, dt = \int_{-2}^{-2} \delta(t + 2) e^{-t} \, dt = e^2 = 7.3891
\]

(c)

\[
\int_{-\infty}^{\infty} \delta(1 - t)(t^3 + 4) \, dt = \int_{-\infty}^{\infty} \delta(t - 1)(t^3 + 4) \, dt \quad \text{since } \delta(t) \text{ is an even function}
\]

\[
= \int_{1}^{1} \delta(t - 1)(t^3 + 4) \, dt = 1^3 + 4 = 5
\]

3. (a) For function \( x(t) \), consider the function \( s(t) \), where

\[
s(t) = \begin{cases} 
A & \frac{T}{2} \leq t \leq \frac{T}{2} \\
0 & \text{otherwise.}
\end{cases}
\]

Then,

\[
S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} \, dt = \int_{-\frac{T}{2}}^{\frac{T}{2}} A e^{-j2\pi ft} \, dt = AT \text{sinc}(fT)
\]

where \( \text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \). By the time-shifting property of the Fourier transform,

\[
\mathcal{F}(s(t - t_0)) = e^{-j2\pi f_0} S(f)
\]

Now,

\[
x(t) = s(t - \frac{1}{2})|_{T=1} + 2s(t - 2)|_{T=2} + s(t - \frac{7}{2})|_{T=1}
\]
i. Let $A = 1$ and $T = 1$. Then, $S(f) = \text{sinc}(f)$. Furthermore, with these values of $A$ and $T$, $\mathcal{F}(s(t - \frac{1}{2})) = e^{-j\pi f}\text{sinc}(f)$

ii. Let $A = 2$ and $T = 2$. Then, $S(f) = 4\text{sinc}(2f)$. With these values of $A$ and $T$, $\mathcal{F}(s(t - 2)) = 4e^{-j4\pi f}\text{sinc}(2f)$

iii. Let $A = 1$ and $T = 1$. Then, $S(f) = \text{sinc}(f)$. With these values of $A$ and $T$, $\mathcal{F}(s(t - \frac{7}{2})) = e^{-j7\pi f}\text{sinc}(f)$

By the linearity property of the Fourier transform, we have

$$X(f) = e^{-j\pi f}\text{sinc}(f) + 4e^{-j4\pi f}\text{sinc}(2f) + e^{-j7\pi f}\text{sinc}(f)$$

(b) For function $y(t)$, consider the function $g(t)$, where

$$g(t) = \begin{cases} 
  t + 1 & -1 \leq t < 0 \\
  -t + 1 & 0 \leq t \leq 1 \\
  0 & \text{otherwise.}
\end{cases}$$

From Fourier transform tables, $G(f) = \text{sinc}^2(f)$

Now, $y(t) = g(t + 4) + g(t - 4)$

By the time-shifting and linearity properties of the Fourier transform,

$$Y(f) = e^{j8\pi f}G(f) + e^{-j8\pi f}G(f) = 2\cos(8\pi f)\text{sinc}^2(f)$$

4. (a) For function $x(t)$, we use the fact that multiplication in the time domain is equivalent to convolution in the frequency domain. We have,

$$x(t) = w(t)\cos(20t)$$

where

$$w(t) = \begin{cases} 
  1 & \frac{-T}{2} \leq t \leq \frac{T}{2} \\
  0 & \text{otherwise.}
\end{cases}$$

with $T = \frac{\pi}{20}$. Hence,

$$X(f) = W(f) * \mathcal{F}(\cos(20t))$$
where

\[ F(\cos(20t)) = \int_{-\infty}^{\infty} \frac{e^{j20t} + e^{-j20t}}{2} e^{-j2\pi ft} \, dt \]

\[ = \frac{1}{2} \delta(f - \frac{20}{2\pi}) + \frac{1}{2} \delta(f + \frac{20}{2\pi}) \]

and where \( W(f) = \frac{2\pi}{5} \text{sinc}(\frac{2\pi f}{5}) \). This yields

\[ X(f) = \frac{1}{2}[W(f - \frac{20}{2\pi}) + W(f + \frac{20}{2\pi})] \]

\[ = \frac{\pi}{5} \text{sinc}[\frac{2\pi}{5}(f - \frac{20}{2\pi})] + \frac{\pi}{5} \text{sinc}[\frac{2\pi}{5}(f + \frac{20}{2\pi})] \]

