Lecture 20
Deformation

Xin (Shane) Li

What is Deformation

- A useful interactive technique for
  - Geometric modeling and editing
  - Movie and game models animation
  - Physical simulation in virtual environments (more torus, liquid, smoke)

- Input: 1) A set of points with/without connectivity
  with 2) Parts of points moved to new positions
  or 2*) External forces
- Output: The deformed points set, the shape change is physically natural

SIGGRAPH 2005
Deformation Papers Digest

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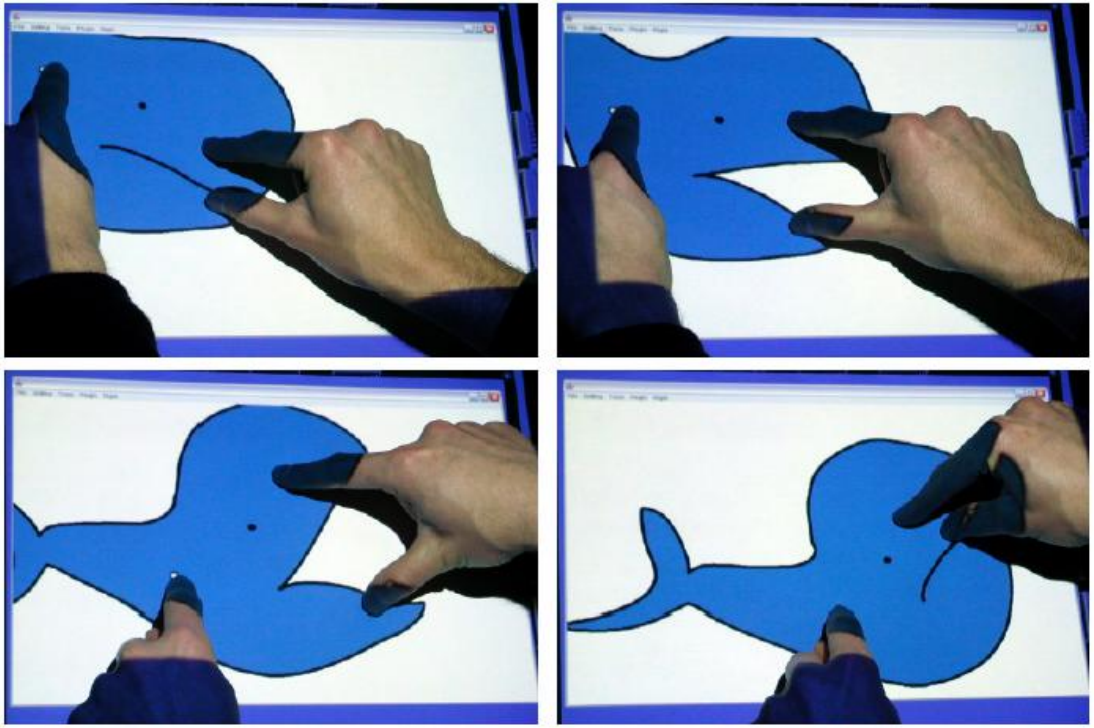
Contents

- Skinning Animation
- Meshless Deformations
- Linear Rotation-Invariant Coordinates
- Sketch interface
- VGL
- As-Rigid-as-Possible

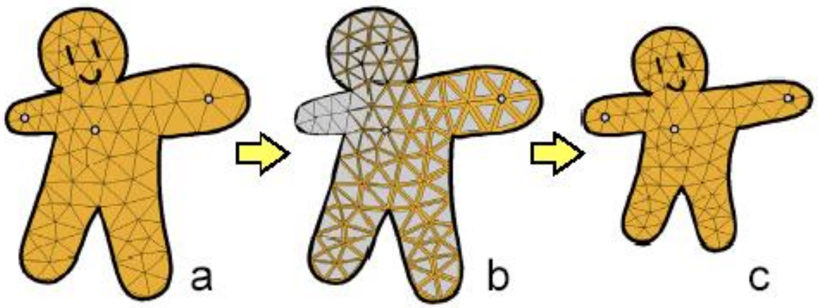
As-Rigid-As-Possible Shape Manipulation

- Takeo Igarashi (University of Tokyo)
- Tomer Moscovich (Brown University)
- John F. Hughes (Brown University)



Overview

- They present an interactive system that lets a user move and deform a 2D shape represented by a triangle mesh.



