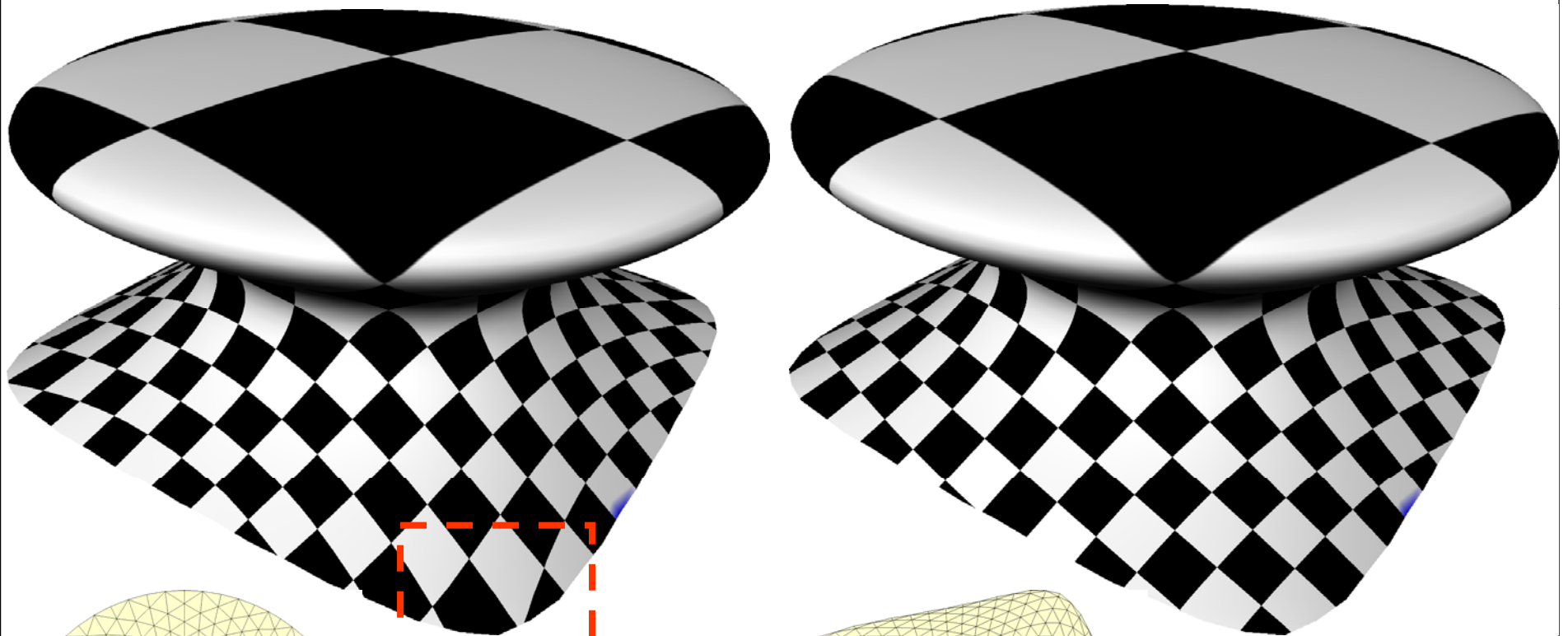


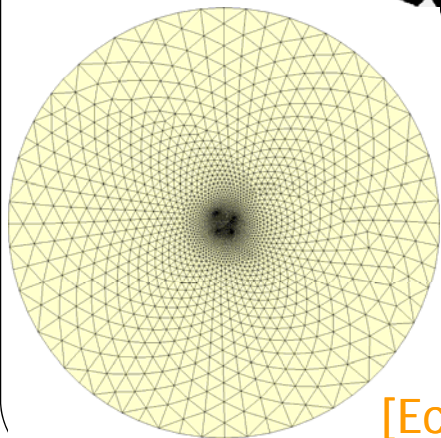
# (Re)Meshing

**Nov. 17, 19, 2009**

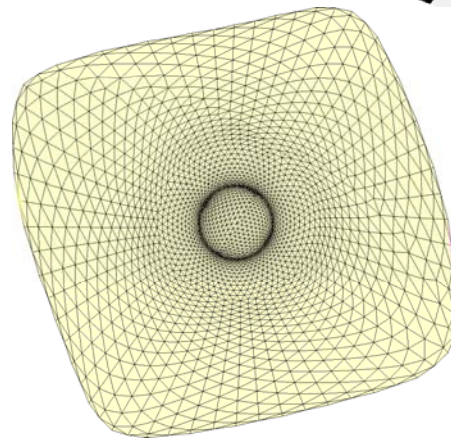
# Conformal: fixed vs free boundary



distortion

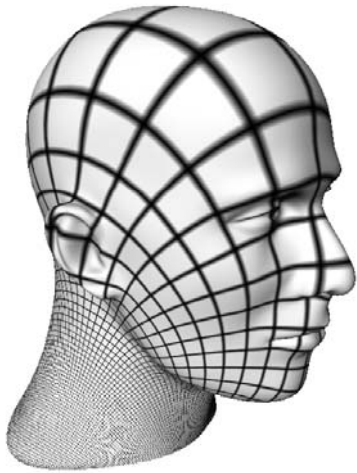


[Eck et al. 95]

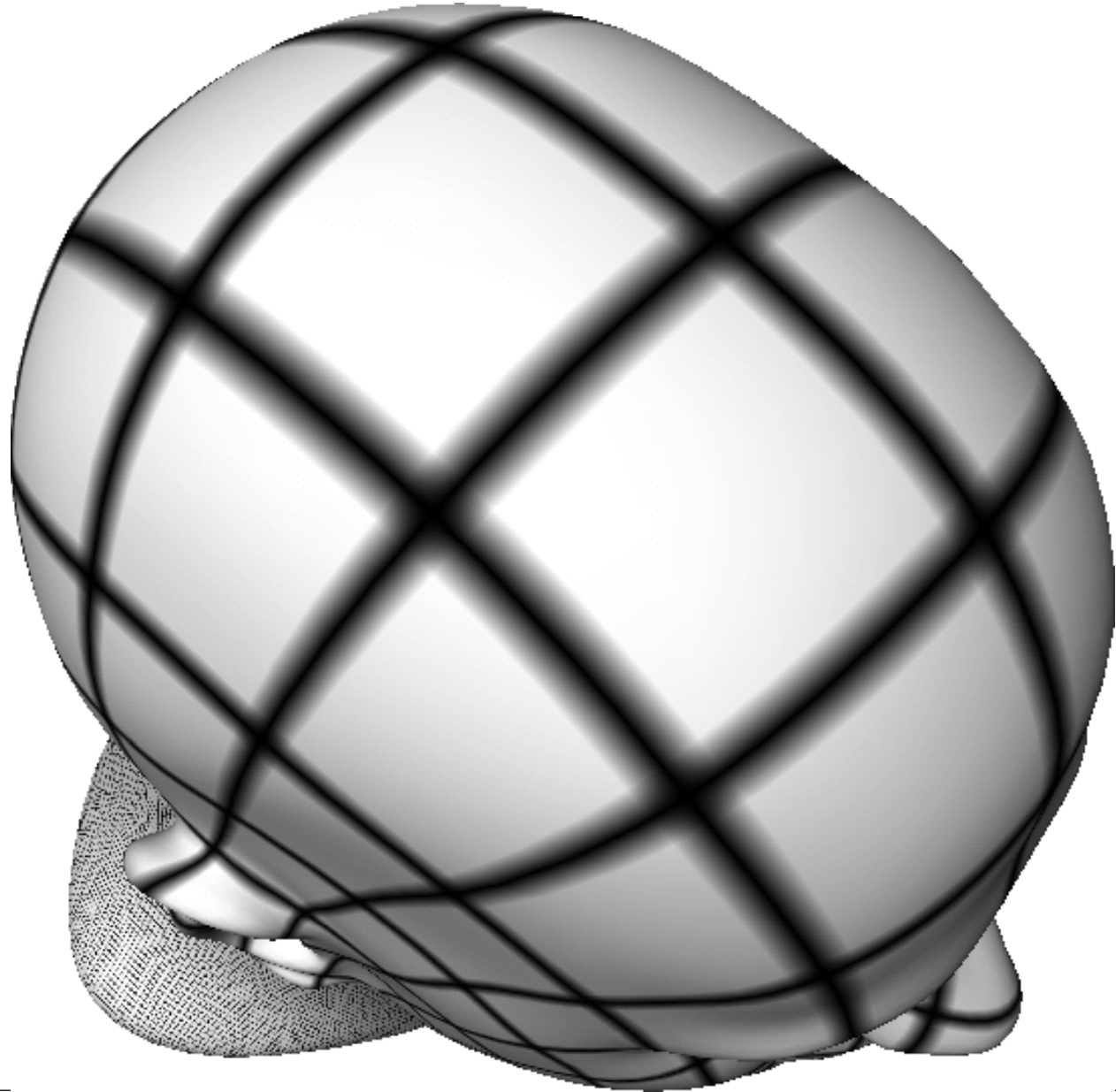


[Levy et al. '02,  
Desbrun et al. '02]

# Preservation of angles

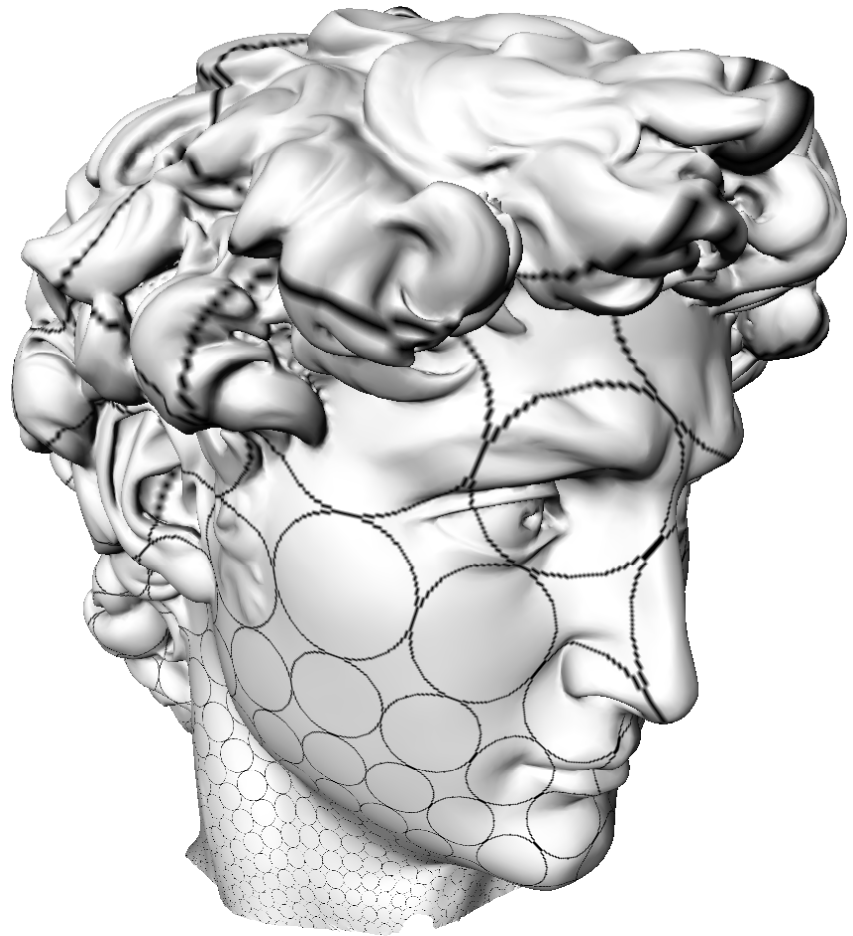


Isoparametric lines

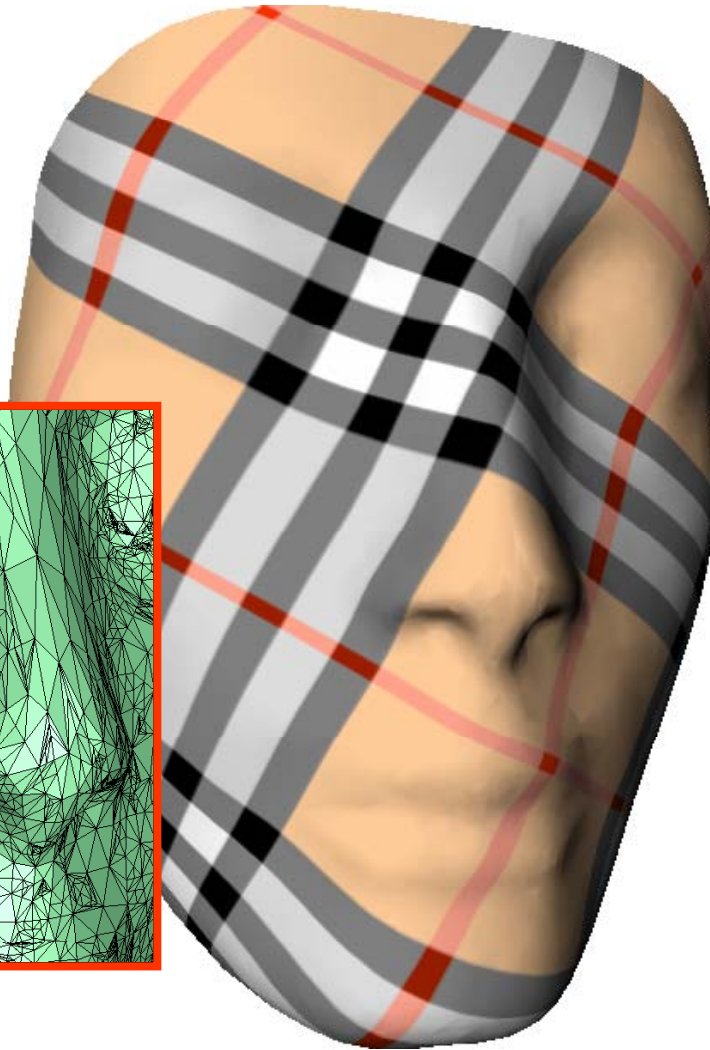
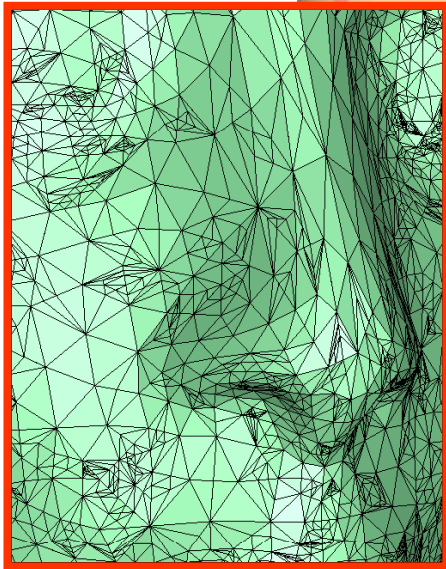
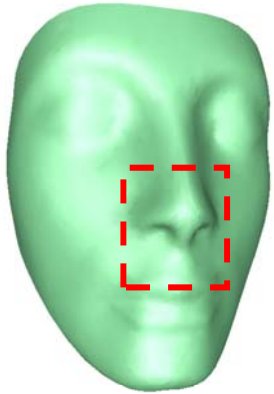


# Conformality

Angle-preserving + locally isotropic



# Behavior w.r.t. sampling

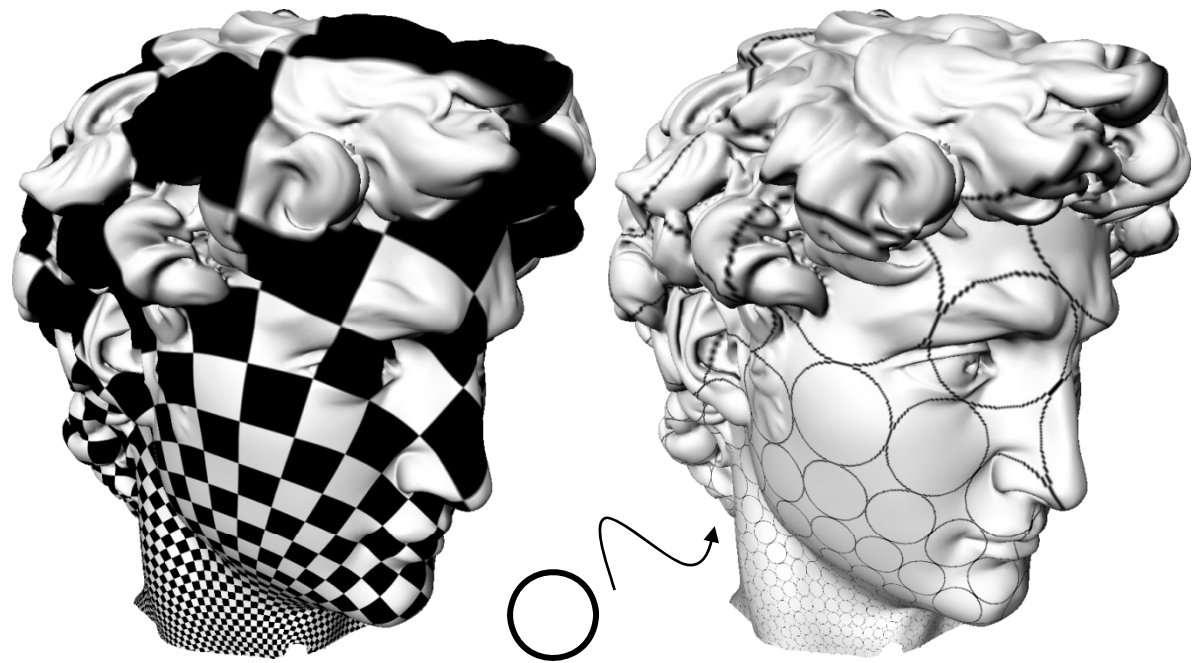


Conformal



Tutte

"A **well-shaped** element in parameter space will not be deformed too much once lifted in embedding space"



Motivation

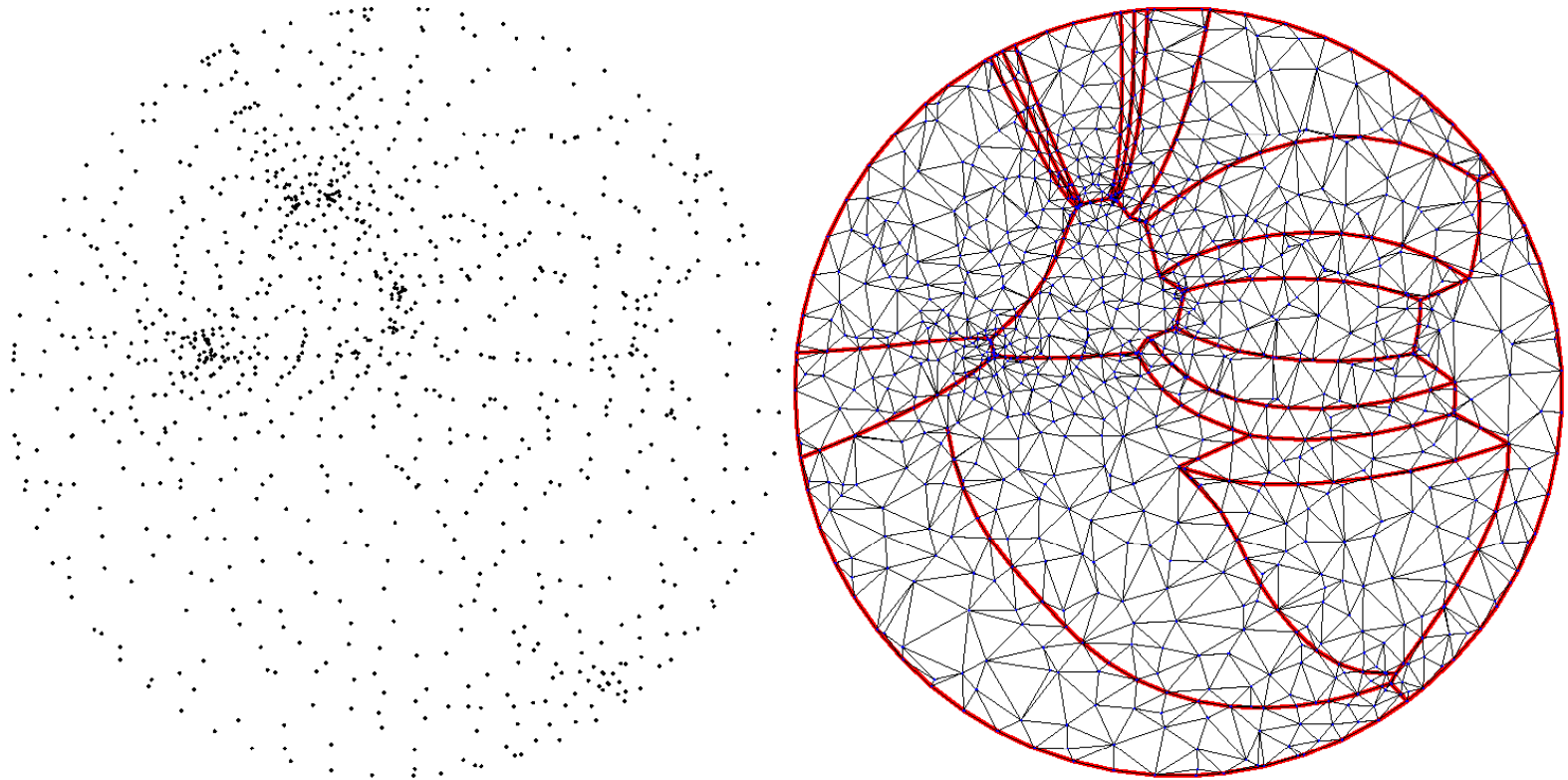
Previous work

Contributions

Algorithm

- sample repartition *error diffusion*
- parameterization *conformal*
- **meshing**
- sample placement

# Meshing



Constrained Delaunay triangulation  
in parameter space

[CGAL] -> solves robustness issues



Motivation

Previous work

Contributions

Algorithm

- sample repartition *error diffusion*
- parameterization *conformal*
- meshing *Delaunay*
- **sample placement**

