Progress on High Performance Robust and Fault Tolerant Control

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Example: Robust Stabilization

\[ P(\Delta) \text{ stable for all } \Delta \]

- \[ P(\Delta) = P_0 + W_1 \Delta W_2 \]
- \[ P(\Delta) = P_0(I + W_1 \Delta W_2) \]
- \[ P(\Delta) = P_{11} + P_{12} \Delta (I - P_{22} \Delta)^{-1} P_{21} \]
  \[ \|P_{22} \Delta\|_{\infty} < 1 \]

\[
\max_{K \text{ Stabilizing}} \{ \gamma : \|\Delta\|_{\infty} < \gamma \}
\]

\[ K_{opt} = 0 \]

Performance: None
Suppose $K_0$ is a stabilizing controller and let

$$K_0 = \tilde{V}^{-1}\tilde{U}, \quad P_0 = \tilde{M}^{-1}\tilde{N}.$$ 

Then every stabilizing controller can be written as:

$$K = (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M})$$

for some $Q \in \mathcal{H}_\infty$

Then $u = -Ky$

$f = 0$ if the plant model is perfect, i.e., if $P = P_0$. 

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Controller Design

===Separation of Performance and Robustness===

A high performance robust system can be designed in two steps:

(a) Design $K_0 = \tilde{V}^{-1}\tilde{U}$ to satisfy the system performance specifications with a nominal plant model $P_0$;

(b) Design $Q$ to satisfy the system robustness requirements.

Note that the controller $Q$ will not affect the system nominal performance.

$K_0$ can be any stabilizing controller: PI, lead-lag, LQG, $H_\infty$, ...
Estimation Error $f$

\[ P_0 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \tilde{M}^{-1} \tilde{N} \]

\[
\begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} = \begin{bmatrix} A + LC & B + LD & L \\ C & D & I \end{bmatrix}.
\]

Denote the state vector of \[ \begin{bmatrix} \tilde{N} & \tilde{M} \end{bmatrix} \] by \( \hat{x} \) and note that

\[ f = \tilde{N}u - \tilde{M}y. \]

Then we have

\[
\dot{\hat{x}} = (A + LC)\hat{x} + (B + LD)u - Ly
\]

\[
f = (C\hat{x} + Du) - y
\]

i.e., $f$ is the estimated output error.
When the plant itself is stable, we can take

\[ \begin{align*}
\tilde{N} &= P_0, & \tilde{M} &= I.
\end{align*} \]

\( f = (P_0 - P)u \) is the error between the output of the nominal model and the output of the true system.
When $\tilde{U}$ is minimum phase, take $Q = \tilde{U}\tilde{Q}$ for some stable $\tilde{Q}$. Then the controller can be written as

$$K = (I - K_0\tilde{Q}\tilde{N})^{-1}(K_0 + K_0\tilde{Q}\tilde{M})$$

If $P_0$ is also stable, then
Example: Robust Stabilization Again

Optimal $\bar{Q} = -I$.

Open Loop control with

\[ u = 0 \quad y \]

\[ u = Kr \]

with

\[ K = K_0(I + P_0K_0)^{-1} \]
Robustness Example from $\mu$-Toolbox

Nominal plant

$$ P = \frac{1}{s - 1}. $$

True plant in a multiplicative set

$$ \mathcal{M}(P, W_u) := \left\{ P(1 + \Delta W_u) : \max_\omega |\Delta(j\omega)| \leq 1 \right\} $$

with

$$ W_u = \frac{1}{4} \left( \frac{1}{2} s + 1 \right) \frac{1}{32 s + 1} $$

such that $P(1 + \Delta W_u)$ and $P$ have the same number of unstable poles.
LFT Form and Performance

\[
G = \begin{bmatrix}
0 & 0 & W_u \\
W_p P & W_p & W_p P \\
P & I & P
\end{bmatrix}.
\]

closed-loop stability and disturbance rejection up to 0.6 rad/sec, with at least 100 : 1 at DC for all possible models.

approximately:

\[
\|T_{ed}\|_\infty = \left\| \frac{W_p}{1 + \tilde{P}K} \right\|_\infty \leq 1
\]

for all \( \tilde{P} \in \mathcal{M}(P, W_u) \) with the weighting

\[
W_p = \frac{1}{4} s + 0.6 \quad \frac{1}{s + 0.006}.
\]
Let $M$ be
\[
\begin{bmatrix}
    z \\
    e
\end{bmatrix} = M(s) \begin{bmatrix} w \\ d \end{bmatrix} = \begin{bmatrix}
    M_{11} & M_{12} \\
    M_{21} & M_{22}
\end{bmatrix} \begin{bmatrix} w \\ d \end{bmatrix}
\]

Nominal performance (i.e., when $\Delta = 0$):
\[
\|T^0_{ed}\|_\infty := \|T_{ed}|_{\Delta=0}\|_\infty = \|M_{22}\|_\infty = \left\| \frac{W_p}{1 + PK} \right\|_\infty
\]

Robust stability margin:
\[
\|T_{zw}\|_\infty = \|M_{11}\| = \left\| \frac{W_uPK}{1 + PK} \right\|_\infty.
\]