- The user moves several vertices of the mesh and the system computes positions of all other points in real time.

Algorithms

1. Scale-free construction

Allow rotation and uniform scaling

$$E_{\{v_2\}} = \left\| v_2^{\text{desired}} - v_2' \right\|^2$$

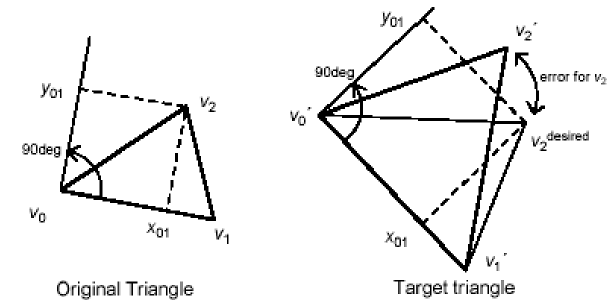
2. Scale adjustment

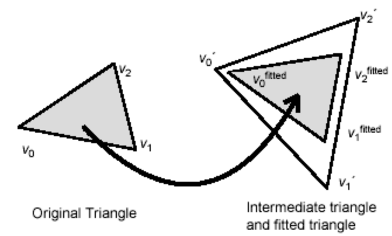
2.1 Fit the original triangle to the triangles computed above

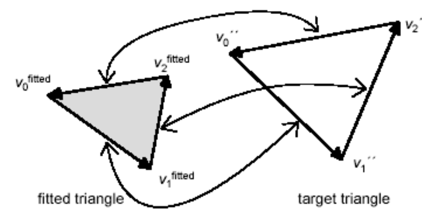
$$E_{f\{v_0^{\text{fitted}}, v_1^{\text{fitted}}, v_2^{\text{fitted}}\}} = \sum_{i=1,2,3} \left\| v_i^{\text{fitted}} - v_i' \right\|^2$$

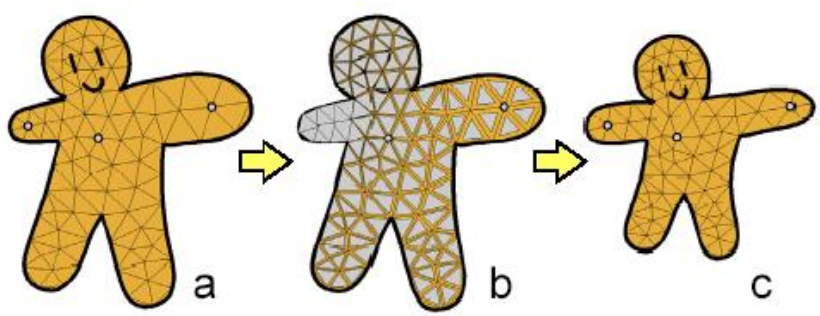
2.2 Generate the final result using the fitted triangles

$$E_{2\{v_0'', v_1'', v_2''\}} = \sum_{(i,j)\in\{(0,1),(1,2),(2,0)\}} \left\| \overrightarrow{v_i'' v_j''} - \overrightarrow{v_i^{\text{fitted}} v_j^{\text{fitted}}} \right\|^2$$