# Sample placement

Given a bounded domain and a density function,

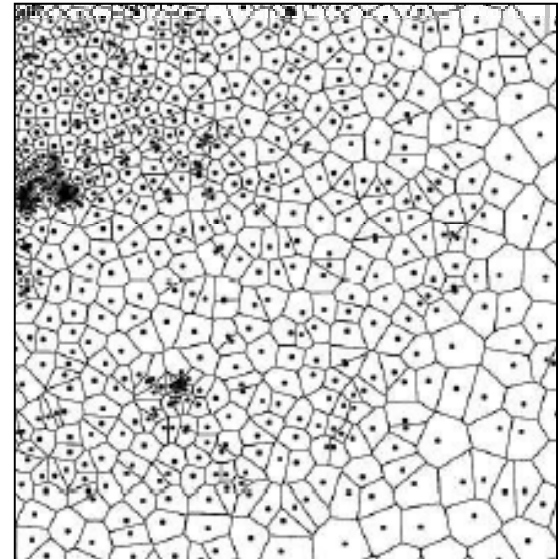
sampling

=

- partitioning the domain
- repartitioning the density function among a set of samples

# Sample placement

- partitioning the domain  
-> Voronoi tessellation
- repartitioning the density function among a set of samples  
= Equal-mass enclosing



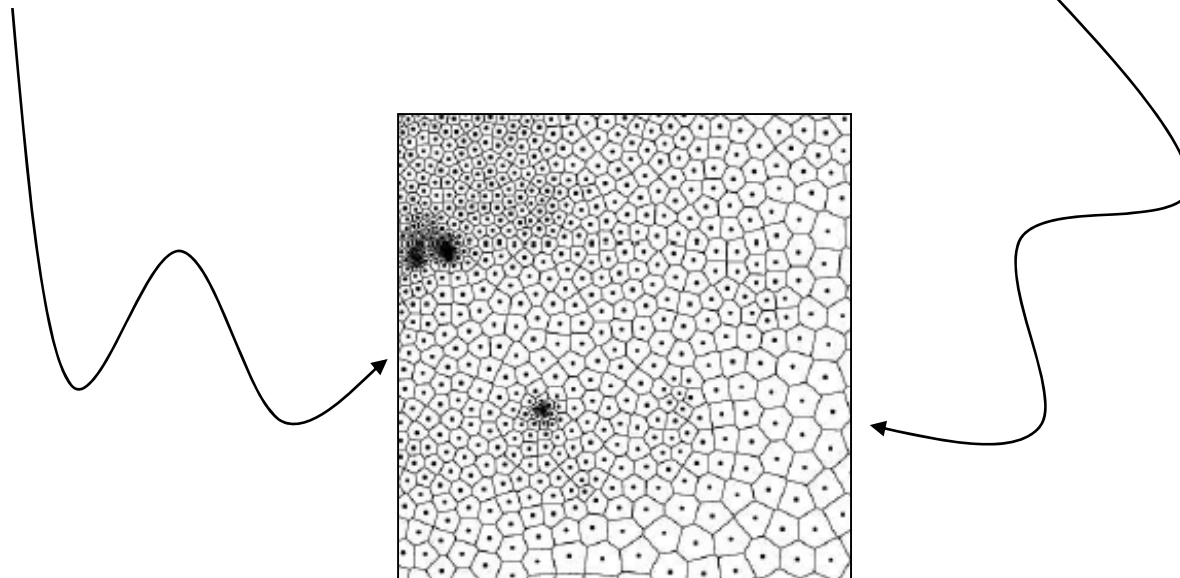
# Isotropic sampling

- **sampling**

- partitioning
- equal-mass enclosing

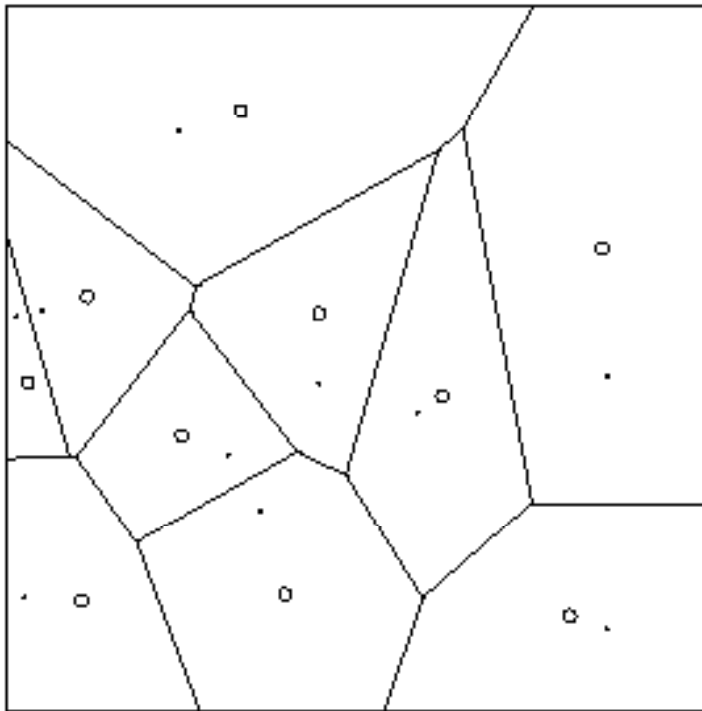
- **isotropic sampling**

- each tiles as compact as possible

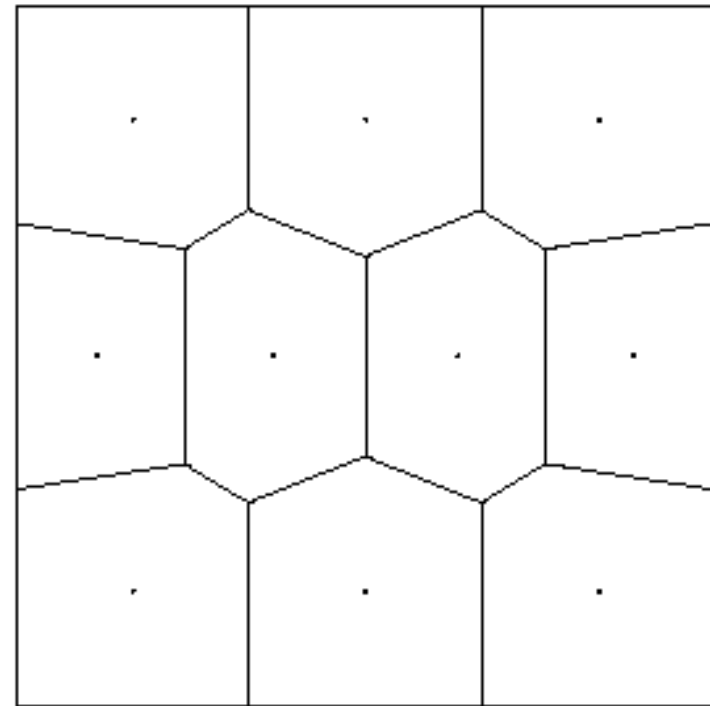


centroidal Voronoi tessellation satisfies both conditions

# Centroidal Voronoi diagram



Ordinary Voronoi diagram

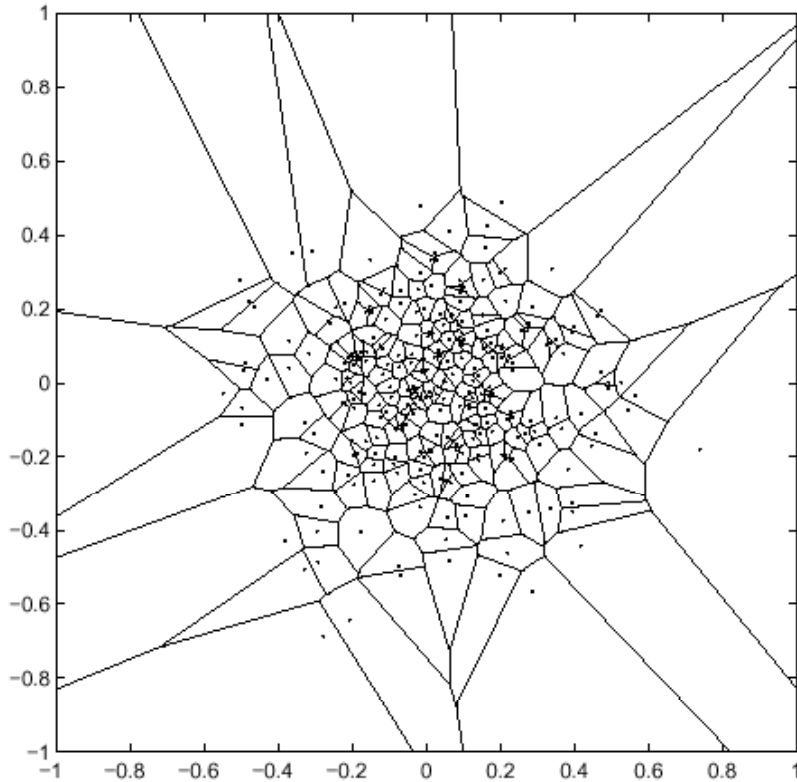


Centroidal Voronoi diagram

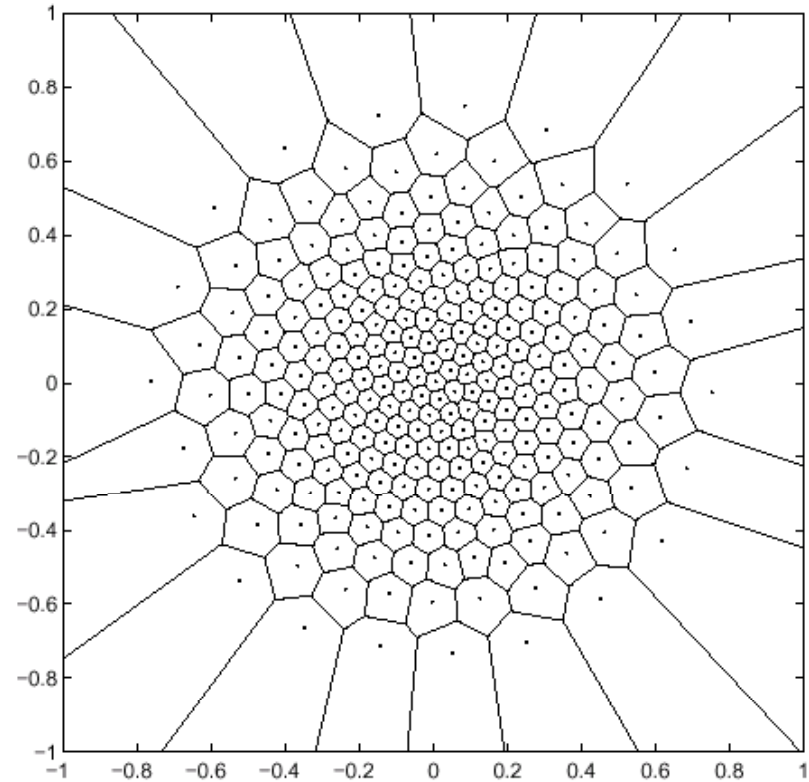
- sites
- centroids

Sites **coincide** with centroids  
(center of mass)

# Weighted Centroidal Voronoi diagram



Monte-Carlo



WCVD

Non-uniform density

# Centroidal Voronoi diagram

## Used for:

- optimal clustering
- optimal repartition of resources
- quantization
- tiling, etc. [Du *et al.* 01]

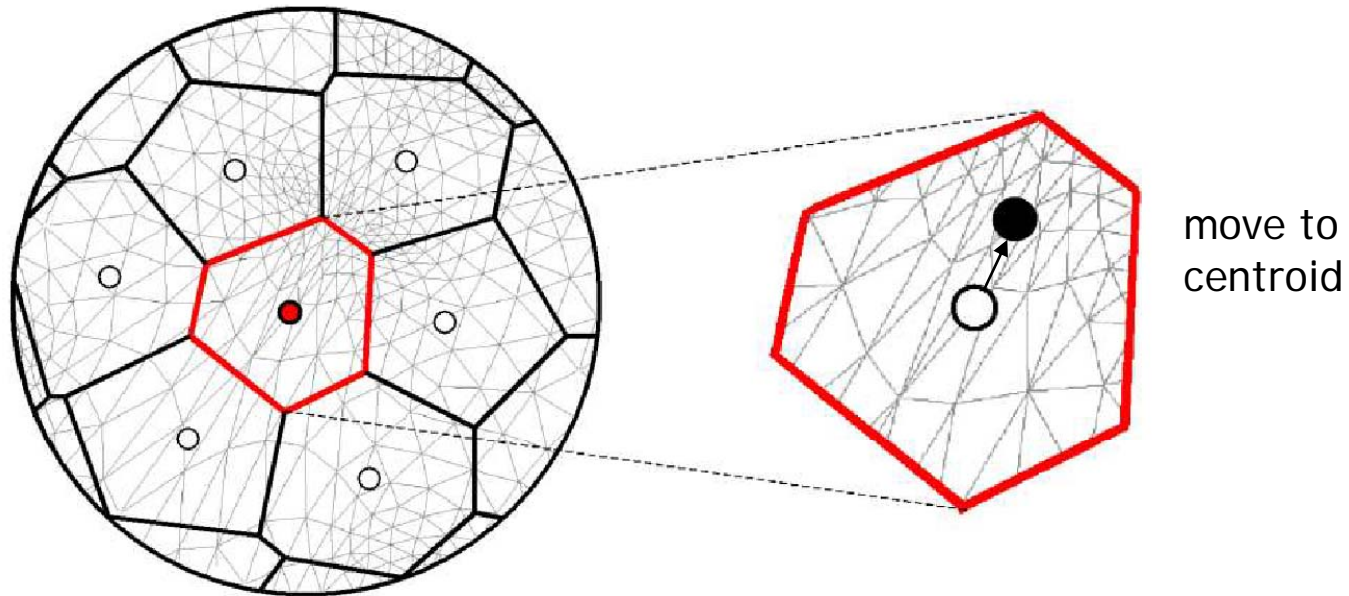
## Note:

- special configuration, not algorithm
- several algorithms: Lloyd, k-means, etc.
- works in  $nD$

# Sample placement

Two process sorted by increasing degrees of freedom:

1. build 1D WCVD
2. build 2D WCVD via Lloyd relaxation





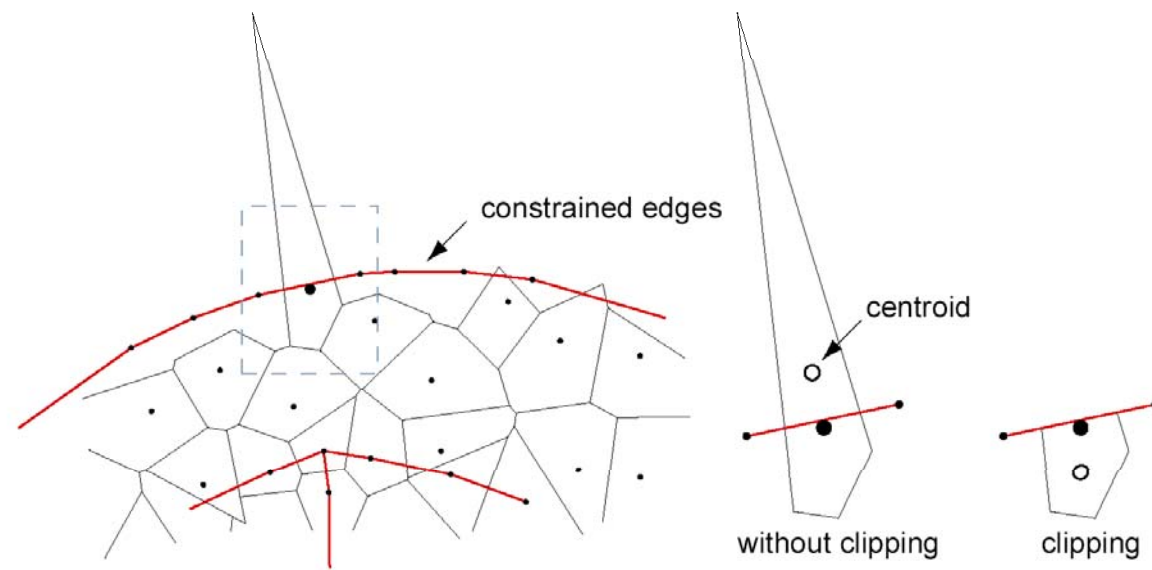
# Asymmetric behavior

## Three types of samples

0. Corner

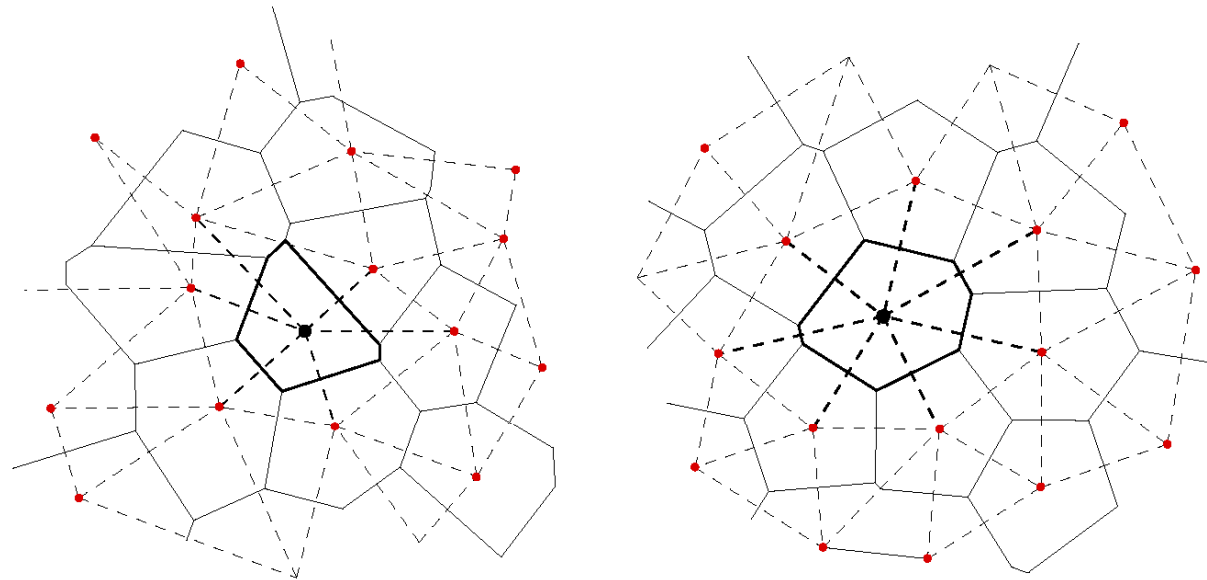
1. Feature, repulse surface samples via cell clipping

2. Surface, reverse not true



# Density approximation

- Piecewise linear on **new** samples
- Low pass filter density function for undersampling
- Exploit “de Gabriel” properties for better efficiency



