EE 3755
Computer Arithmetic
Handout #10
Ripple Carry Adder

Consider two n-bit binary numbers
A = (an-1 an-2 ... a1 a0)₂ ; B = (bn-1 bn-2 ... b1 b0)₂

One two-operand adder (adding A and B) is the ripple carry adder shown below:

\[ \begin{align*}
\text{an-1} & \quad \text{bn-1} & \quad \text{an-2} & \quad \text{bn-2} \\
\text{Cout} & \quad \text{S}_{n-1} & \quad \text{S}_{n-2} & \quad \ldots \quad \text{S}_2 & \quad \text{S}_1 & \quad \text{S}_0 \\
\end{align*} \]

where

FA indicates Full Adder

The delay through an n-bit ripple carry adder is \( n \times D_{FA} \) where \( D_{FA} \) is the delay through a Full Adder.
Unsigned array multipliers

Consider two 5-bit unsigned numbers \( A = a_4 a_3 a_2 a_1 a_0 \) and \( B = b_4 b_3 b_2 b_1 b_0 \) where \( a_4 \) and \( b_4 \) are the most significant bits of \( A \) and \( B \) while \( a_0 \) and \( b_0 \) are the least significant bits of \( A \) and \( B \) respectively.

The table below describes the multiplication operation.

\[
\begin{array}{cccc}
  a_4 & a_3 & a_2 & a_1 & a_0 \\
  \times & b_4 & b_3 & b_2 & b_1 & b_0 \\
  a_{46} & a_{3b} & a_{2b} & a_{1b} & a_{0b} \\
  a_{4b} & a_{3b} & a_{2b} & a_{1b} & a_{0b} \\
  a_{4b} & a_{3b} & a_{2b} & a_{1b} & a_{0b} \\
  a_{4b} & a_{3b} & a_{2b} & a_{1b} & a_{0b} \\
  + & a_{4b} & a_{3b} & a_{2b} & a_{1b} & a_{0b} \\
  P_9 & P_8 & P_7 & P_6 & P_5 & P_4 & P_3 & P_2 & P_1 & P_0 &= P \\
\end{array}
\]

Summand matrix describing the add-shift operations in a 5-by-5 unsigned multiplication

• In the above, every term \( a_i b_j \) denotes the AND operation between bits \( a_i \) and \( b_j \). Each term \( a_i b_j \) is called a summand.

• \( P_9 \) and \( P_0 \) are the most significant and least significant bits of the product respectively.
The delay through an $n$-by-$n$ unsigned array multiplier will be $\lfloor \log_2(n^2) \rfloor \times \text{Delay of FA} + (n-1) \times \text{Delay of FA} = D_{\text{AND}} + 2(n-1) \times D_{\text{FA}}$

A 5-by-5 unsigned array multiplier.

.. The cost of an $n$-by-$n$ unsigned array multiplier will be $n^2$ AND gates plus $2 \times (n-1)$ FAs.
An example of a 5-by-5 unsigned multiplication with $A = (a_4,a_3,a_2,a_1,a_0)$ and $B = (b_4,b_3,b_2,b_1,b_0)$ results in the product $P = A \times B = 27 \times 17 = 459 \text{so } P = (p_4,p_3,p_2,p_1,p_0) = (01100111)_2$. The product is $(110111)_2 = 27$ and $B = (b_4,b_3,b_2,b_1,b_0)$ = (1001)_2 = 17.