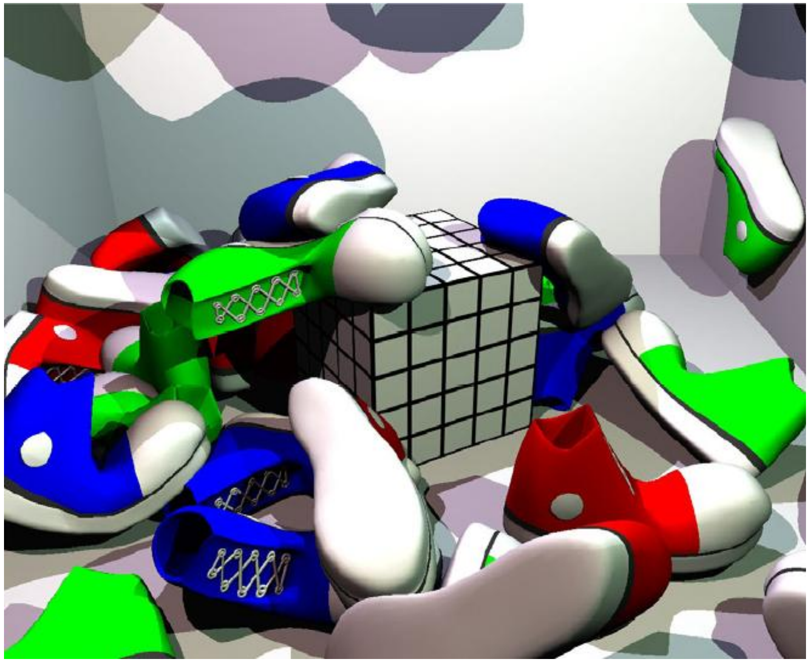


• Video

- Skinning Animation
- Meshless Deformations
- Linear Rotation-Invariant Coordinates
- Sketch interface
- VGL
- As-Rigid-as-Possible

Meshless Deformations Based on Shape Matching

- Matthias Müller(NovodeX/AGEIA & ETH Zürich)
- Bruno Heidelberger (ETH Zürich),
- Matthias Teschner (University of Freiburg)
- Markus Gross (ETH Zürich)



Overview

- Newton's second law of motion
  - Implicit: stable but inefficient
  - Explicit: efficient but unstable
- Idea
  - the energies → geometric constraints
  - forces → distances of current positions to goal positions

- ➢Skinning Animation
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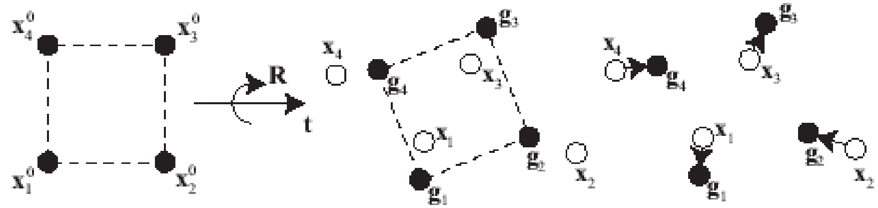
Algorithms



- Shape matching
  - Find the rotation matrix and translation vectors

$$\sum_i w_i(\mathbf{R}(\mathbf{x}_i^0 - \mathbf{t}_0) + \mathbf{t} - \mathbf{x}_i)^2,$$

$$\mathbf{t}_0 = \mathbf{x}_{cm}^0 = \frac{\sum_i m_i \mathbf{x}_i^0}{\sum_i m_i}, \quad \mathbf{t} = \mathbf{x}_{cm} = \frac{\sum_i m_i \mathbf{x}_i}{\sum_i m_i},$$

$$\mathbf{g}_i = \mathbf{R}(\mathbf{x}_i^0 - \mathbf{x}_{cm}^0) + \mathbf{x}_{cm}.$$