Motivation

Previous work

Contributions

Algorithm

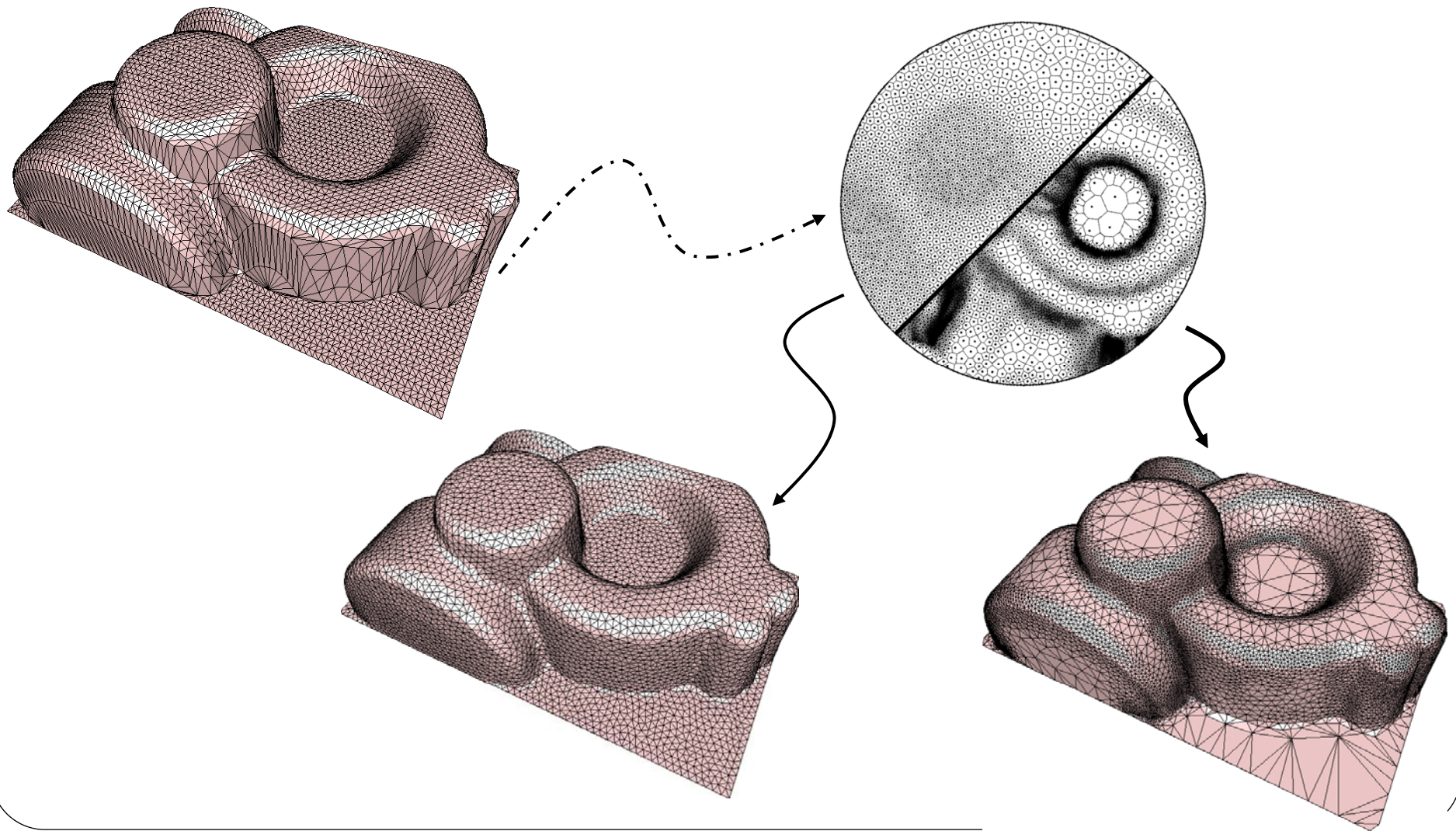
Results

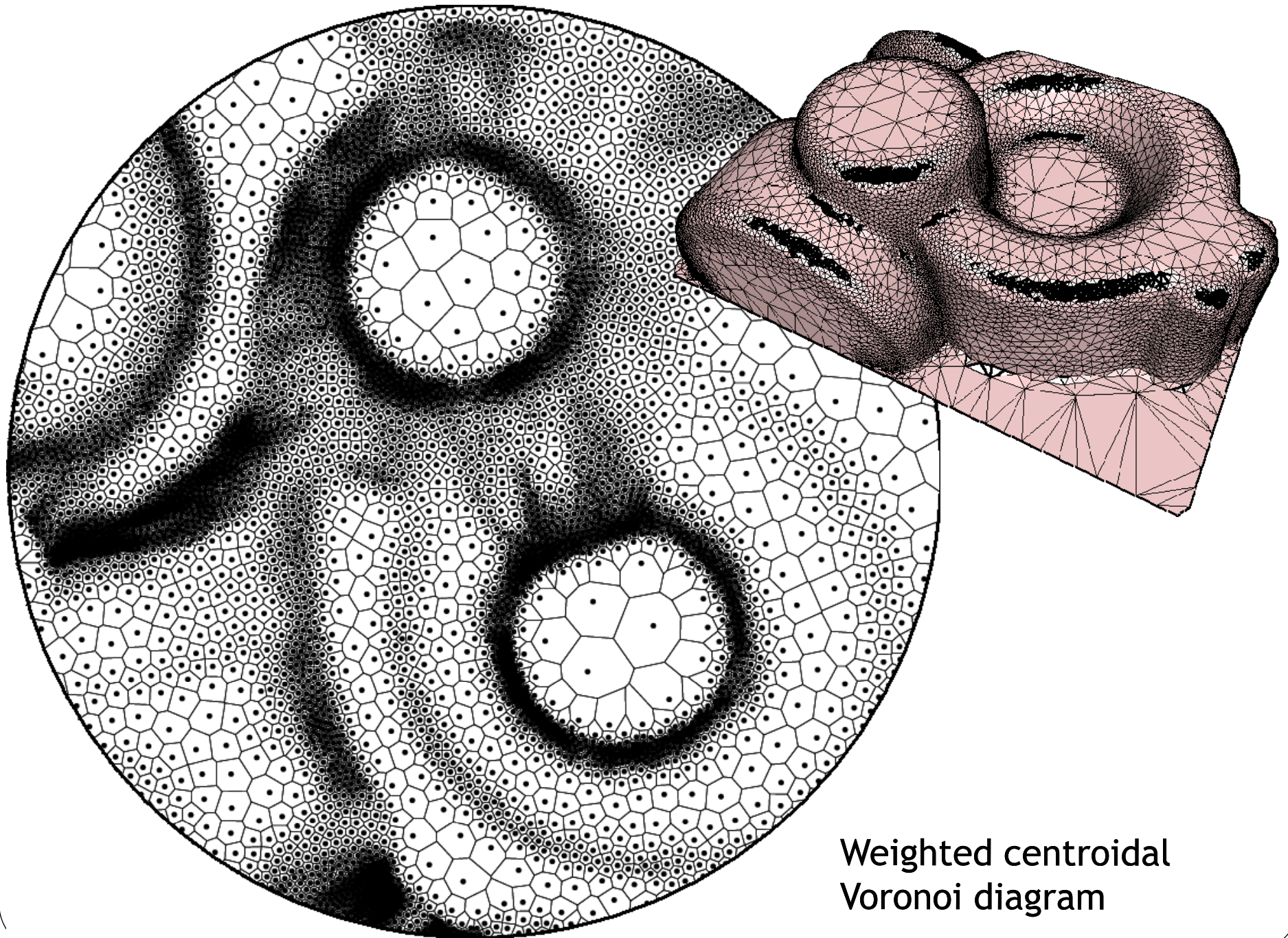
Limitations

Conclusions

Future Work

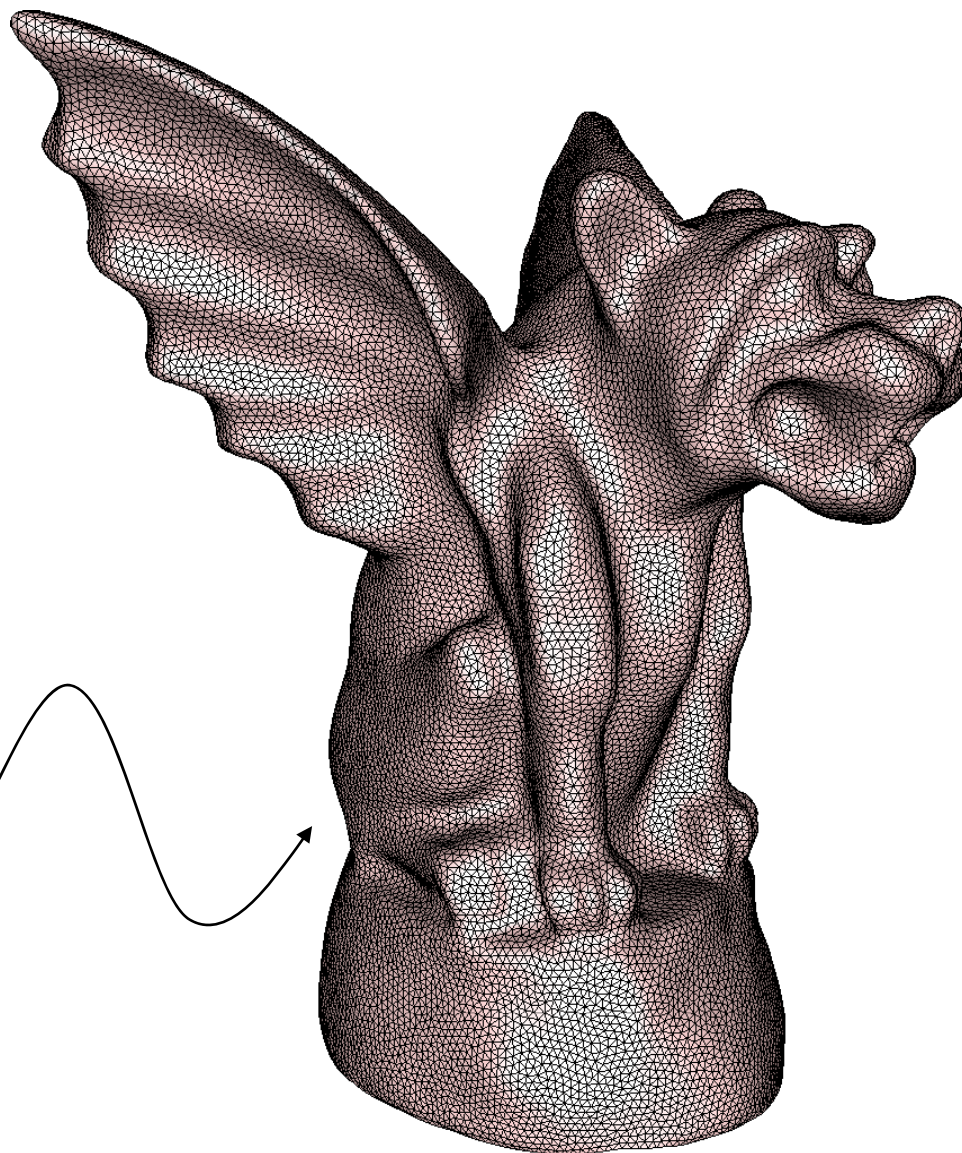
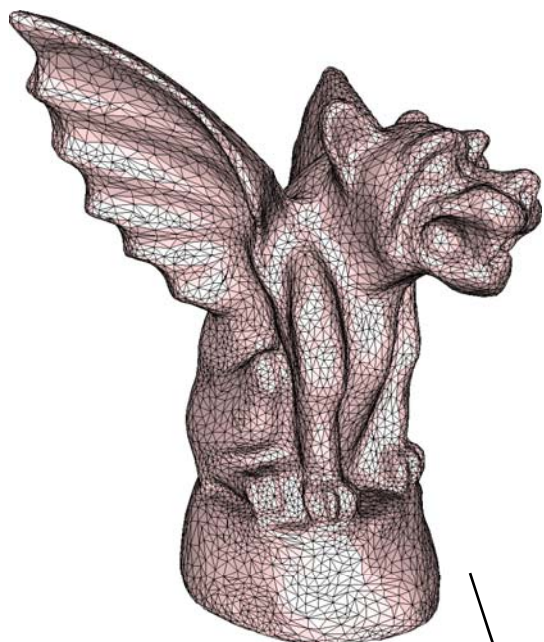
# Uniform vs curvature-adapted



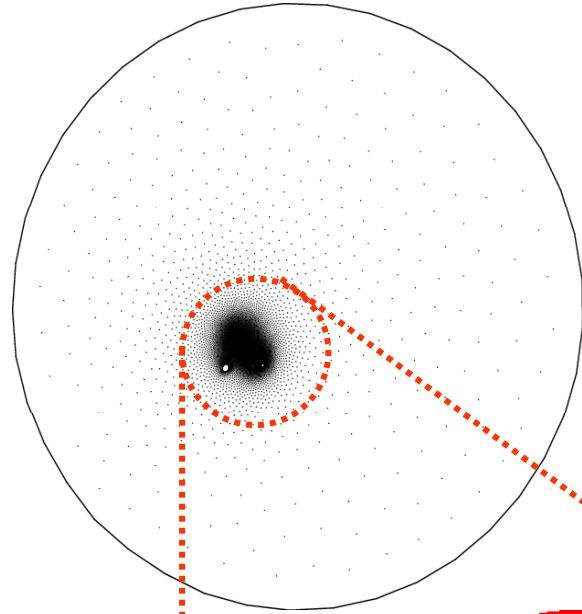
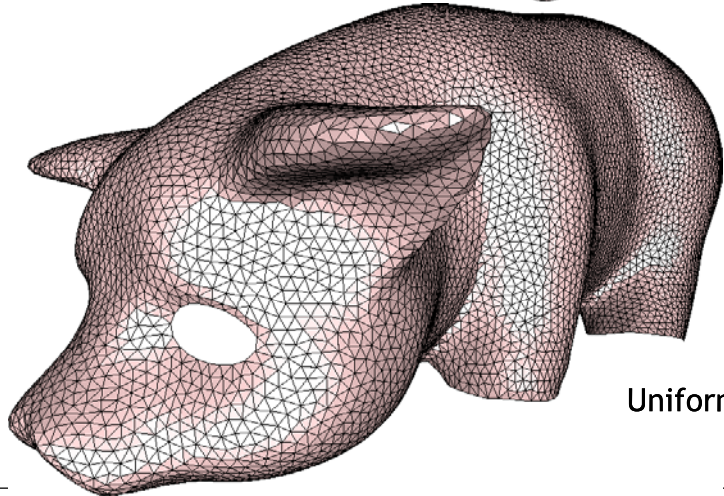
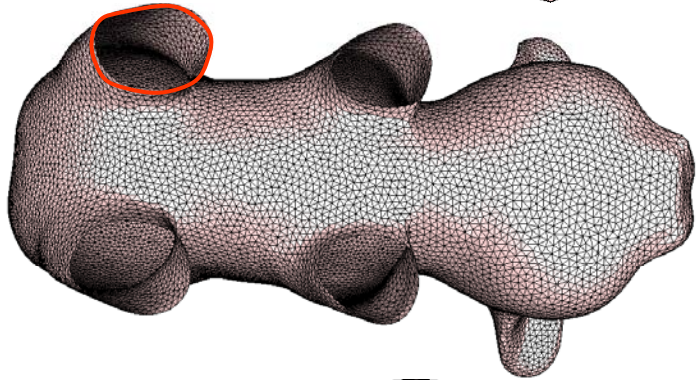
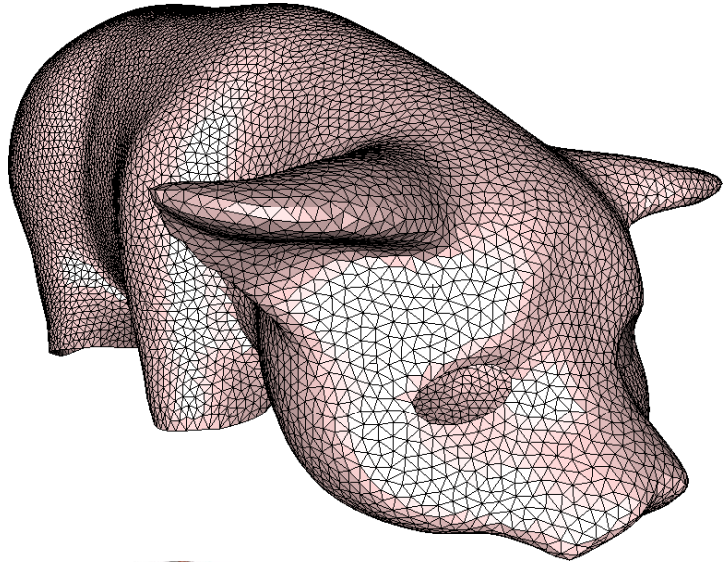


Weighted centroidal  
Voronoi diagram

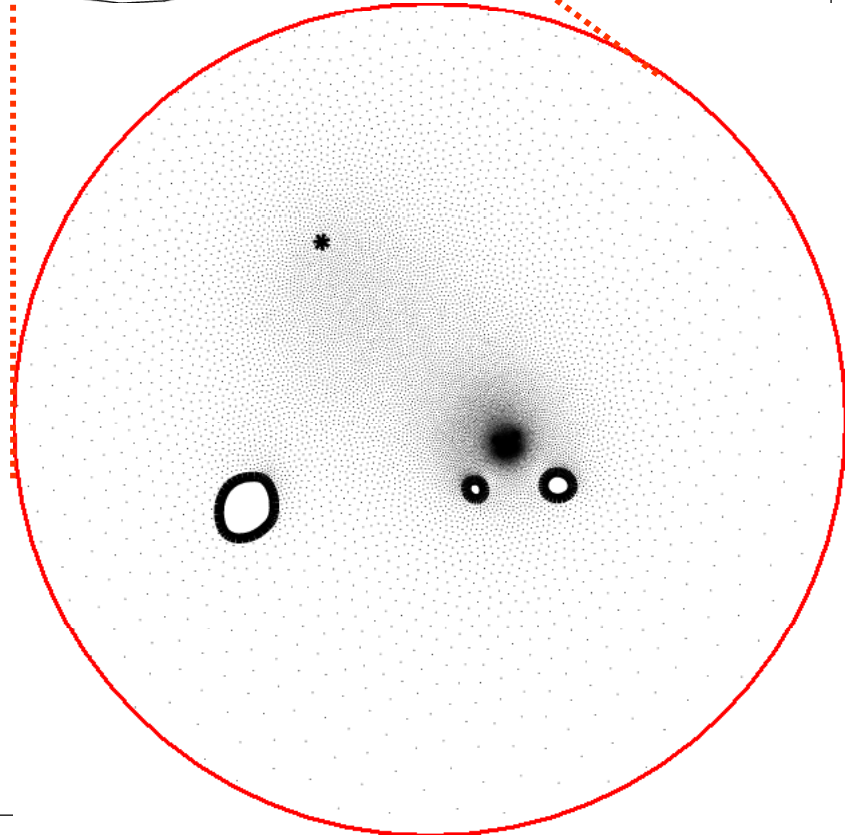
# Gargoyle



uniform - 30,000 vertices

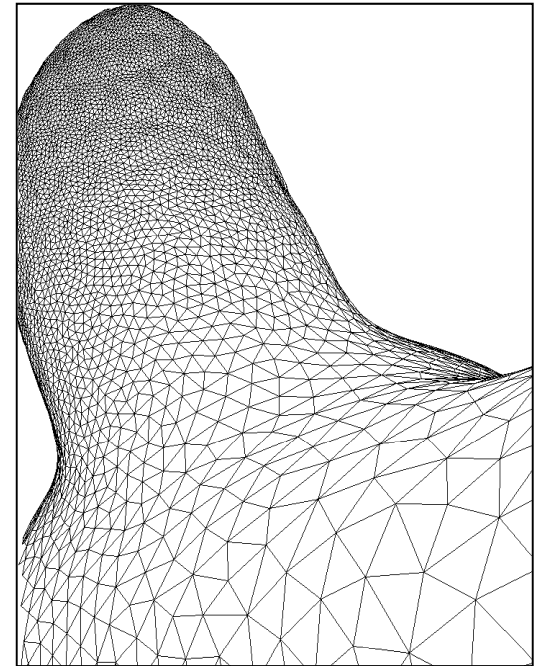
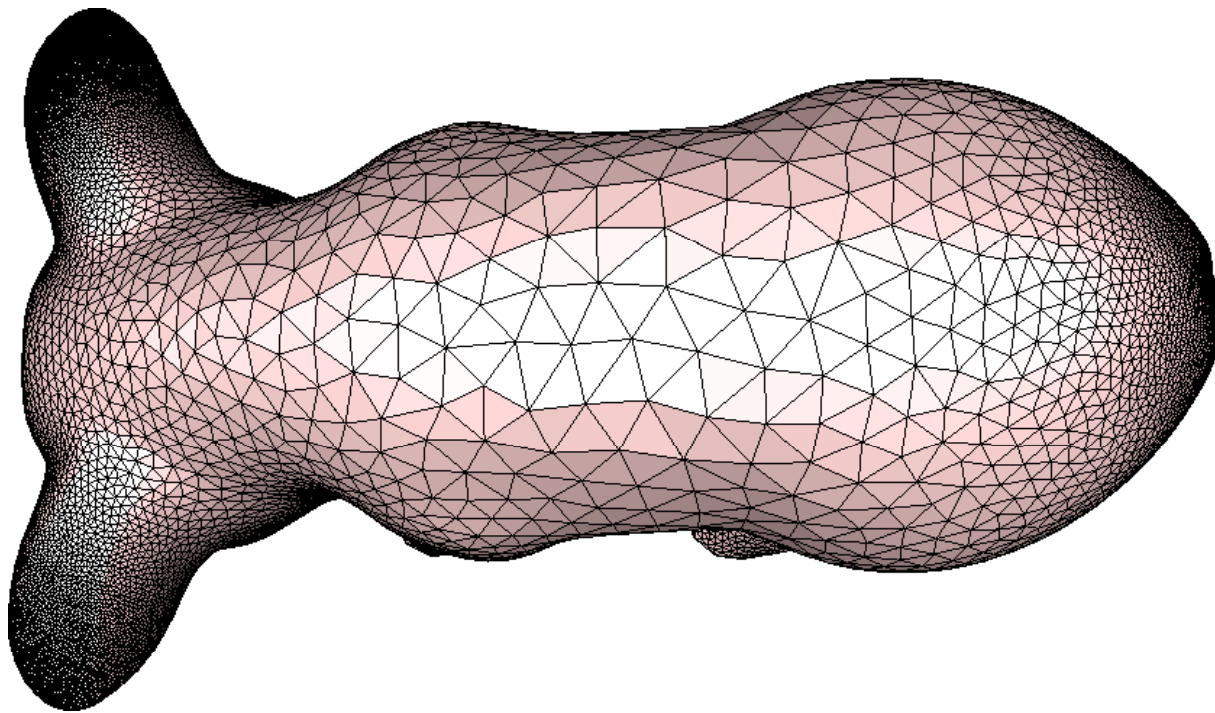