(b) Consider, for the moment, the function \( g(t) \), where

\[ g(t) = \begin{cases} \frac{-40}{9\pi} \cos(20t) & \frac{-9\pi}{40} \leq t \leq \frac{9\pi}{40} \\ \ 0 & \text{otherwise} \end{cases} \]

So

\[ g(t) = w(t) \cos(20t) \]

where

\[ w(t) = \begin{cases} A & \frac{-T}{2} \leq t \leq \frac{T}{2} \\ \ 0 & \text{otherwise} \end{cases} \]

with \( A = \frac{-40}{9\pi} \) and \( T = \frac{9\pi}{20} \). Working along lines of part (a) above, we get

\[ G(f) = \frac{1}{2}[W(f - \frac{20}{2\pi}) + W(f + \frac{20}{2\pi})] \]

where, in this case, \( W(f) = AT \text{sinc}(fT)|_{A=\frac{-40}{9\pi},T=\frac{9\pi}{20}} \)

This yields \( W(f) = \frac{2\pi}{5} \text{sinc}(\frac{2\pi f}{5}) \) and

\[ G(f) = -\text{sinc}[\frac{9\pi}{20}(f - \frac{20}{2\pi})] - \text{sinc}[\frac{9\pi}{20}(f + \frac{20}{2\pi})] \]

Now, the given function \( y(t) \) may be expressed as \( y(t) = tg(t) \), and hence,
\[ Y(f) = \frac{j}{2\pi df} G(f) \]

by the Differentiation in frequency property of the Fourier transform, where \( G(f) \) is as obtained above.

5. (a) \( \mathcal{F}(x(t - 1)) = \int_{-\infty}^{\infty} x(t - 1)e^{-j2\pi ft} dt \). By substituting \( u = t - 1 \), we get

\[ \mathcal{F}(x(t - 1)) = e^{-j2\pi f} \int_{-\infty}^{\infty} x(u)e^{-j2\pi fu} du = e^{-j2\pi f} X(f) \]

(b) Let \( y(t) = x(t - 1) \). Then, \( Y(f) = e^{-j2\pi f} X(f) \) from part (a), and

\[ \mathcal{F}(x(1 - t)) = \mathcal{F}(y(-t)) = Y(-f) = e^{j2\pi f} X(-f) \]

by using the scaling property of the Fourier transform.

(c) We will obtain \( \mathcal{F}(x(t - 2)) = e^{-j4\pi f} X(f) \), by proceeding along similar lines to part (a). In other words, if \( y(t) = x(t - 2) \), then \( Y(f) = e^{-j4\pi f} X(f) \). We then have

\[ \mathcal{F}(tx(t - 2)) = \mathcal{F}(ty(t)) = \frac{j}{2\pi df} Y(f) \]

by the Differentiation in frequency property of the Fourier transform. Hence,

\[ \mathcal{F}(tx(t - 2)) = 2e^{-j4\pi f} X(f) + \frac{j}{2\pi} e^{-j4\pi f} X'(f) \]

(d) Let \( y(t) = \frac{d}{dt} x(t) \). Then, \( Y(f) = j2\pi f X(f) \) by the Differentiation in time property of the Fourier transform.

Then,

\[ \mathcal{F}(t \frac{d}{dt} x(t)) = \mathcal{F}(ty(t)) = \frac{j}{2\pi df} Y(f) \]

by the Differentiation in frequency property of the Fourier transform. This yields

\[ \mathcal{F}(t \frac{d}{dt} x(t)) = -[X(f) + fX'(f)] \]