- Integration

$$\mathbf{v}_i(t+h) = \mathbf{v}_i(t) + \alpha \frac{\mathbf{g}_i(t) - \mathbf{x}_i(t)}{h} + h f_{ext}(t)/m_i$$

$$\mathbf{x}_i(t+h) = \mathbf{x}_i(t) + h\mathbf{v}_i(t+h),$$

- Video
- 2D Sketch Demo

- ➢Skinning Animation
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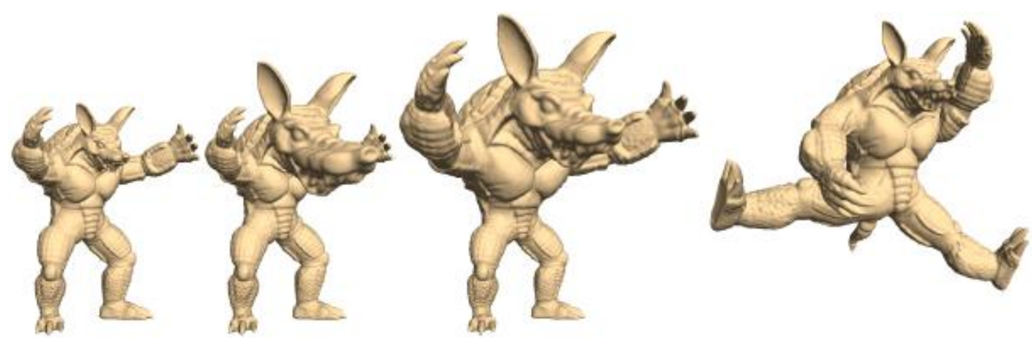
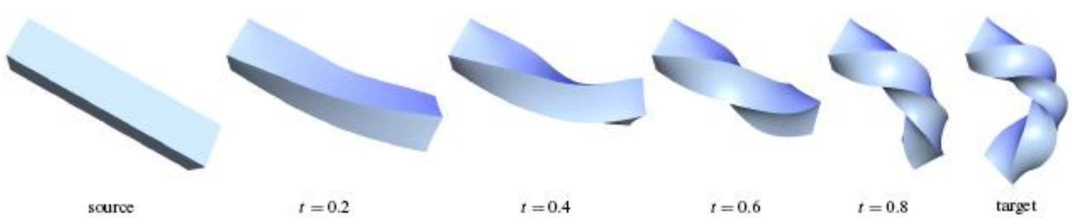
Linear Rotation-Invariant Coordinates for Meshes

- Yaron Lipman
- Olga Sorkine
- David Levin
- Daniel Cohen-Or

(Tel Aviv University)





Overview

- Rigid-motion invariant mesh representation using discrete forms defined at each vertex
- Linear reconstruction scheme that restores the geometry from the discrete forms

Surface Representation

- Discrete forms
  - 1st discrete form (information in tangent plane)
    - Lengths of the projected edges
    - Signed angles between adjacent projected edges
  - 2nd discrete form (information for normal direction)
    - Height function of 1-ring neighborhood of a vertex above the tangent plane
  - Discrete uniquely defines a 1-ring neighborhood
- Discrete frames
  - A triplet forming a right-hand orthonormal basis on one vertex
- Discrete surface equations
  - Encode the difference between adjacent discrete frames

- Given an initial discrete frame at one vertex
  ➔ everything

Mesh editing

- Reconstruct the discrete frames at each vertex by solving the discrete surface equations.

$$\begin{aligned} \delta_j(\mathbf{b}_1^i) &= \mathbf{b}_1^j - \mathbf{b}_1^i \\ \delta_j(\mathbf{b}_2^i) &= \mathbf{b}_2^j - \mathbf{b}_2^i \\ \delta_j(\mathbf{N}^i) &= \mathbf{N}^j - \mathbf{N}^i. \end{aligned}$$

$$\begin{aligned} \delta_j(\mathbf{b}_1^i) &= \Gamma_{j,1}^{i,1}\mathbf{b}_1^i + \Gamma_{j,1}^{i,2}\mathbf{b}_2^i + A_{j,1}^i\mathbf{N}^i \\ \delta_j(\mathbf{b}_2^i) &= \Gamma_{j,2}^{i,1}\mathbf{b}_1^i + \Gamma_{j,2}^{i,2}\mathbf{b}_2^i + A_{j,2}^i\mathbf{N}^i \\ \delta_j(\mathbf{N}^i) &= \Gamma_{j,3}^{i,1}\mathbf{b}_1^i + \Gamma_{j,3}^{i,2}\mathbf{b}_2^i + A_{j,3}^i\mathbf{N}^i. \end{aligned}$$

- Reconstruct the geometry at each vertex from the discrete forms and the discrete frames.