Example of extreme area distortion



Uniform sampling

# Smooth gradation

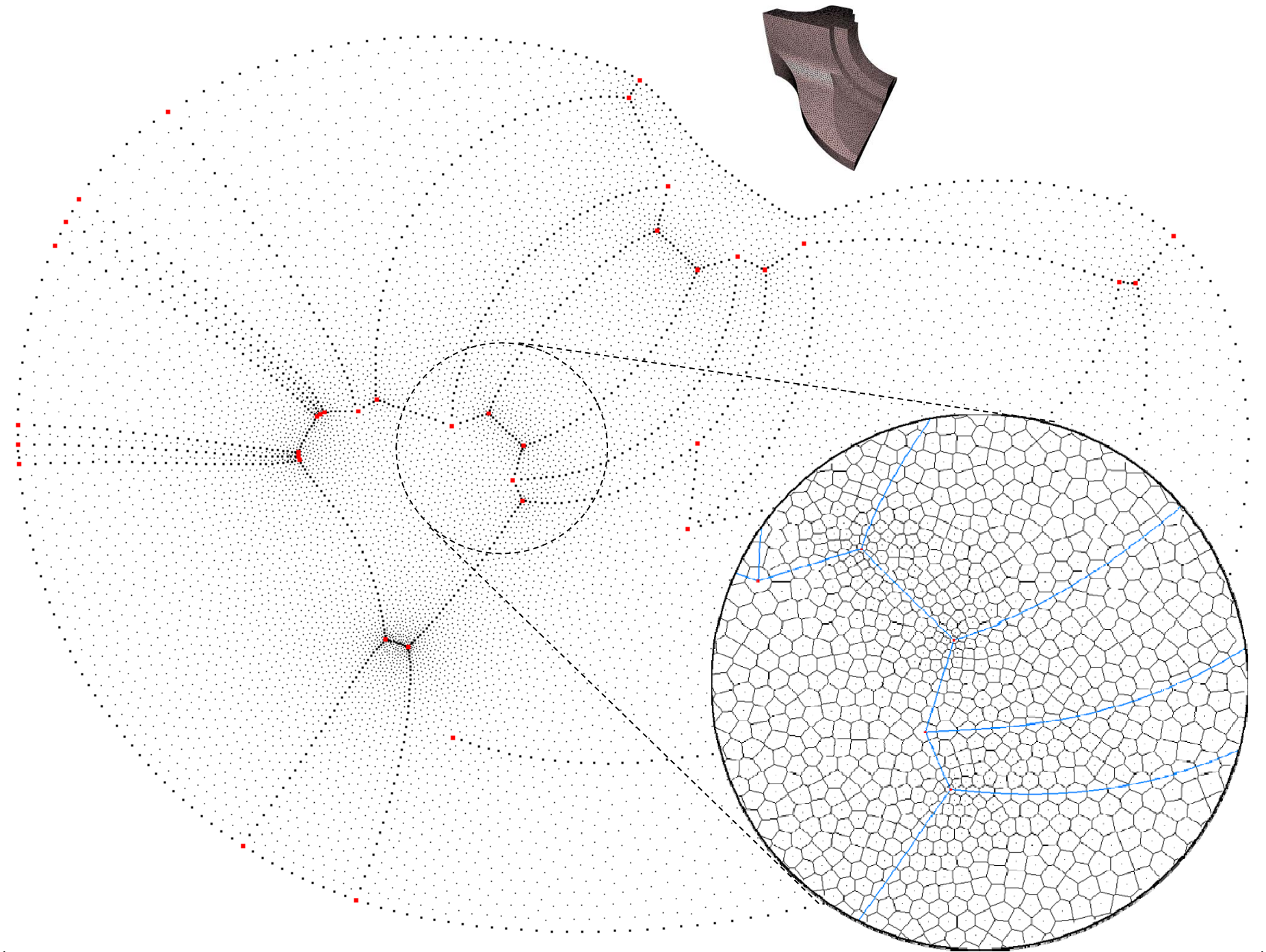




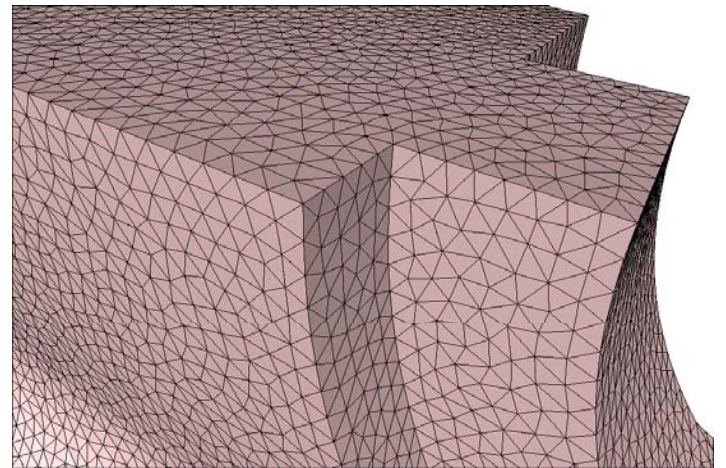
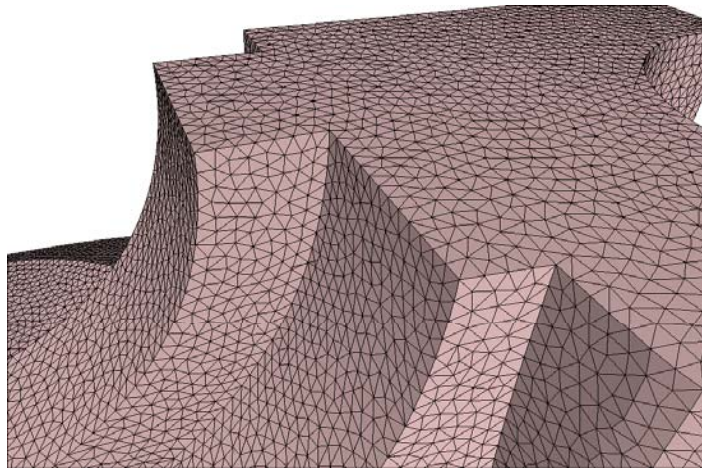
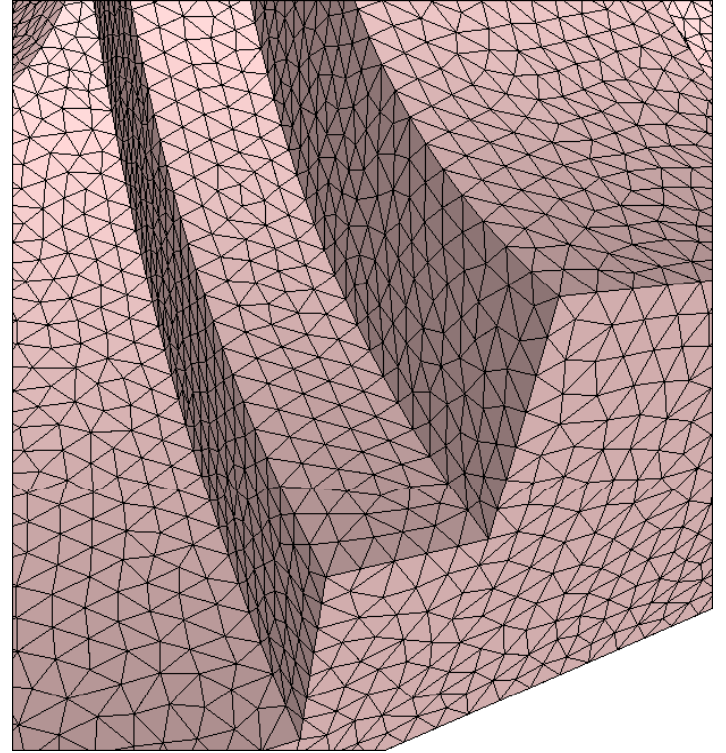
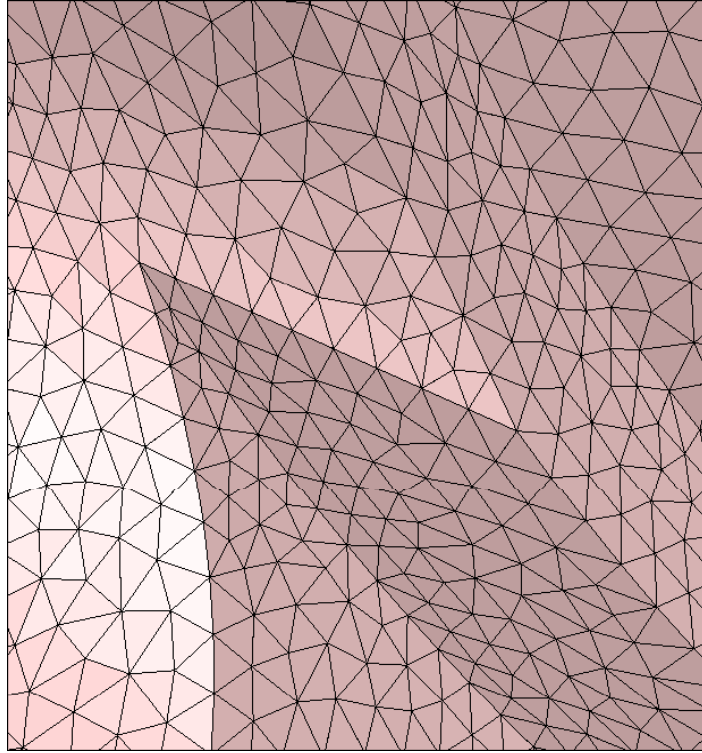
# CAD models

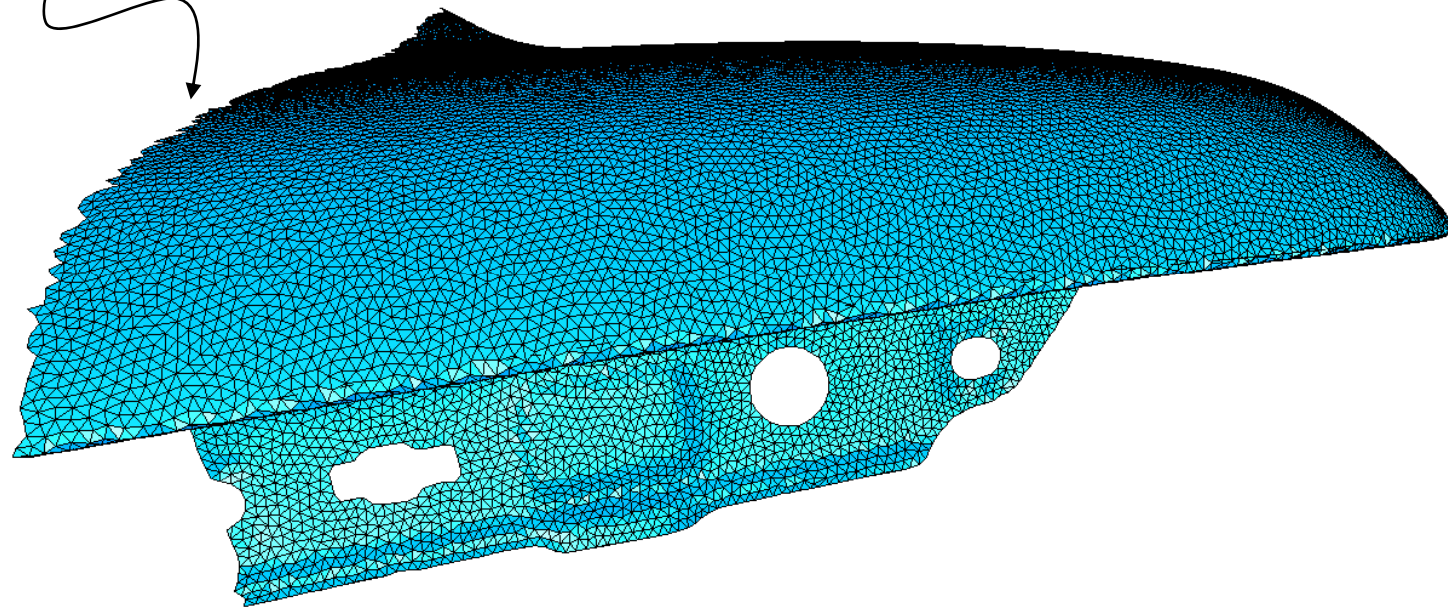
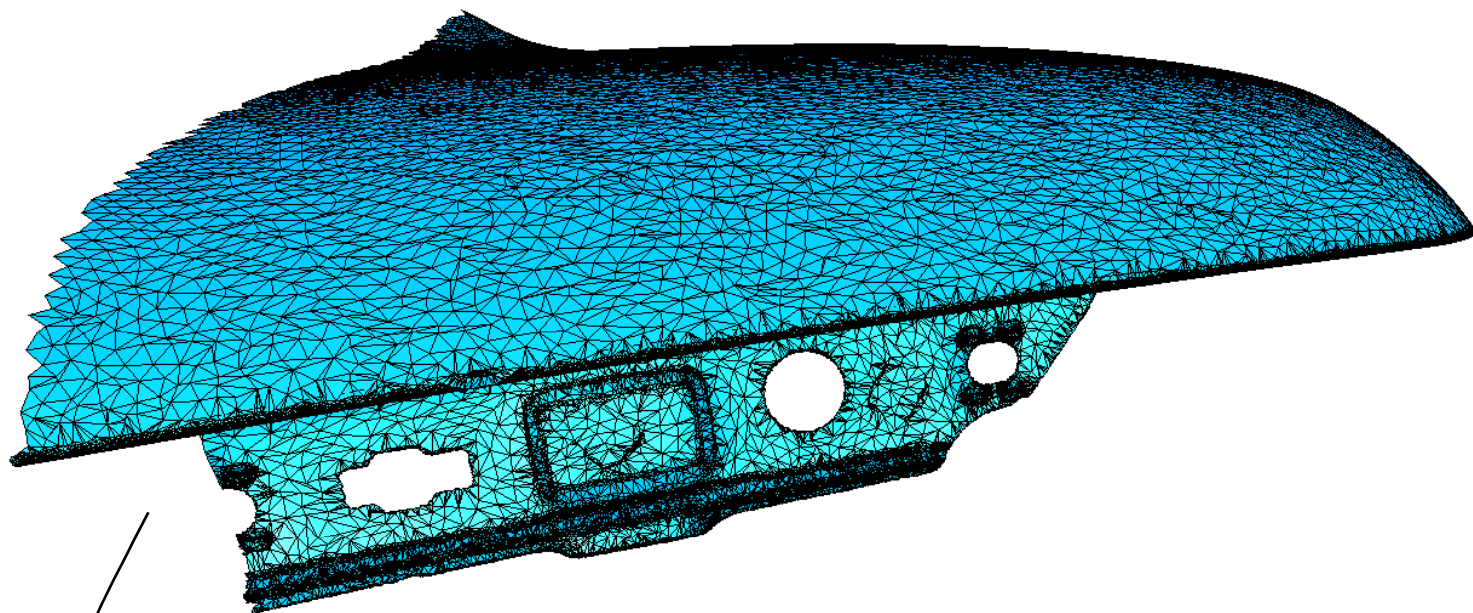
## Feature backbones:

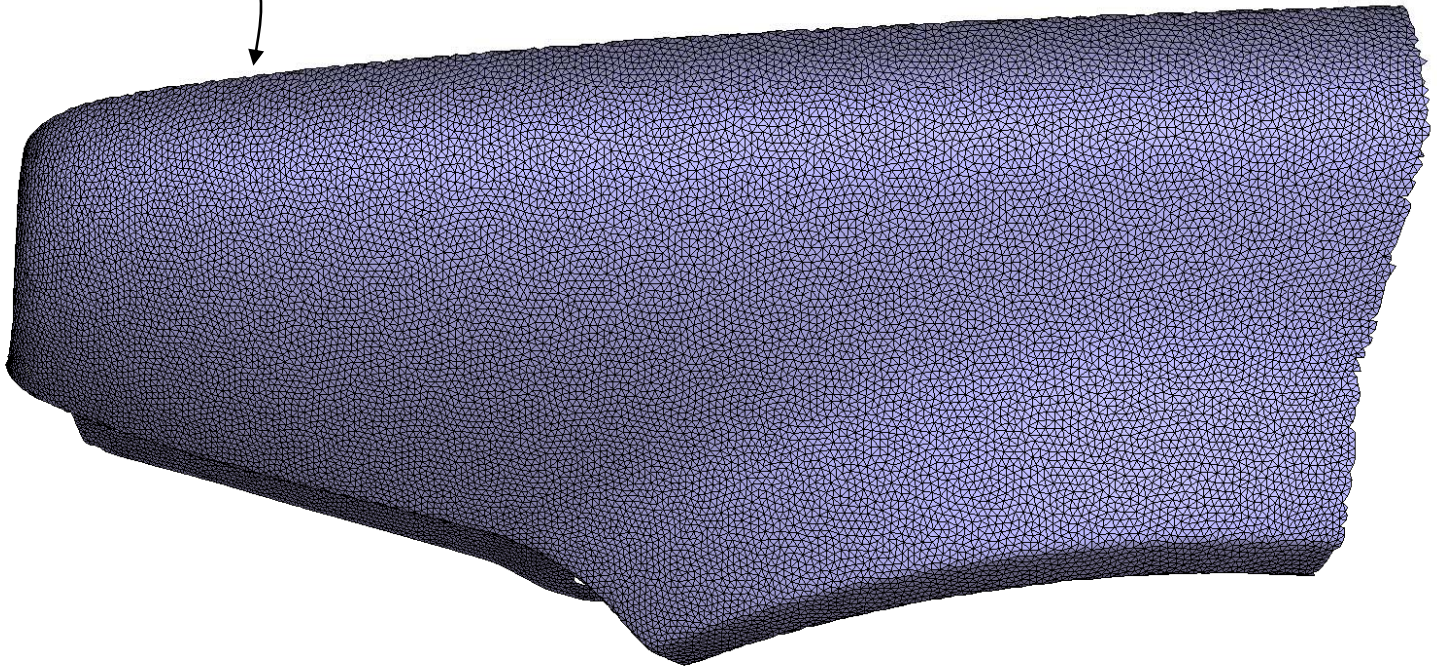
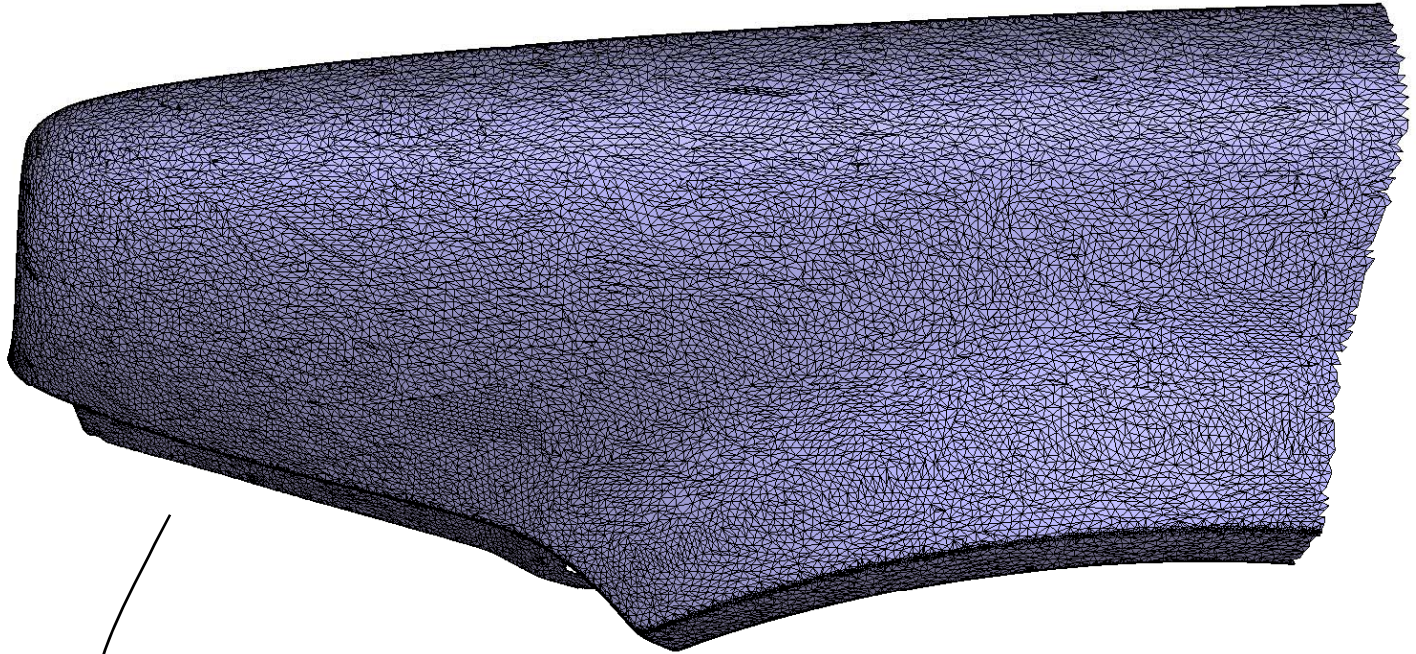
- 1D error diffusion
- arc-length parameterization of backbones
- 1D WCVD





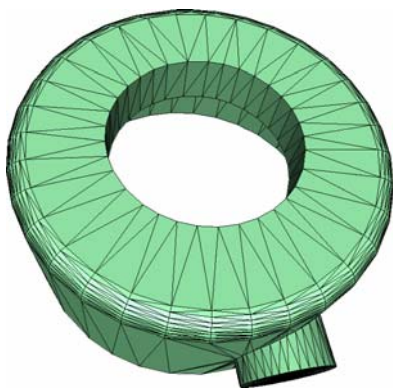




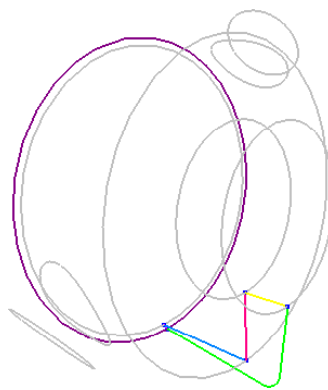


# Genus > 0 model

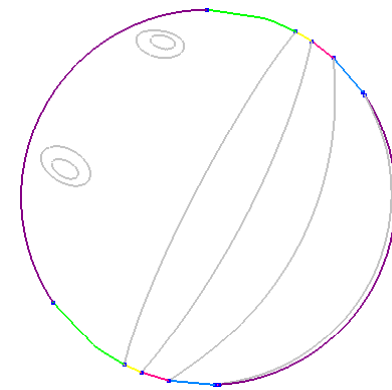
- cutting -> **cut graph**
- add cut graph to feature skeleton  
-> twin backbones associated pairwise
- synchronize sampling along twin backbones to guarantee stitching