Robust performance:
\[
\|T_{ed}\|_\infty = \left\| \frac{W_p}{1 + \bar{P}K} \right\|_\infty \leq 1
\]

is satisfied if and only if
\[
\mu_{\Delta_P}(M(j\omega)) \leq 1, \quad \forall \omega
\]

where $\Delta_P = \text{diag}(\Delta, \Delta_f)$. 
NP of Two PI Controllers

from $\mu$-toolbox:

$$K_1 = \frac{10(0.9s + 1)}{s}, \quad K_2 = \frac{2.8s + 1}{s}.$$ 

Frequency Responses of $T_{ed}^0$ for Nominal Performance: $K_1$ (solid) and $K_2$ (dashed)
RS of Two PI Controllers

from $\mu$-toolbox:

$$K_1 = \frac{10(0.9s + 1)}{s}, \quad K_2 = \frac{2.8s + 1}{s}.$$ 

Frequency Responses of $T_{zw}$ for Robust Stability: $K_1$ (solid) and $K_2$ (dashed)
from $\mu$-toolbox:

\[ K_1 = \frac{10(0.9s + 1)}{s}, \quad K_2 = \frac{2.8s + 1}{s}. \]

Frequency Responses of $\mu_{\Delta P}(M(j\omega))$ for Robust Performance: $K_1$ (solid) and $K_2$ (dashed)
10 plants including the nominal and two "worst-case" plants in the set $\mathcal{M}(P, W_u)$

\[ P = \frac{1}{s - 1} \]
\[ P_1 = \frac{1}{s - 1} \frac{6.1}{s + 6.1} \]
\[ P_2 = \frac{1}{s - 1.425} \]
\[ P_3 = \frac{1}{s - 0.67} \]
\[ P_4 = \frac{1}{s - 1} \frac{-0.07s + 1}{0.07s + 1} \]
\[ P_5 = \frac{1}{s - 1} \frac{70^2}{s^2 + 2 \cdot 0.15 \cdot 70s + 70^2} \]
\[ P_6 = \frac{1}{s - 1} \frac{70^2}{s^2 + 2 \cdot 5.6 \cdot 70s + 70^2} \]
\[ P_7 = \frac{1}{s - 1} \left( \frac{50}{s + 50} \right)^6 \]
\[ P_{wc1} = \frac{1}{s - 1} \frac{-2.9621(s - 9.837)(s + 0.76892)}{(s + 32)(s + 0.56119)} \]
\[ P_{wc2} = \frac{1}{s - 1} \frac{s^2 + 3.6722s + 34.848}{(s + 7.2408)(s + 32)} \]
Step Response with $K_1$ and Various Plants for Standard Feedback Implementation
Step Response with $K_2$ and Various Plants for Standard Feedback Implementation
New Implementation

\[ P = N/M, \quad N = \frac{1}{s+1}, \quad M = \frac{s-1}{s+1}. \]

Then

\[ K_2 = \frac{K_1(1 + \tilde{Q}M)}{1 - K_1\tilde{Q}N} \]

\[ \tilde{Q}(s) = -\frac{0.1s(6.2s + 1)(s + 1)}{(0.9s + 1)(s^2 + 1.8s + 1)}. \]
Step Responses of the Nominal $P$: $K_1$ (solid), $K_2$ (dashed), and GIMC (solid)
Robust Performance

Step Responses of the Closed-loop System with $K_2$ Implemented Using the GIMC Structure Under Various Perturbations
Comparsion of Worst Cases

Step Responses of $P_{wc1}$: $K_1$ (dash-dot), $K_2$ (dashed), and GIMC (solid)

Step Responses of $P_{wc2}$: $K_1$ (dash-dot), $K_2$ (dashed), and GIMC (solid)
Fault Tolerant Control

= Conventional robust control approach =

\[ f \] is the *residual signal* used in fault diagnosis

a possible actuator fault in the first channel of an \( m \) actuator system with \( B = [B_1, B_2, \ldots, B_m] \)
can be represented by introducing an uncertainty in the corresponding input matrix

\[
\dot{x} = Ax + B_1(1+\delta)u_1 + B_2u_2 + \ldots + B_m u_m, \quad \delta \in [-1, 0]
\]

where \( \delta = -1 \) implies a total failure of the actuator and \( \delta = 0 \) implies no actuator failure.

Worst Case Design: a robust controller is designed for this uncertain system

Performance: Too conservative—worst case is rare.
Fault Tolerant Control

Our approach

(a) Design $K_0 = \tilde{V}^{-1}\tilde{U}$ to satisfy the system performance by assuming no faults (and model uncertainties).

(b) Design $Q$ to tolerate possible actuators and/or sensors failures (and model uncertainties). This $Q$ can be designed using standard robust control techniques, fuzzy control methods, adaptive control techniques, etc.
Connections with 2DOF

\[
\begin{bmatrix} K_1, & K_2 \end{bmatrix} = (V - QN)^{-1} \begin{bmatrix} R & U + QM \end{bmatrix}
\]

All 2DOF controllers

Now take \( R = UR \) for any \( R \in \mathcal{H}_\infty \)
Experimental Study

== A Gyroscope System ==

Sensor faults: $\tilde{y} = (I + \Delta)y$, $\Delta = \begin{bmatrix} \Delta_1 & \Delta_2 \end{bmatrix}$
Simulation with Fault

Sensor fault at 8 sec.

Our new controller performs exactly the same as the nominal LQG controller when there is no fault and maintains stability when there is a fault.

$H_{\infty}$ controller is robust but slow.
Our new controller performs closely to the LQG controller. The difference is caused by the model uncertainties.

$H_{\infty}$ controller fails to perform because of dead zone.
Experimental response with sensor fault at 2.2 sec.

Experimental data is consistent with the simulation data.
Detection and Switching

Robust controller $Q$ is off when there is no fault.

$Q$ is switched on when the error exceeds a threshold.
Experimental response: switching on the robustness signal $q$ after detecting sensor failure (6 sec.)
Experimental response: switching on the robustness signal $q$ after detecting sensor failure (6 sec.) with delay 0.35 sec