- Video

$$\hat{\mathbf{x}}^j - \hat{\mathbf{x}}^i = \tilde{\mathbf{x}}_k^i + \tilde{L}_k^i\mathbf{N}^i = \langle\tilde{\mathbf{x}}_k^i, \mathbf{b}_1^i\rangle\,\mathbf{b}_1^i + \langle\tilde{\mathbf{x}}_k^i, \mathbf{b}_2^i\rangle\,\mathbf{b}_2^i + \tilde{L}_k\,\mathbf{N}^i, \quad \forall (i,j) \in E$$

➢Skinning Animation

➢Meshless Deformations

➢Linear Rotation-Invariant Coordinates

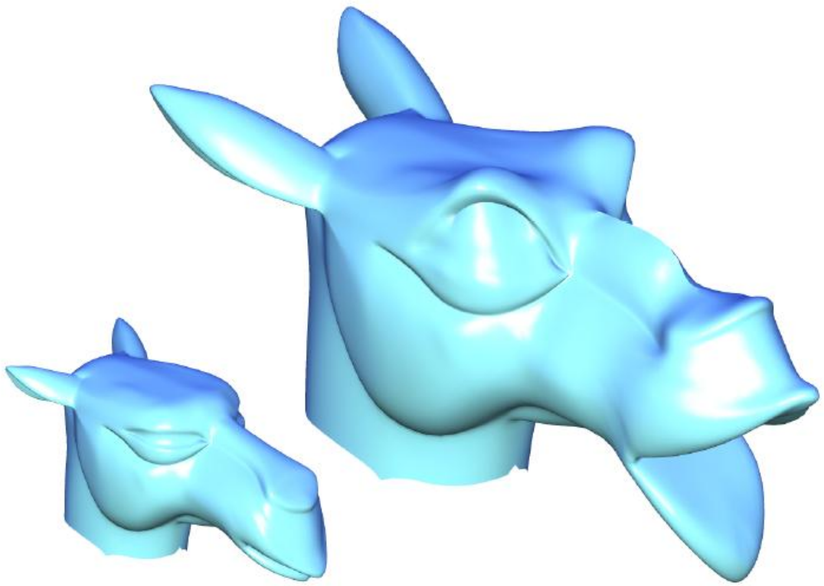
➢Sketch interface

➢VGL

➢As-Rigid-as-Possible

A Sketch-Based Interface for Detail-Preserving Mesh Editing

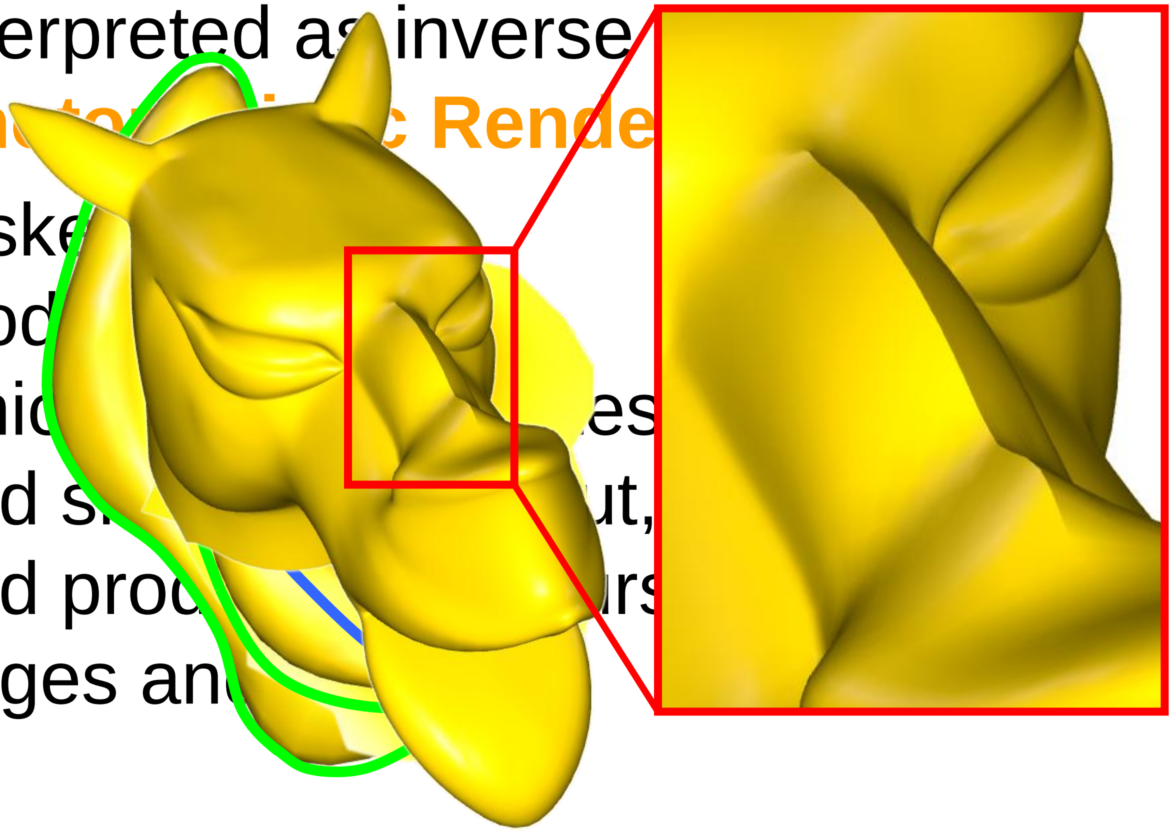
- Andy Nealen (Technische University Darmstadt)
- Olga Sorkine (Tel Aviv University)
- Marc Alexa (Technische University Darmstadt)
- Daniel Cohen-Or (Tel Aviv University)



Ideas and Contributions

- Silhouette sketching