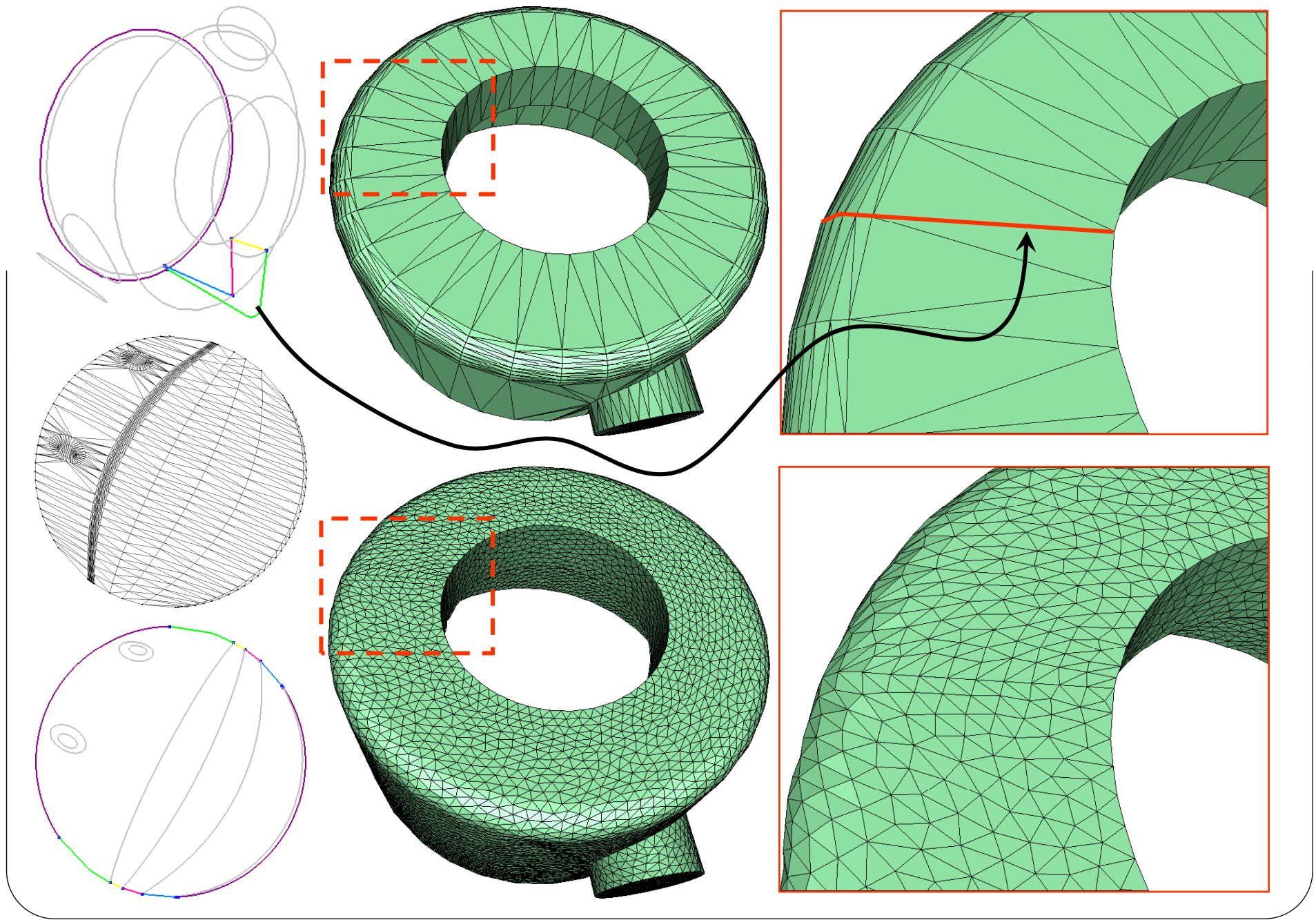
genus 1



cut graph

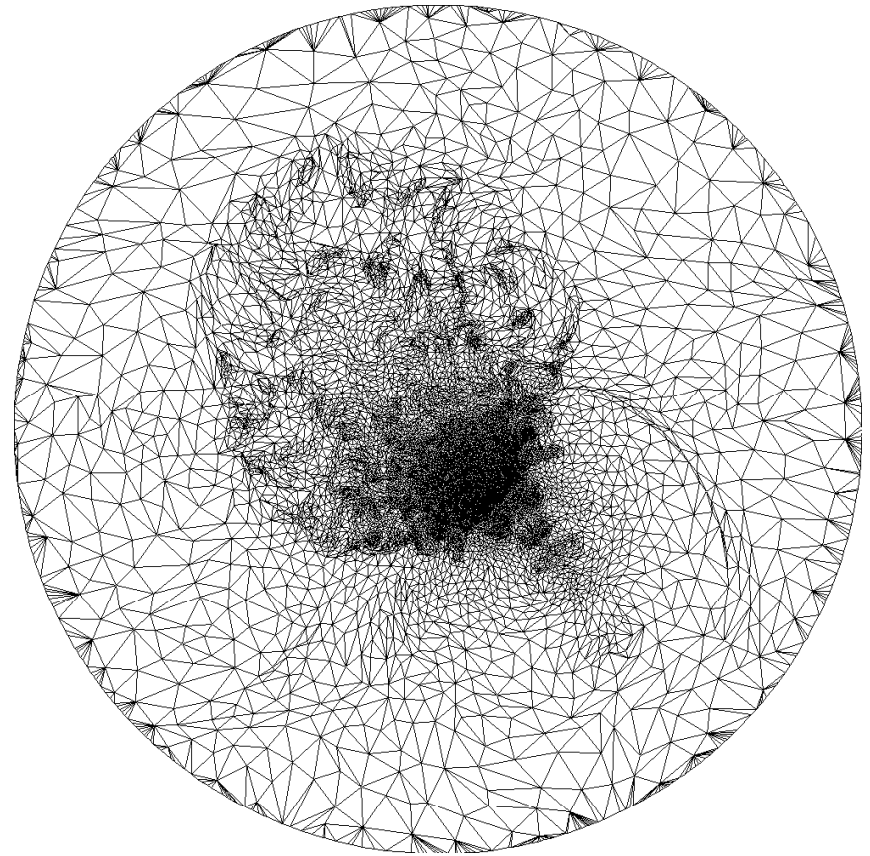
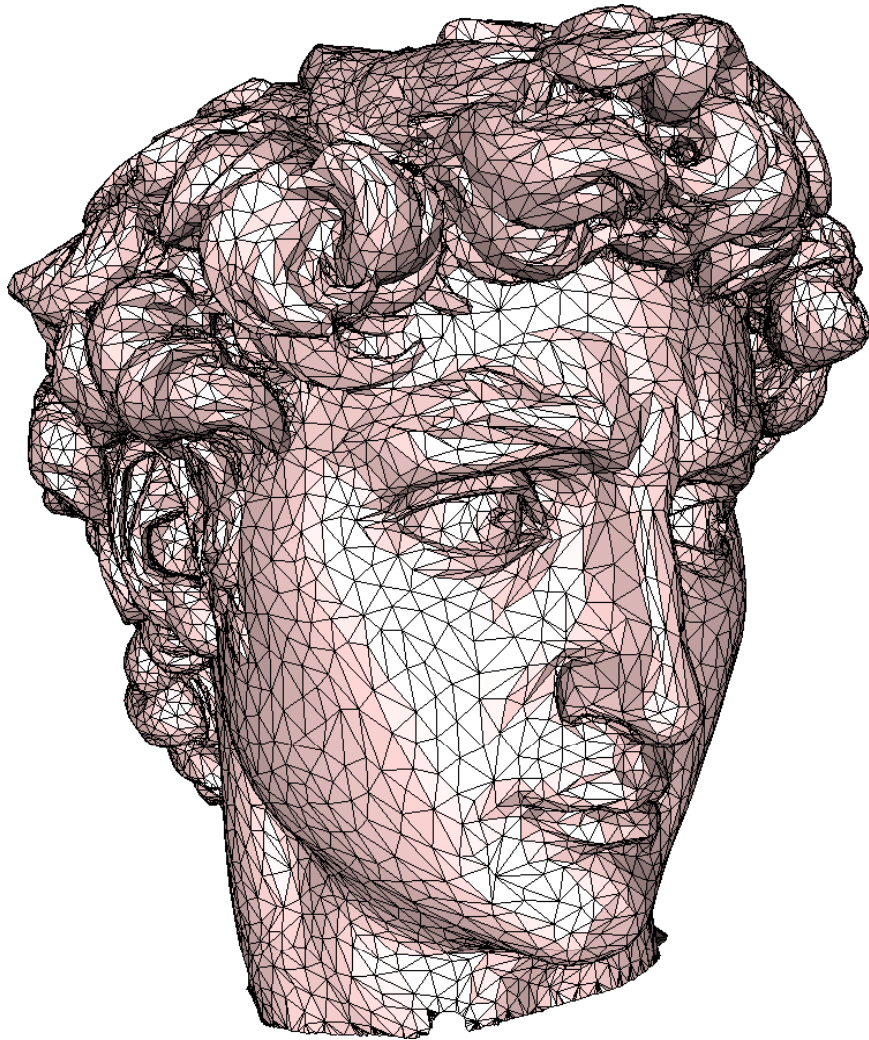


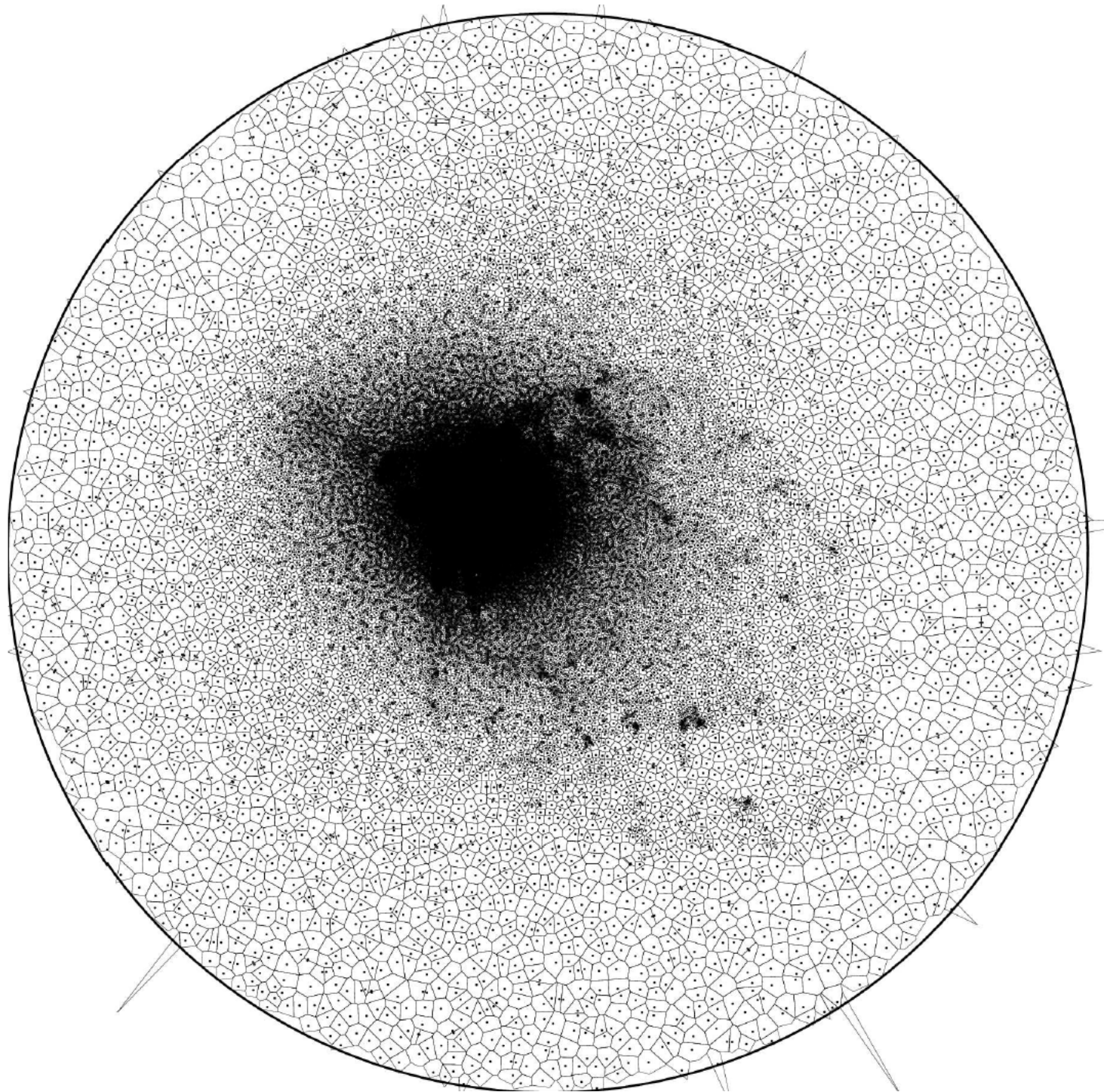
parameter space



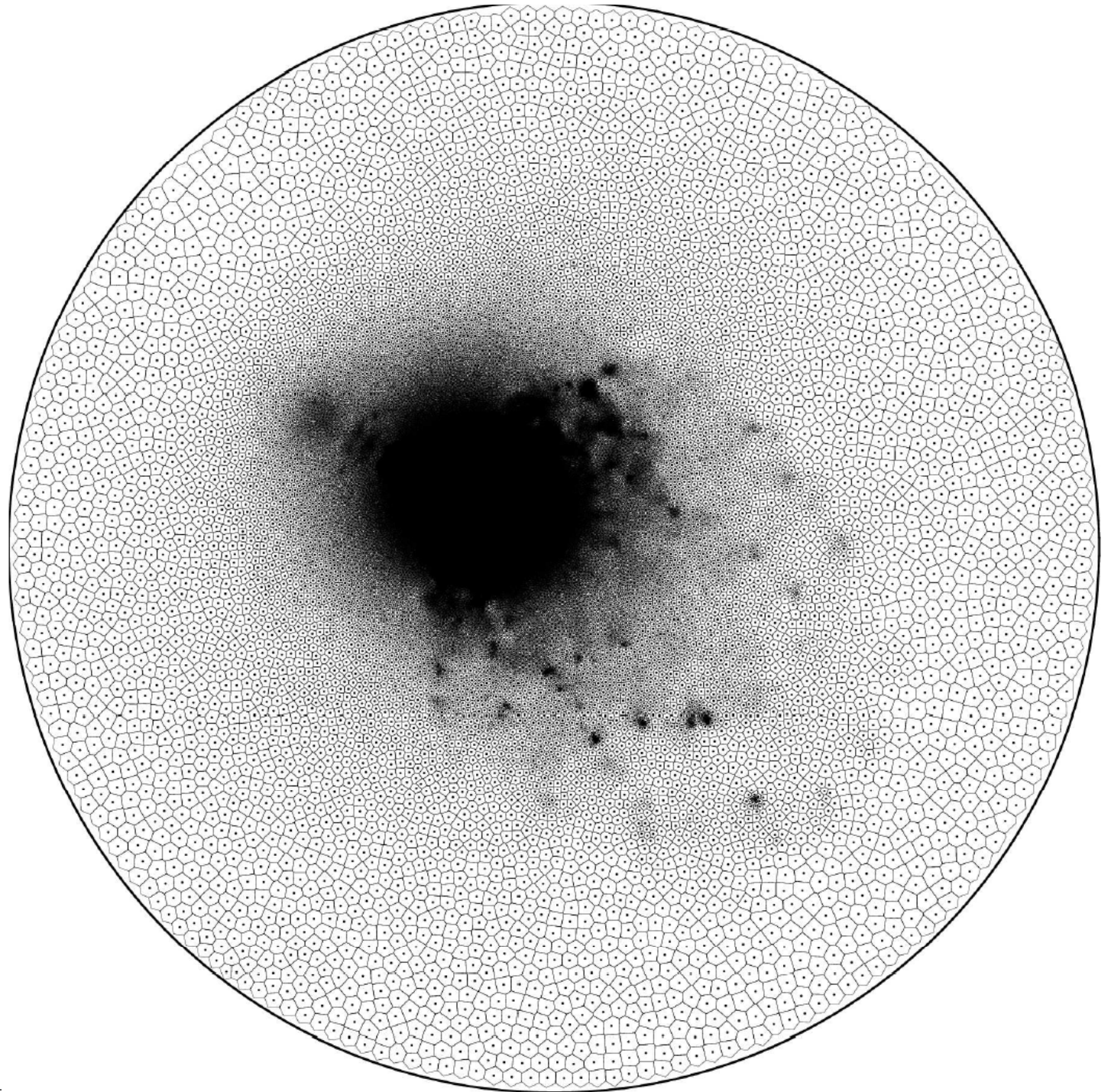


# David model [Digital Michelangelo]



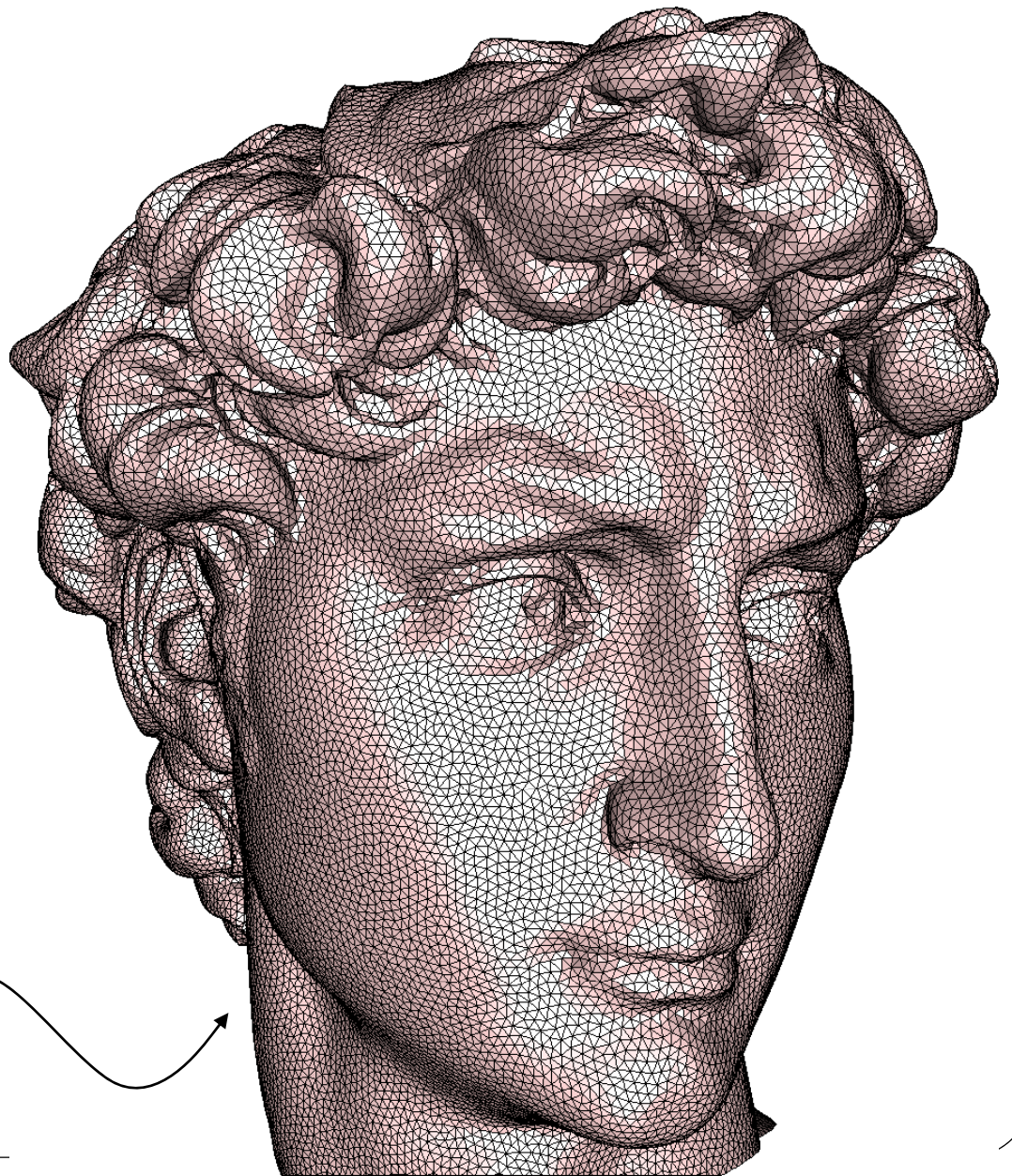
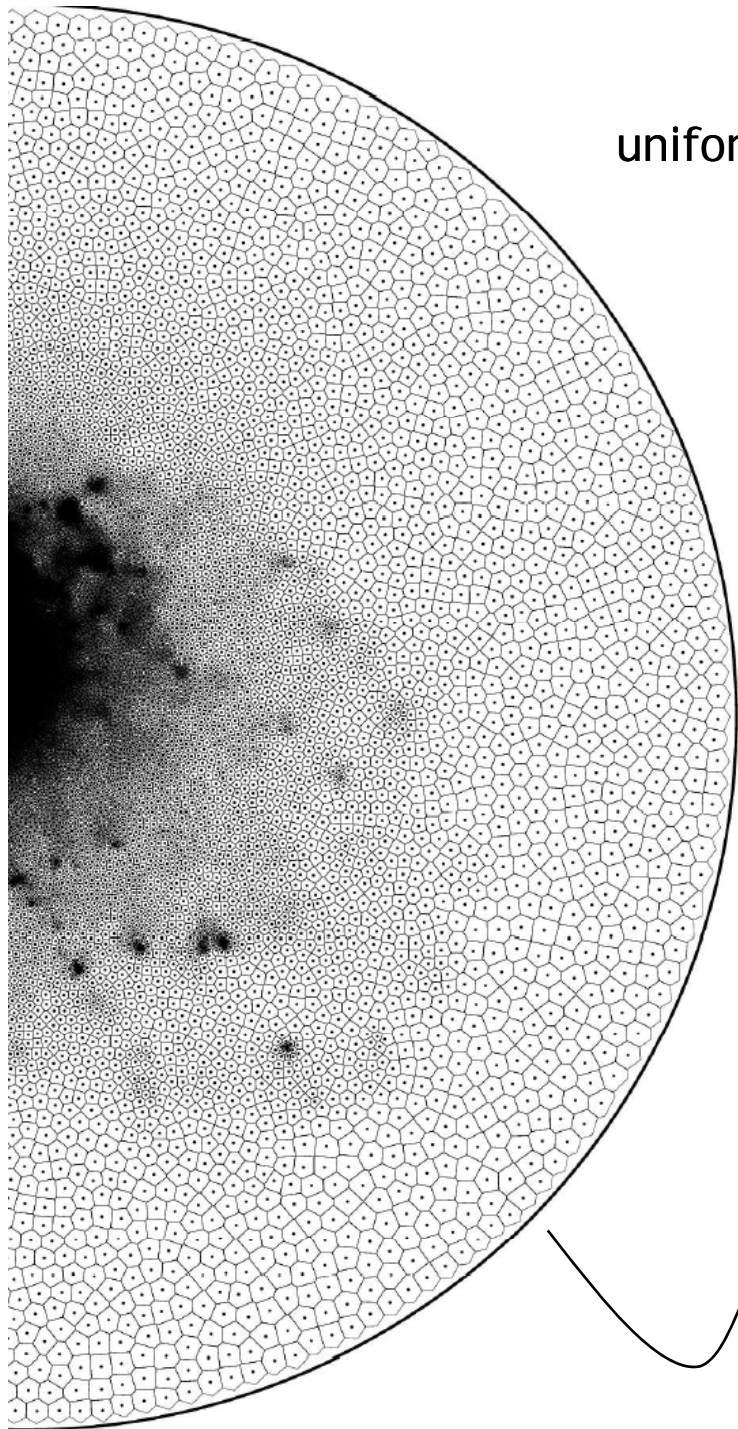


