Problem 1 (35 Points)

1. Consider the message signal $m(t)$ whose spectrum is shown in Figure 1. This signal together with the carrier wave $A_c \cos(2\pi f_c t)$ is applied to a product modulator producing a DSB-SC waveform $s(t)$. The modulated waveform $s(t)$ is then applied to a coherent demodulator. Assuming perfect synchronism between the carriers of the modulator and demodulator, determine the spectrum of the demodulator’s output when

(a) $f_c = 4$ kHz.

(b) $f_c = 7.5$ kHz.

What is the lowest carrier frequency $f_c$ for which $m(t)$ can be obtained by demodulating the modulated wave $s(t)$.

2. Repeat the above for the waveform $m(t)$ whose spectrum is shown in Figure 2.
Problem 2 (30 Points)

A message signal $m(t)$ is DSB-SS modulated using the carrier signal $A_c \cos(2\pi f_c t)$.

1. The received signal is coherently demodulated using a local oscillator that produces the signal $A_r \cos(2\pi f_c t + \phi)$. In other words there is a phase offset $\phi$ between the oscillators in the transmitter and the receiver. Find the output of the detector in the receiver and comment on the effect of $\phi$ on the detected signal.

2. The received signal is coherently demodulated using a local oscillator that produces the signal $A_r \cos[2\pi(f_c + \Delta f)t]$. In other words there is a frequency offset $\Delta f$ between the oscillators in the transmitter and the receiver. Find the output of the detector in the receiver and comment on the effect of $\Delta f$ on the detected signal.

Problem 3 (35 points)

A carrier wave of frequency 105 MHz is frequency modulated by a sine wave of amplitude 5 V (volts) and frequency 15 kHz. The frequency sensitivity, $k_f$, of the modulator is 10 kHz/V.

1. Determine the approximate bandwidth of the FM wave using the Carson’s rule.

2. Determine the bandwidth if we only transmit those side-frequencies with amplitudes that exceed 1% of the unmodulated carrier amplitude. You can use the universal curve provided below.

3. Repeat parts (1) and (2) above assuming that the amplitude of the modulating wave is doubled.

4. Repeat parts (1) and (2) above assuming that the frequency of the modulating wave is doubled.
by the significant side frequencies drops toward that over which the carrier frequency actually deviates. This means that small values of the modulation index $\beta$ are relatively more extravagant in transmission bandwidth than are the larger values of $\beta$.

Consider next the more general case of an arbitrary modulating signal $m(t)$ with its highest frequency component denoted by $W$. The bandwidth required to transmit an FM signal generated by this modulating signal is estimated by using a worst-case tone-bandwidth analysis. Specifically, we first determine the so-called deviation ratio $D$, defined as the ratio of the frequency deviation $\Delta f$, which corresponds to the maximum possible amplitude of the modulation signal $m(t)$, to the highest modulation frequency $W$; these conditions represent the extreme cases possible. The deviation ratio $D$ plays the same role for nonsinusoidal modulation that the modulation index $\beta$ plays for the case of sinusoidal modulation. Then, replacing $\beta$ by $D$ and replacing $f_m$ with $W$, we may use Carson’s rule given by Equation (2.55) or the universal curve of Figure 2.26 to obtain a value for the transmission bandwidth of the FM signal. From a practical viewpoint, Carson’s rule somewhat underestimates the bandwidth requirement of an FM system, whereas using the universal curve of Figure 2.26 yields a somewhat conservative result. Thus, the choice of a transmission bandwidth that lies between the bounds provided by these two rules of thumb is acceptable for most practical purposes.

**Example 2.3**

In North America, the maximum value of frequency deviation $\Delta f$ is fixed at 75 kHz for commercial FM broadcasting by radio. If we take the modulation frequency $W = 15$ kHz, which is typically the “maximum” audio frequency of interest in FM transmission, we find that the corresponding value of the deviation ratio is

$$D = \frac{75}{15} = 5$$

Using Carson’s rule of Equation (2.55), replacing $\beta$ by $D$, and replacing $f_m$ by $W$, the approximate value of the transmission bandwidth of the FM signal is obtained as

$$B_T = 2(75 + 15) = 180 \text{ kHz}$$