- Sketching a shape can be interpreted as inverse **Pho... ...c Rende...**
- A ske... mod... whic... ...es and s... ...ut, and proc... ...rs ridges an...



Algorithms

- Silhouette sketching
  - Define a region of interest on the surface
  - Compute the corresponding silhouette
    - Edge silhouettes
    - Smooth surface silhouettes [Hertzmann and Zorin 2000]
  - Sketch a new silhouette curve
  - Transform the surface
- Feature and contour sketching
  - Sketch the feature curve on screen and prepare the corresponding curve on surface
  - Create a sharp feature along the edge path
- Mesh smoothing
  [Laplacian surface editing. Sorkine et al. EG2004]

Video

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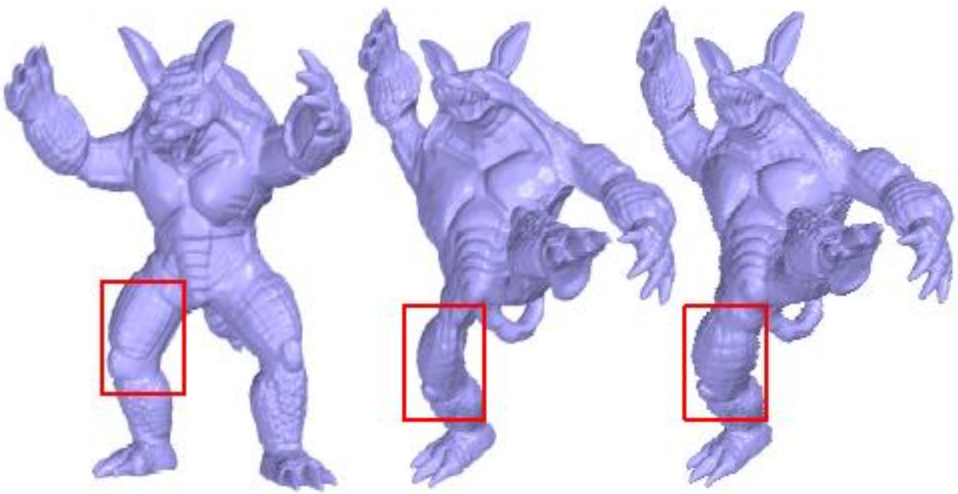
Large Mesh Deformation Using the Volumetric Graph Laplacian