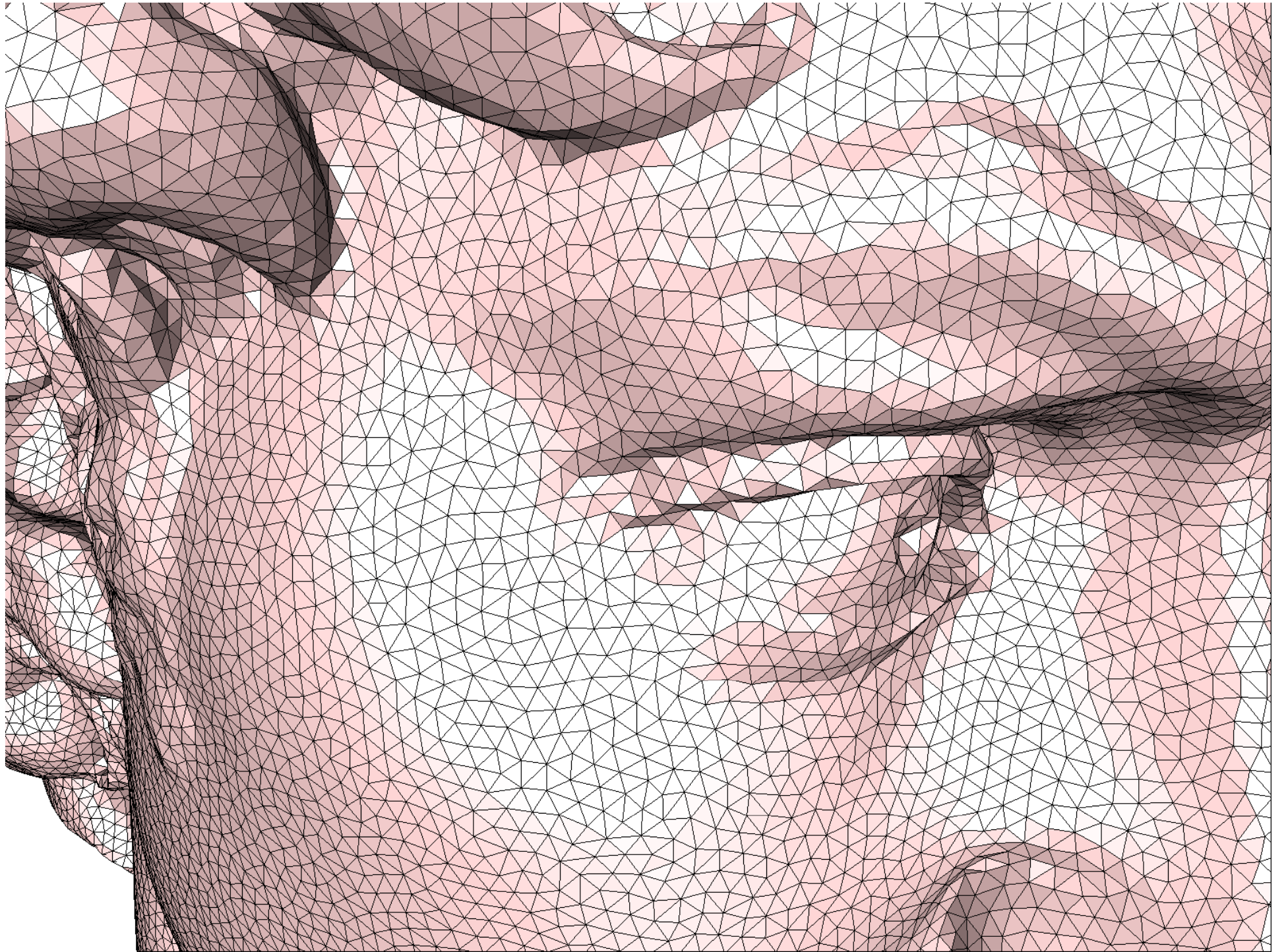
Sample repartition  
by error diffusion  
(50,000 vertices)



Sample placement  
by WCVD

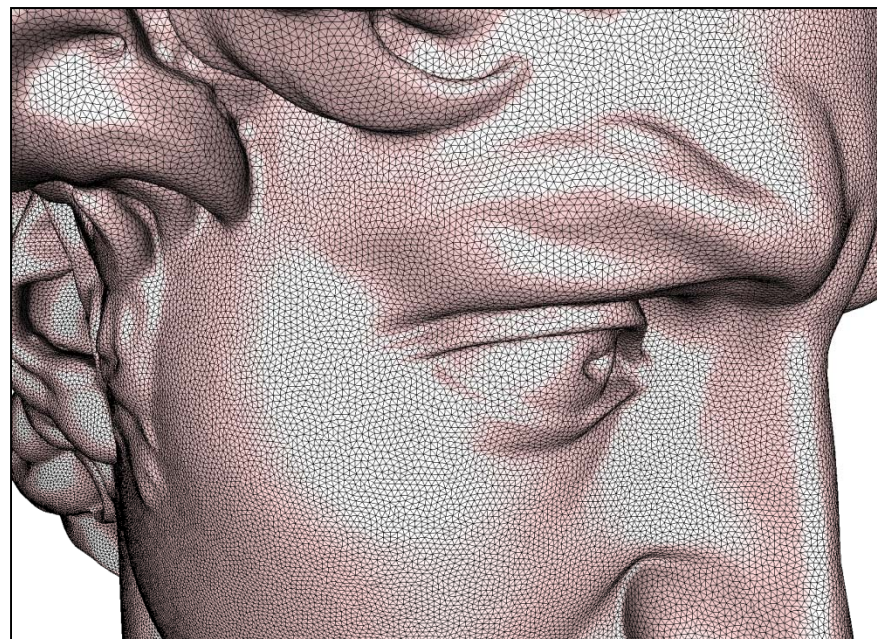
uniform sampling

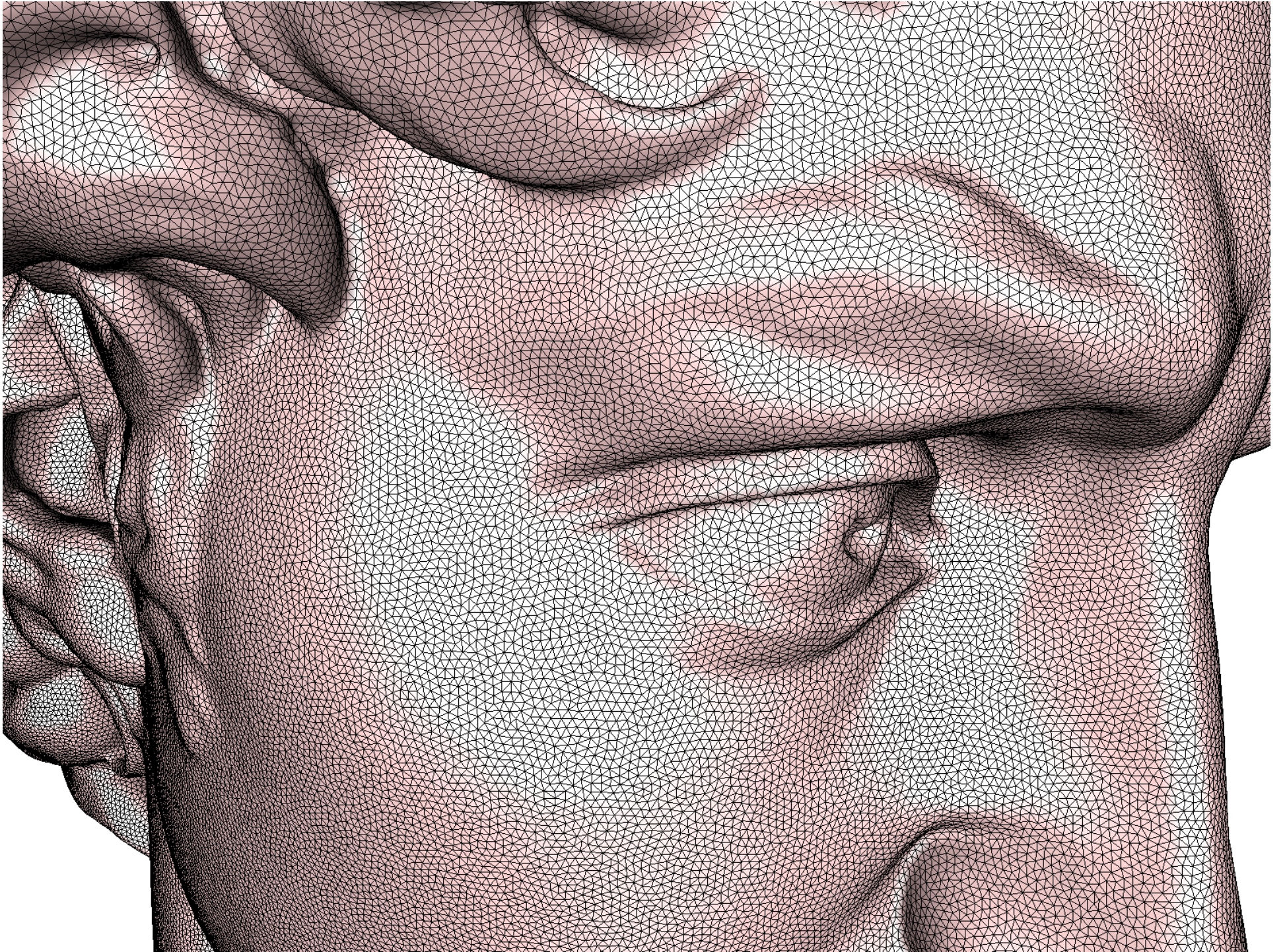




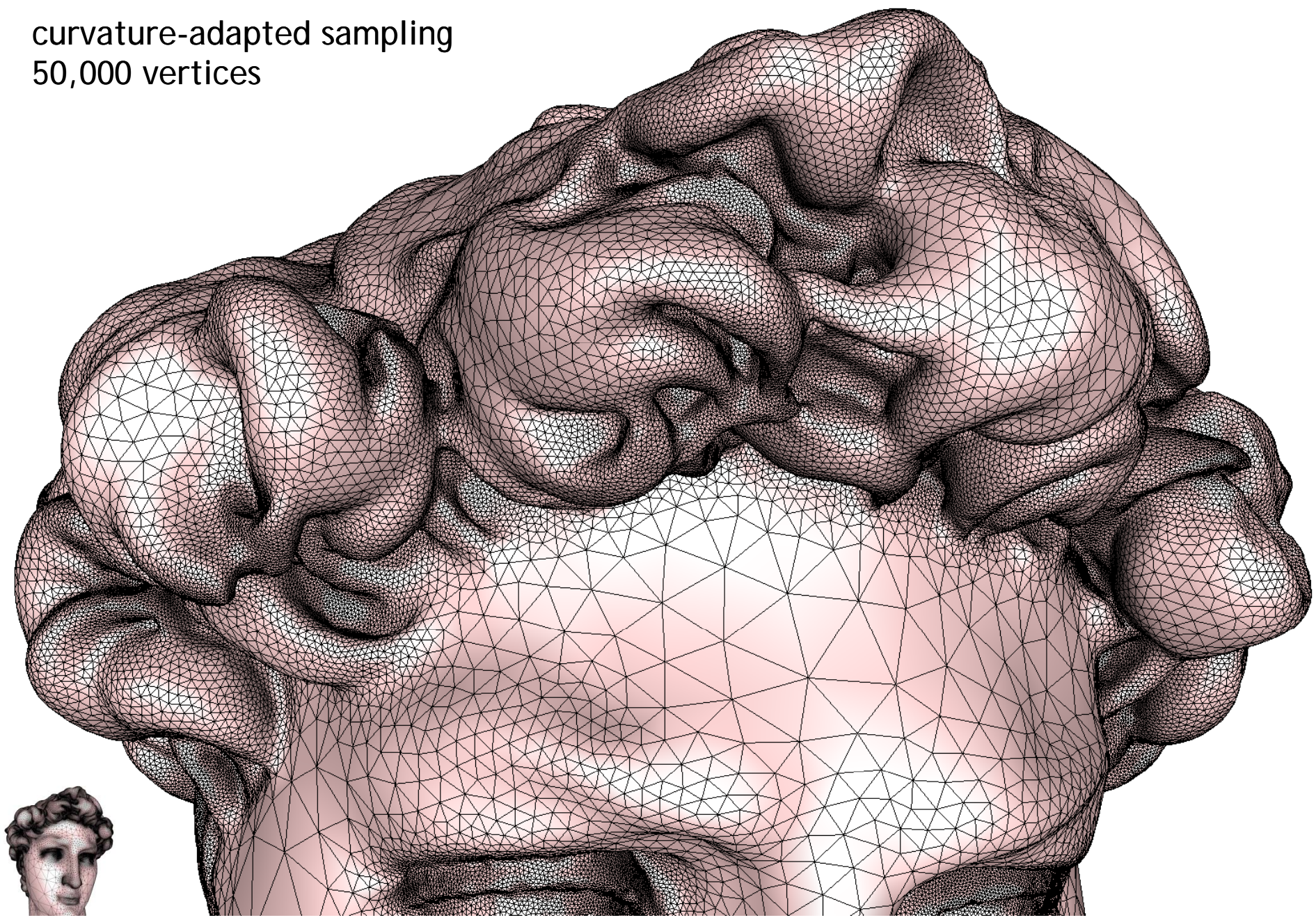


uniform sampling  
300,000 vertices

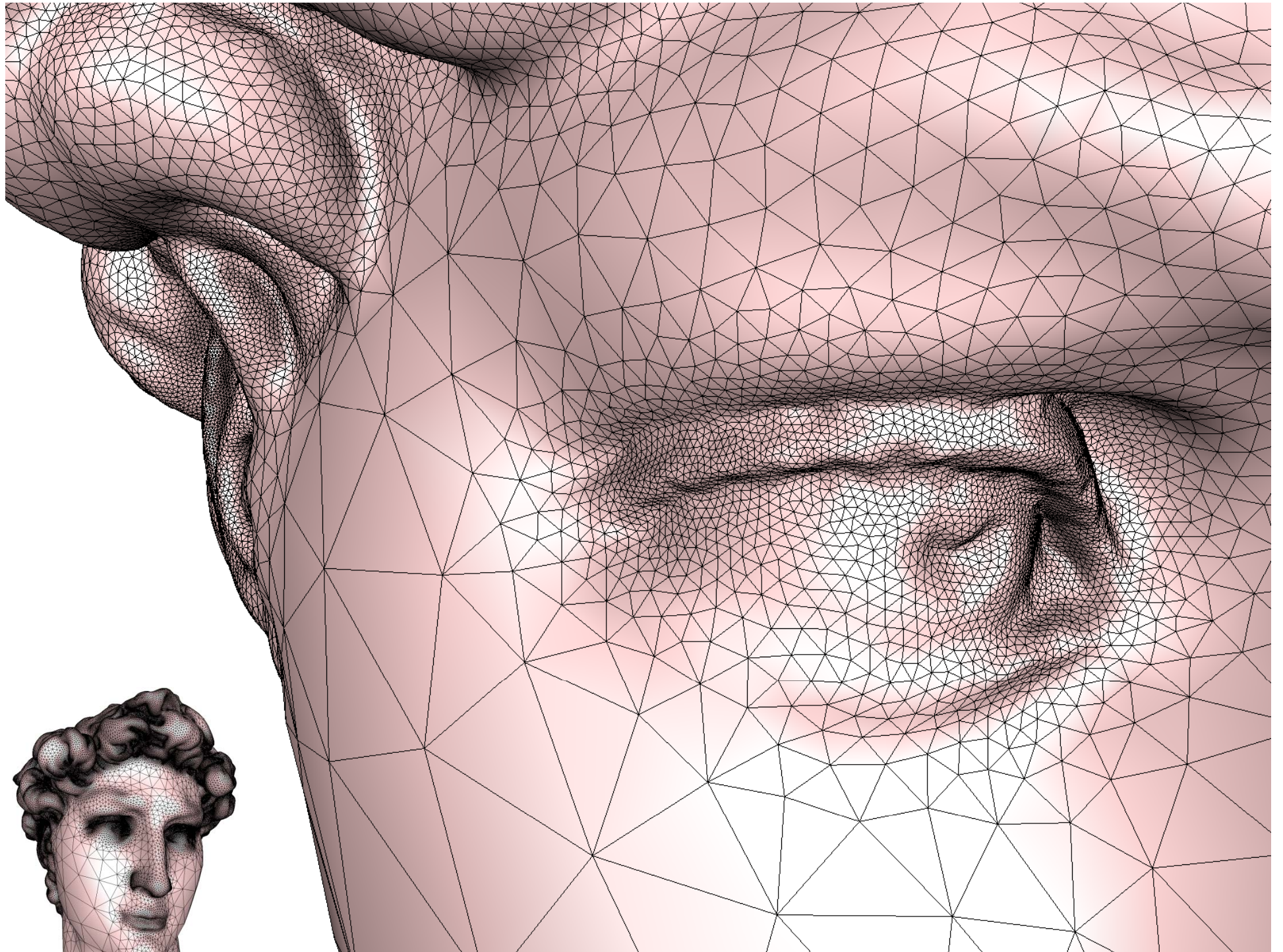


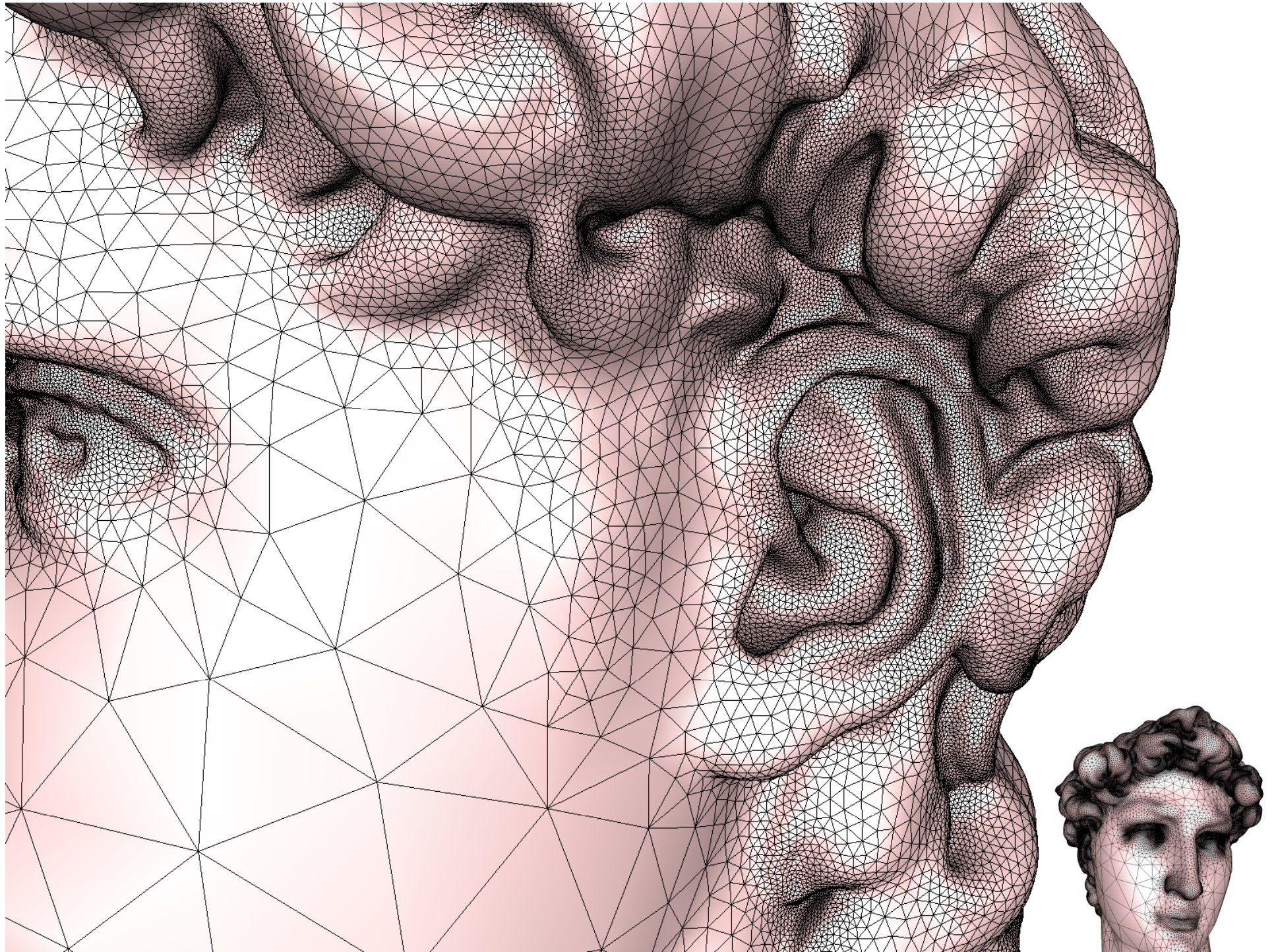


curvature-adapted sampling  
50,000 vertices









Motivation

Previous work

Contributions

Algorithm

Results

Limitations

Conclusions

Future Work

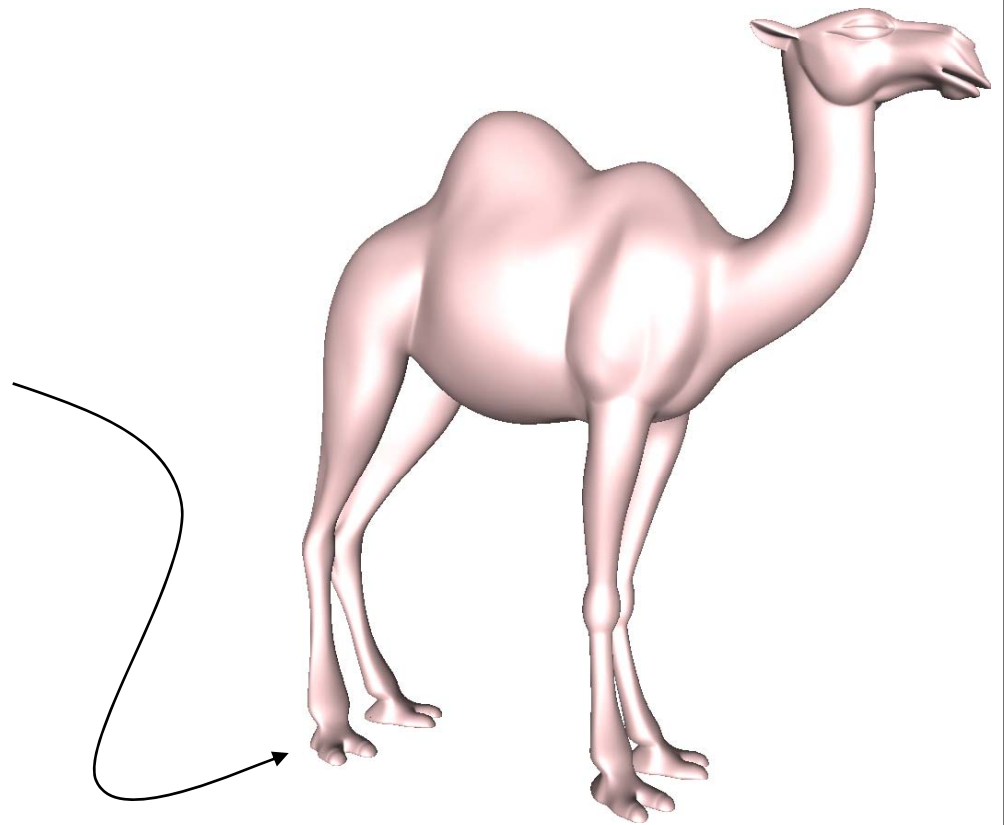
# Limitations

- **Parameterization**
  - still some numerical issues for huge models
  - quality of sampling is *very dependent* on the quality of the parameterization
- **Complex genus or closed surface**
  - requires surface cutting (difficult task)
  - process "curve sampling" along the cut graph
  - makes the implementation trickier (seaming backbones, twin samples to synchronize for stitching, branching vertices, etc.)

# Limitations

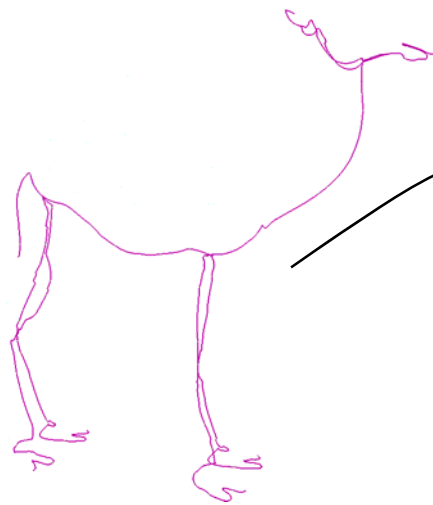
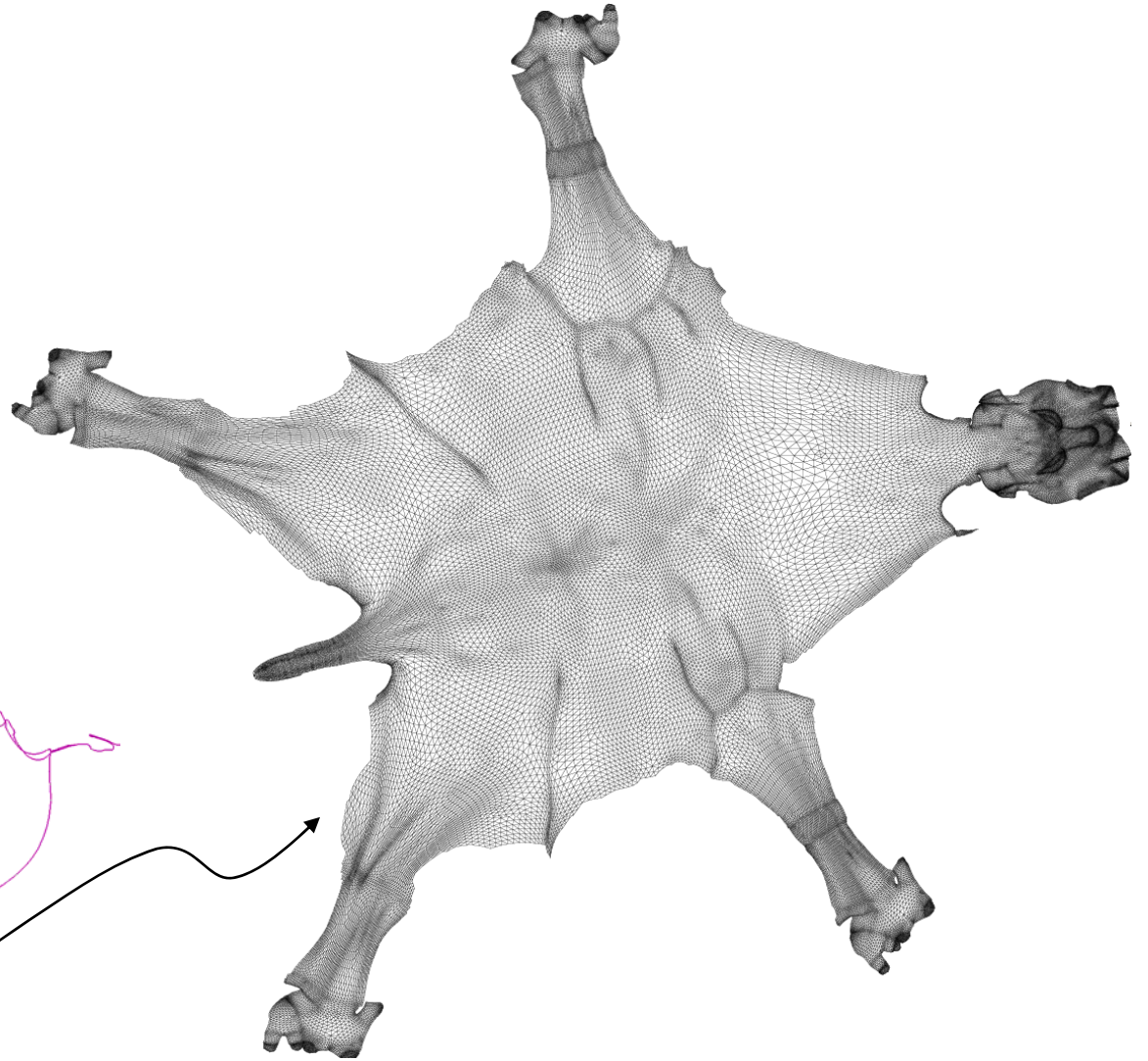
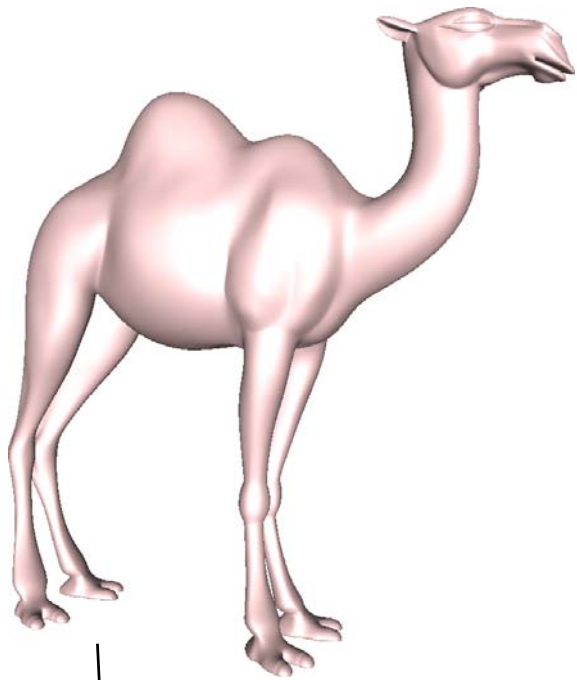
## The Camel

- Closed
- Genus 0
- Sock-like shapes



how to cut it? ->

# Camel pelting

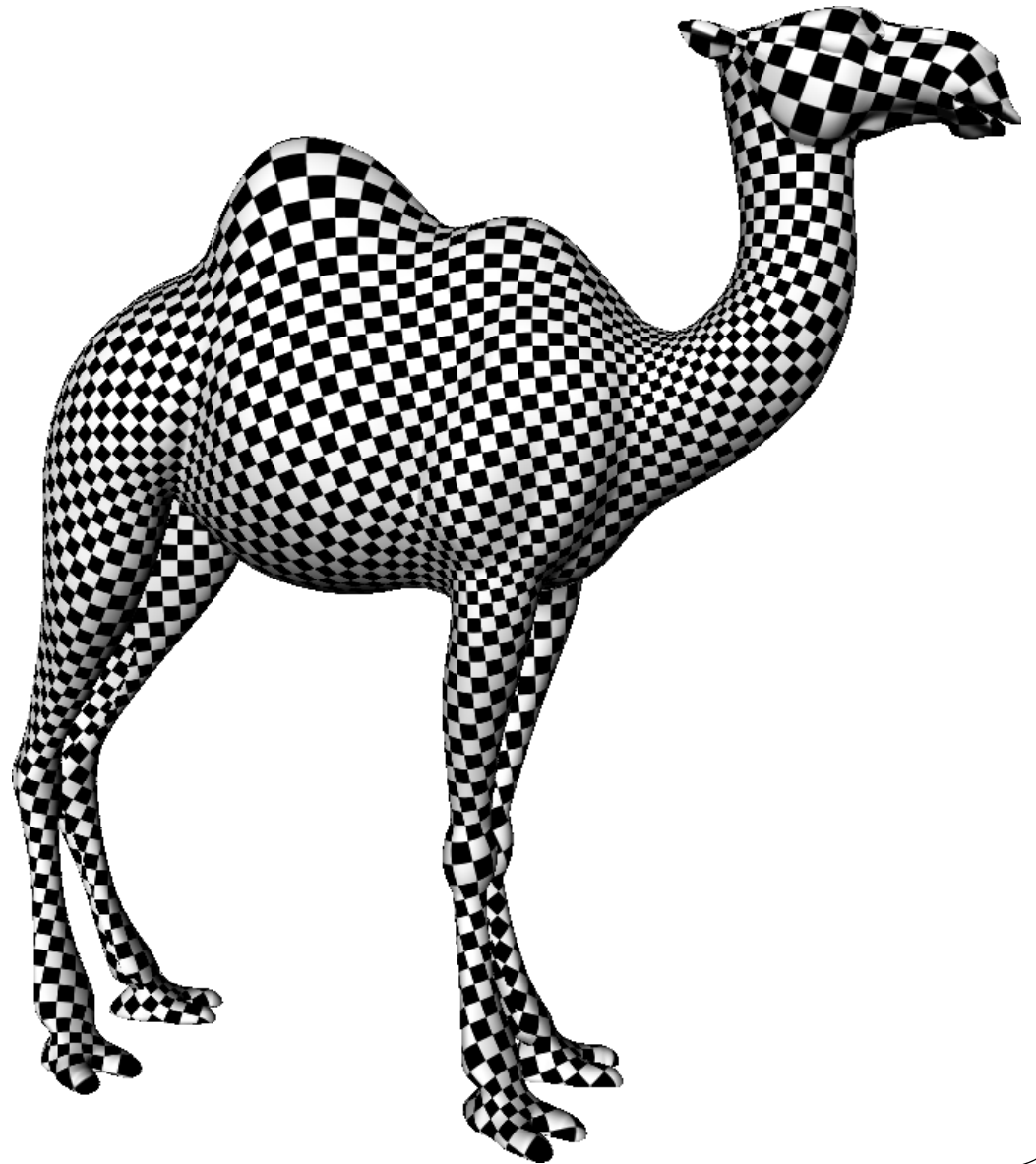


smart cut by [Sheffer]

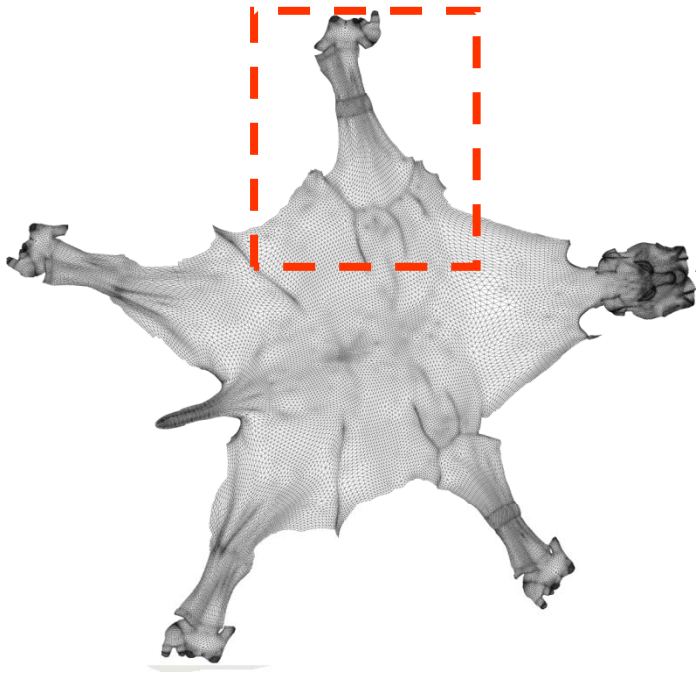
# Smart pelting & Parameterization



[Sheffer]

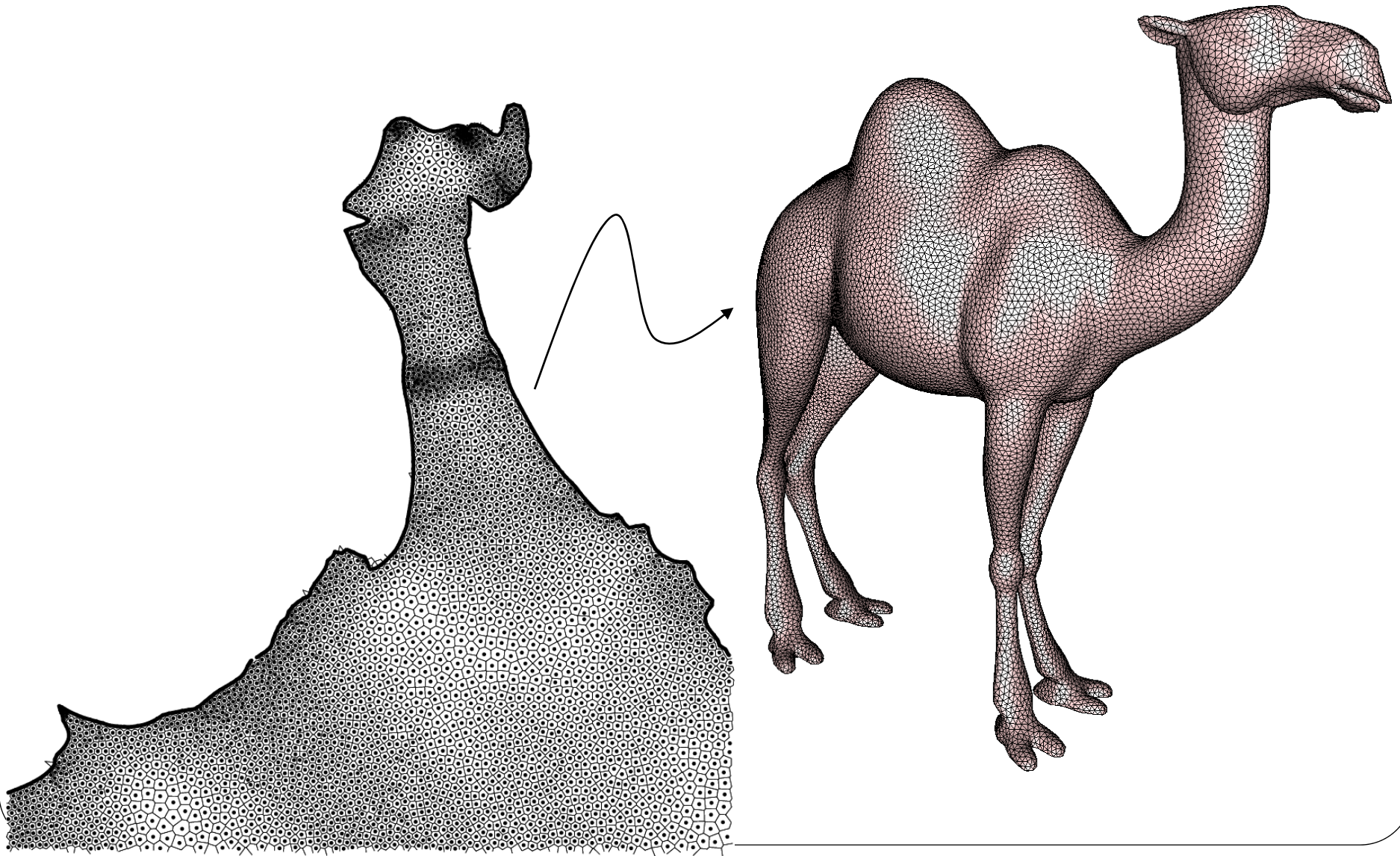


# Uniform remeshing

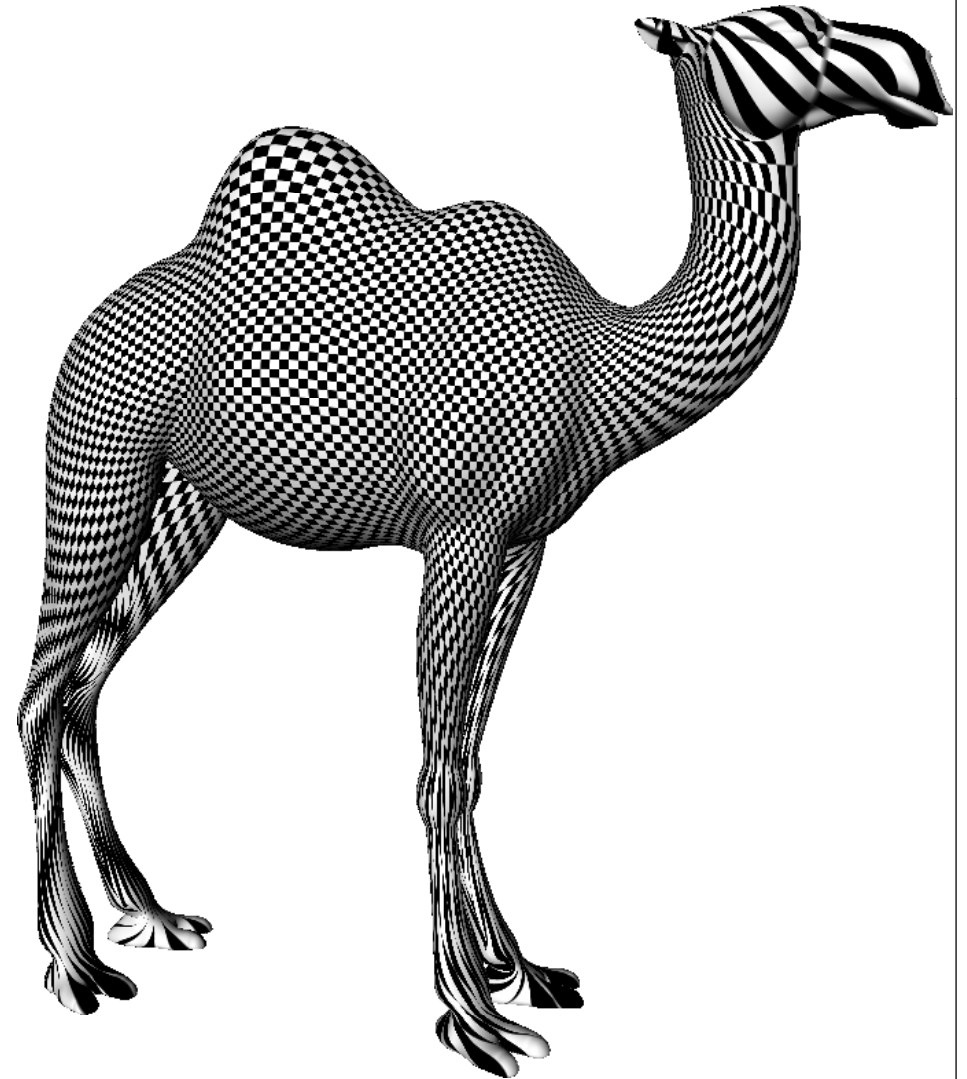
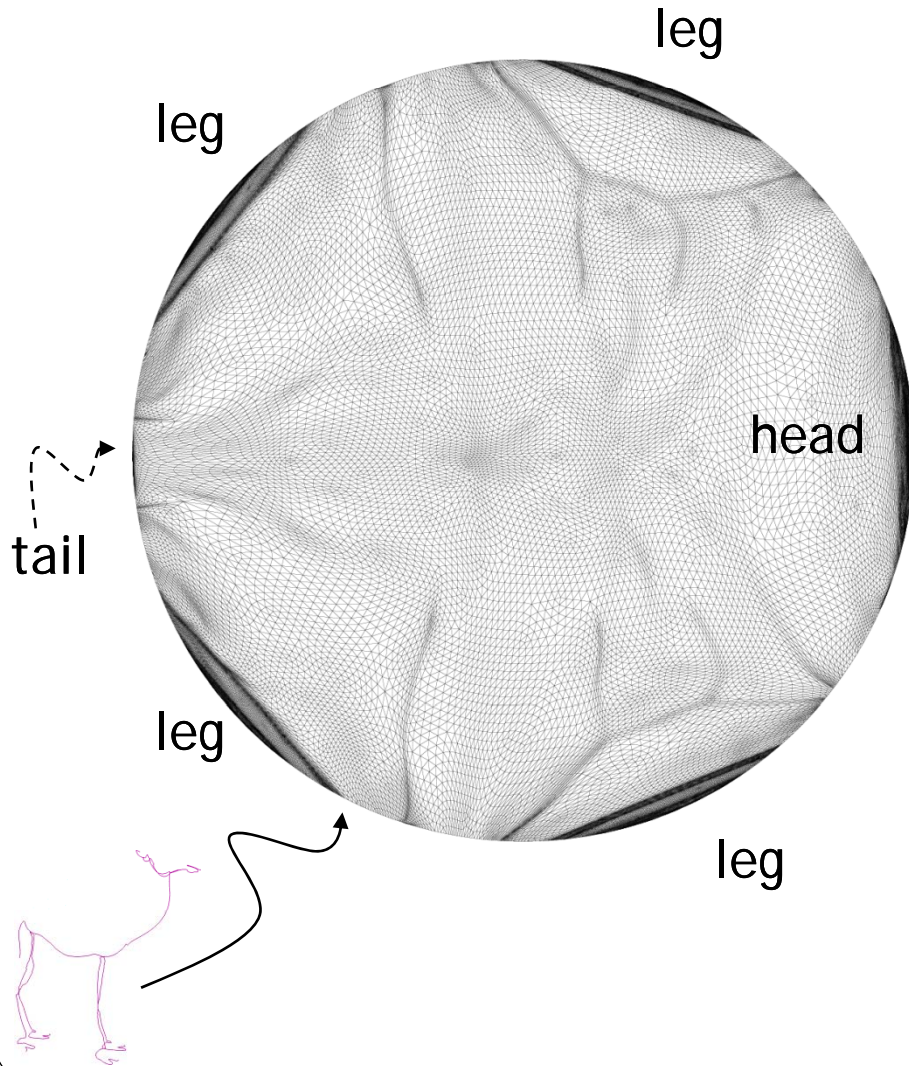