Kun Zhou, Jin Huang, John Snyder, Xinguo Liu, Hujun Bao, Baining Guo, Heung-Yeung Shum
(Zhejiang University, Microsoft Research Asia)



Overview

- Prevent volume changes in large deformation:
  - Construct a graph representing the volume inside the input mesh
  - Minimize a quadric energy function representing volumetric detail preservation

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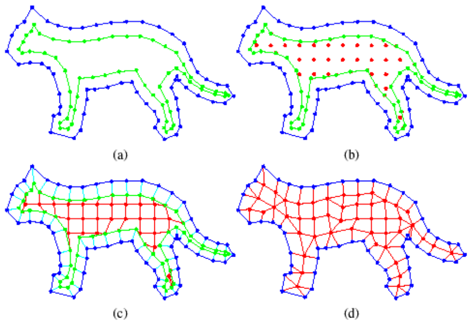
Algorithms

- Construct the volumetric graph
  - Inner shell
    - → lattice sampling
    - → edge connection
    - → graph simplify



- Deform the volumetric graph

$$\sum_{i=1}^{n}\|\mathcal{L}_M(p_i')-\varepsilon_i'\|^2+\alpha\sum_{i=1}^{m}\|p_i'-q_i\|^2+\beta\sum_{i=1}^{N}\|\mathcal{L}_{G'}(p_i')-\delta_i'\|^2$$

$$\delta_i=\mathcal{L}_G(p_i)=p_i-\sum_{j\in\mathcal{N}(i)}w_{ij}p_j,$$

- Propagation of Local Transforms
  - WIRE deformation method

    [Wires: A geometric deformation technique. Singh et al. SIGGRAPH 98]

$$p'=C'(u_p)+R(u_p)\left(s(u_p)(p-C(u_p))\right)$$

$$T_p=f(p)\tilde{T}_p+(1-f(p))I$$

- Video

Skinning Mesh Animations

- Doug L. James
- Christopher D. Twigg

(Carnegie Mellon University)

Overview

$$\mathbf{P} = (\mathrm{p}^1, \mathrm{p}^2, \ldots, \mathrm{p}^S), \text{ where } \mathrm{p}^t \in \mathbb{R}^{3N} \text{ for } N \text{ vertices}$$

$$\mathrm{p}^t \approx \mathrm{T}^t \tilde{\mathrm{p}}, \quad t = 1 \ldots S,$$

$$\mathrm{T}_i^t = \sum_{b \in \mathscr{B}_i} w_{ib} \bar{\mathrm{T}}_b^t,$$

- Use mean shift clustering of mesh rotation sequences to identify bones, and determine bone transformations
- Find bone-vertex influence sets, and vertex weights using least squares methods

Algorithms

- Compute the triangle rotation sequences

$$\mathrm{z}_j = \left(\mathrm{vec}(R_j^1), \ldots, \mathrm{vec}(R_j^S)\right)$$

- Cluster the rotation sequences
  - Find similar rotation sequence points
  - Associate triangles to bones
- Estimate the average bone rotations and translations
  - Area weighted

  [Moakher. "Means and averaging in the group of rotations." SIAM 2002]
- Estimate bone influence range
  - Picks the best n bones
- Estimate vertex weights

$$\sum_{b \in \mathscr{B}_i} (\bar{\mathrm{T}}_b^t \tilde{\mathrm{p}}_i) w_{ib} = \mathrm{p}_i^t,$$

- Video