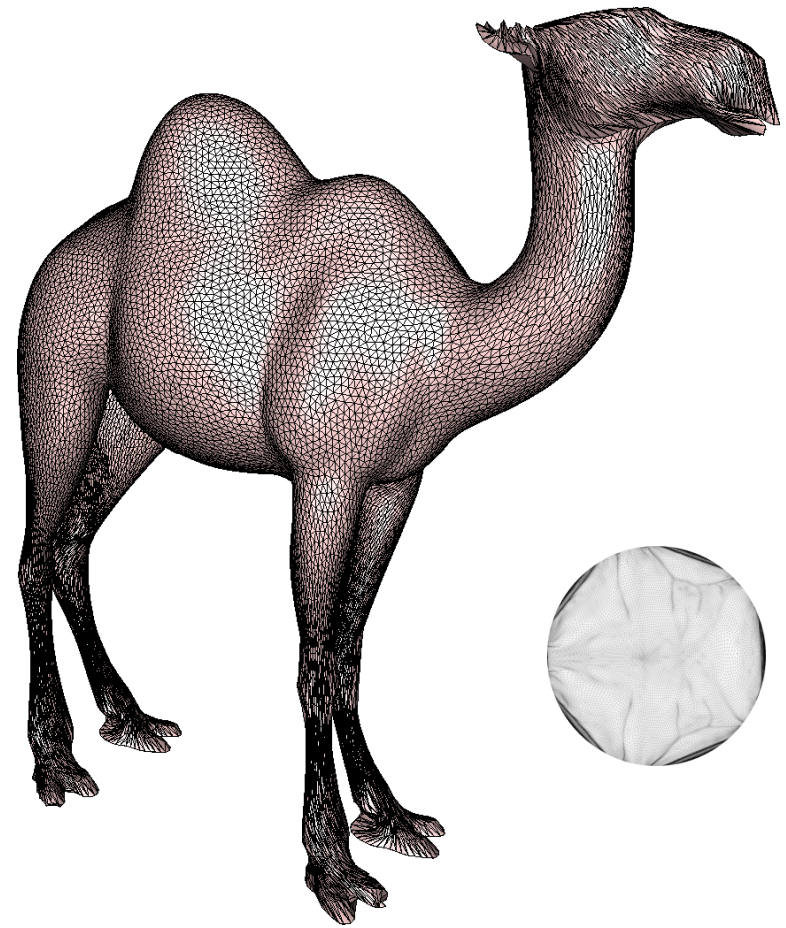
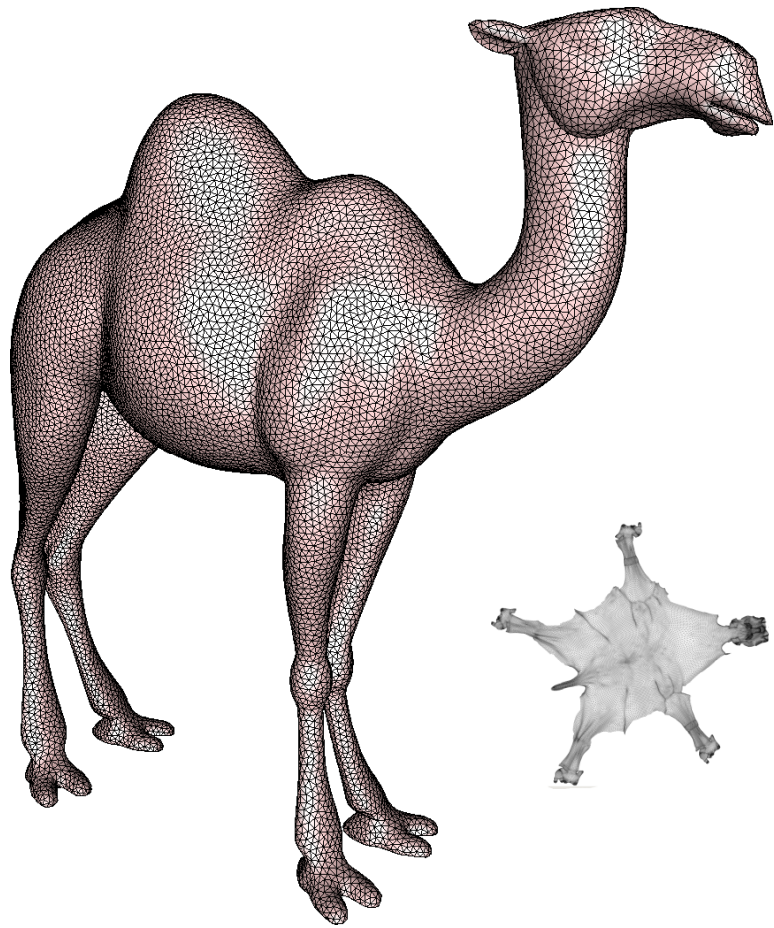
# Uniform remeshing



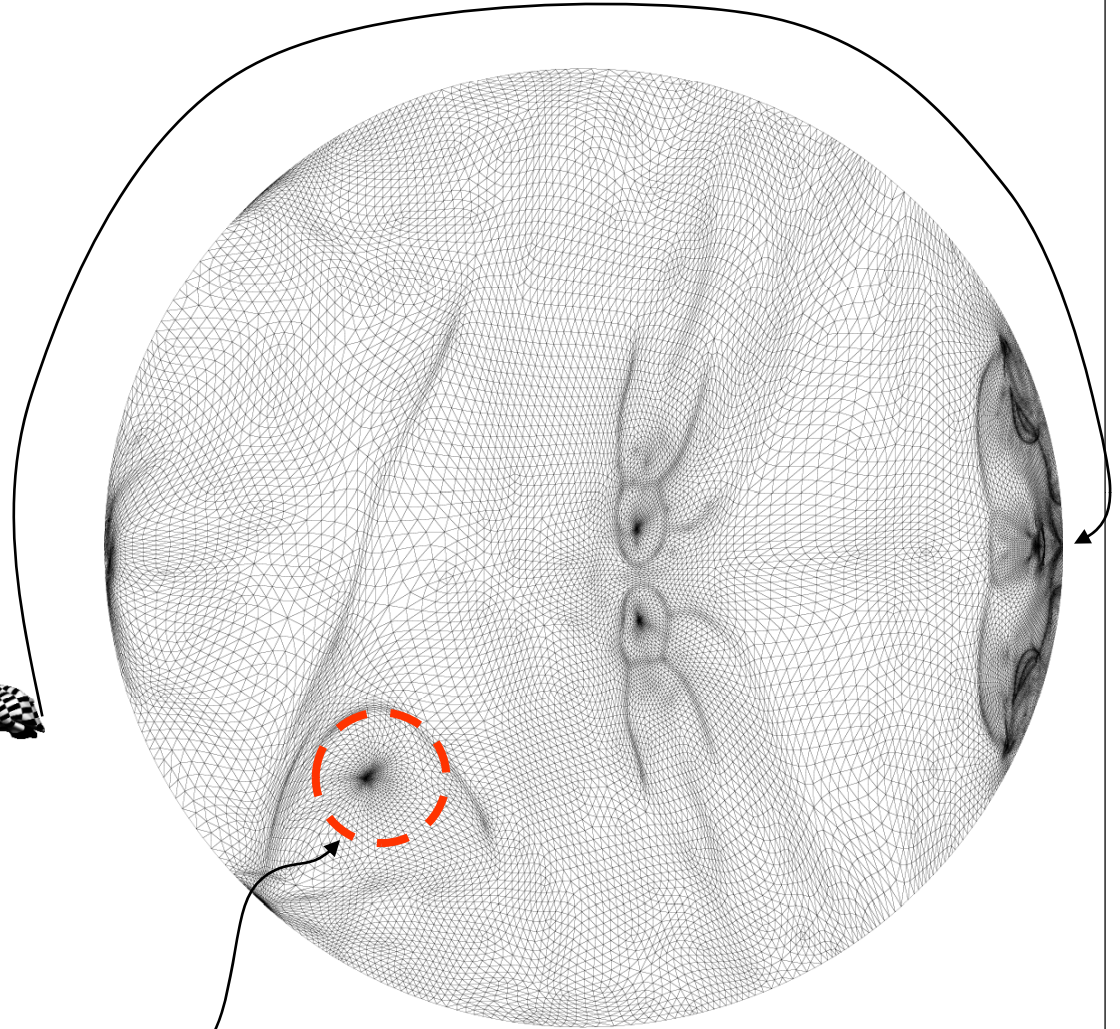
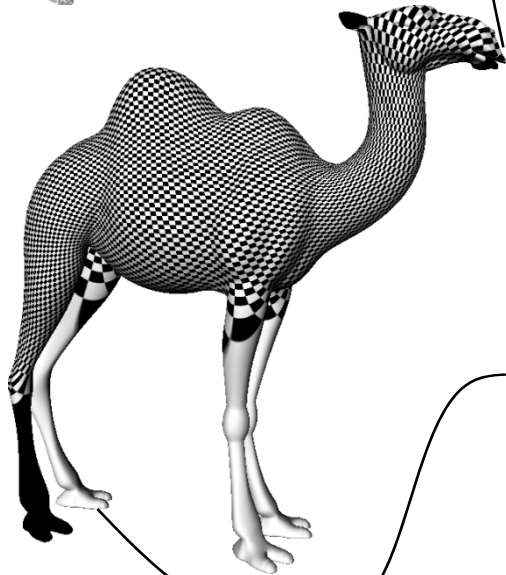
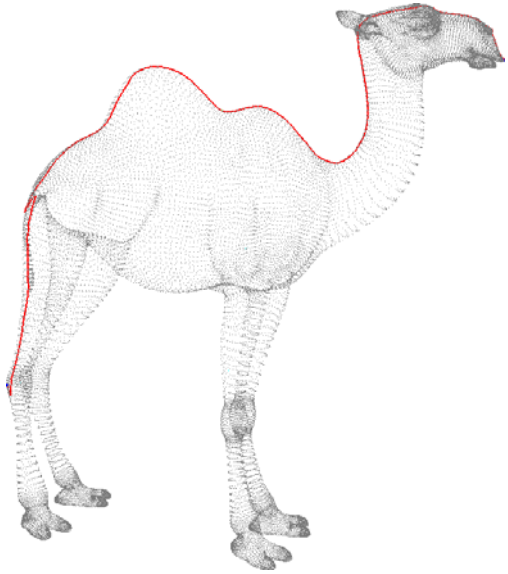
# Same cut, fixed boundary ?



# Remeshing with Free vs fixed boundary

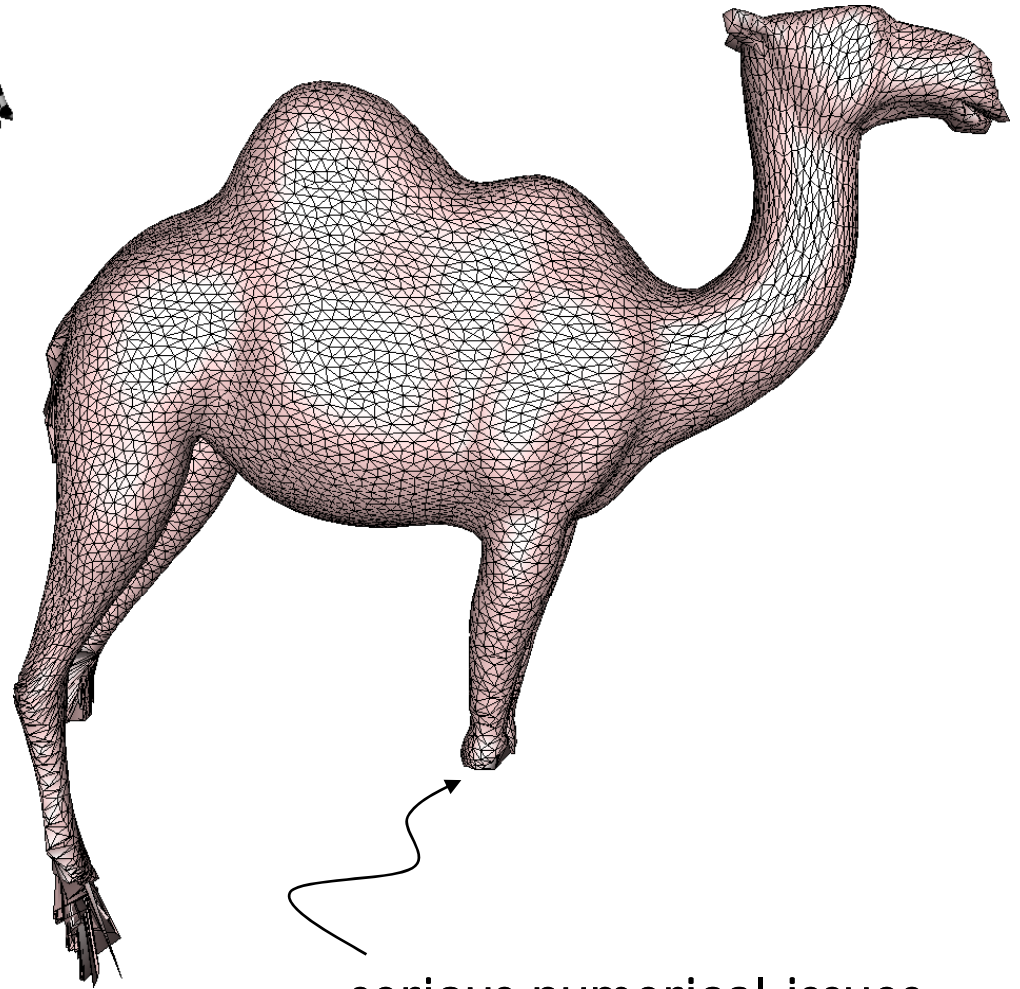
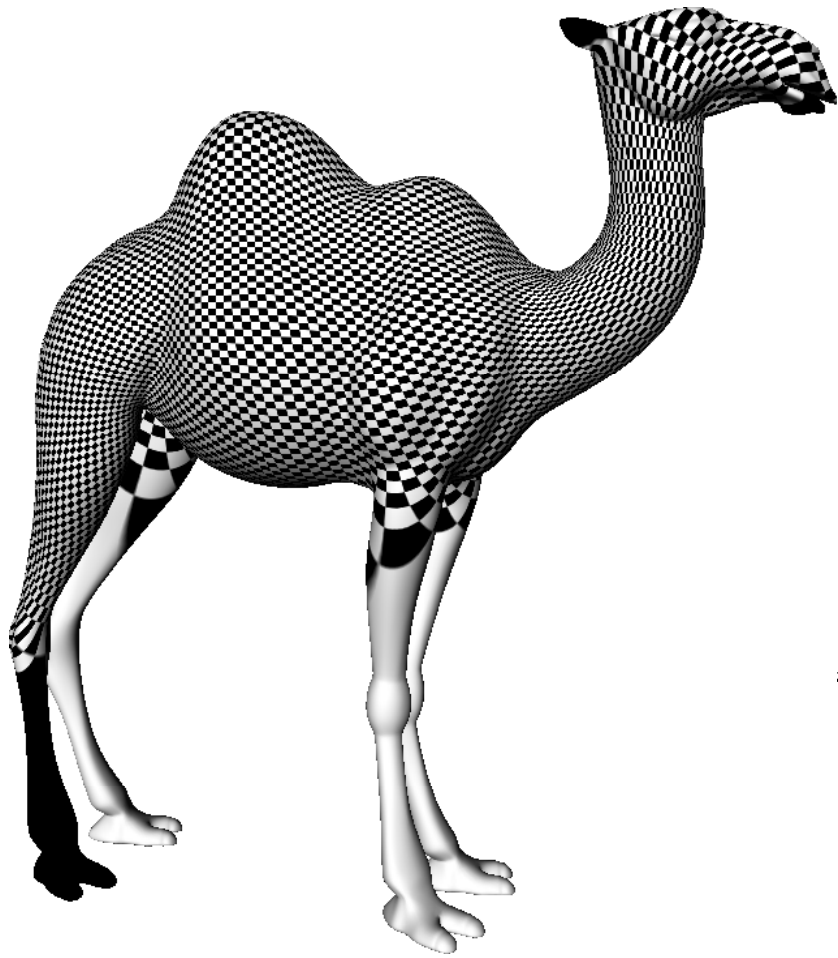


# A naive cut...



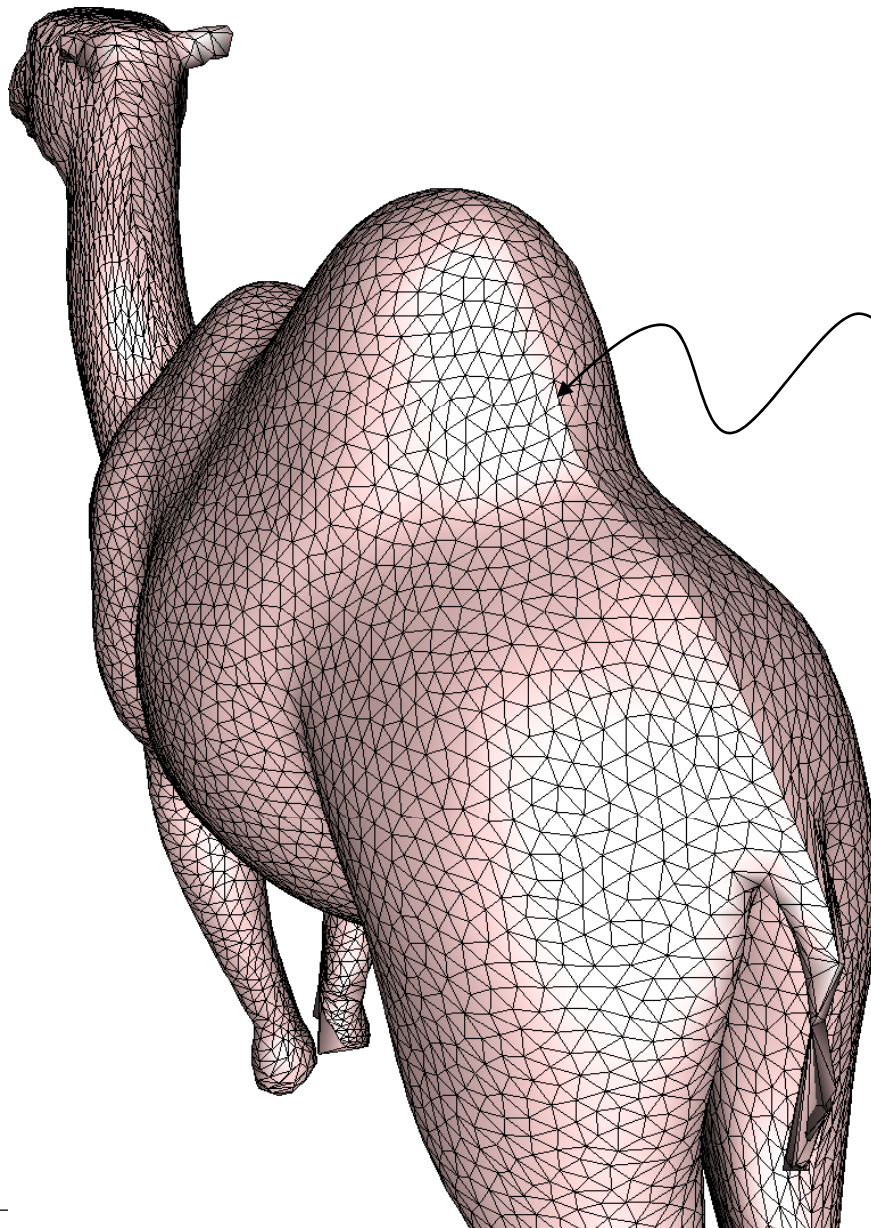
huge isoperimetric distortion

# Numerical issues



serious numerical issues  
with sock-like shapes

Moreover...



visible seam (cut graph  
has been sampled like a  
set of curves)

Motivation

Previous work

Contributions

Algorithm

Results

Limitations

Conclusions

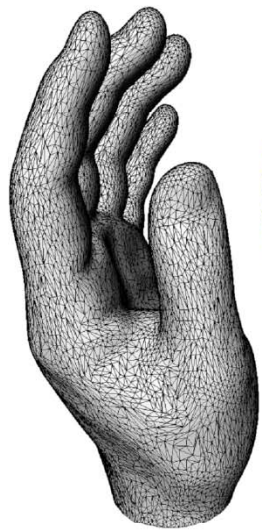
Future Work

# Conclusion

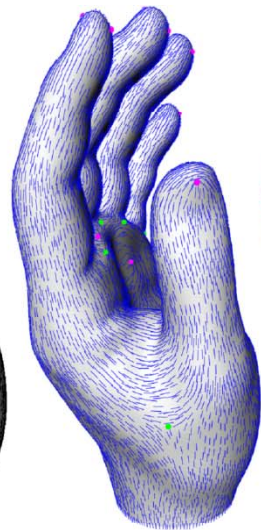
- **Guarantee**: vertex budget
- **Centroidal Voronoi diagram**: captures the essence of isotropic sampling
- **Flexible** design through density
- Handle **features**
- Handle important area distortion
- Still some limitations



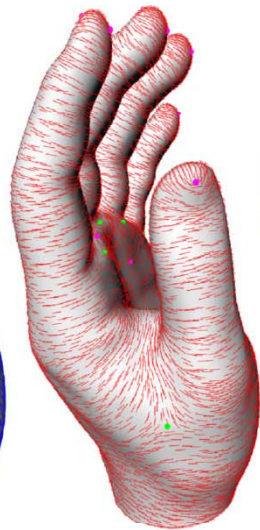
# Anisotropic Remeshing



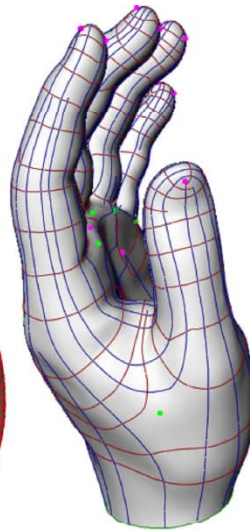
*input mesh*



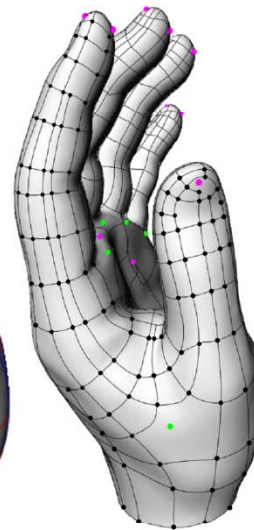
*direction fields*



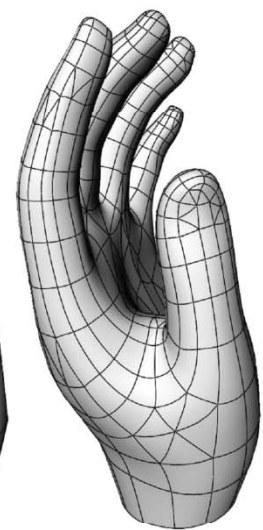
*sampling*



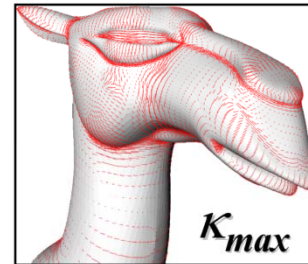
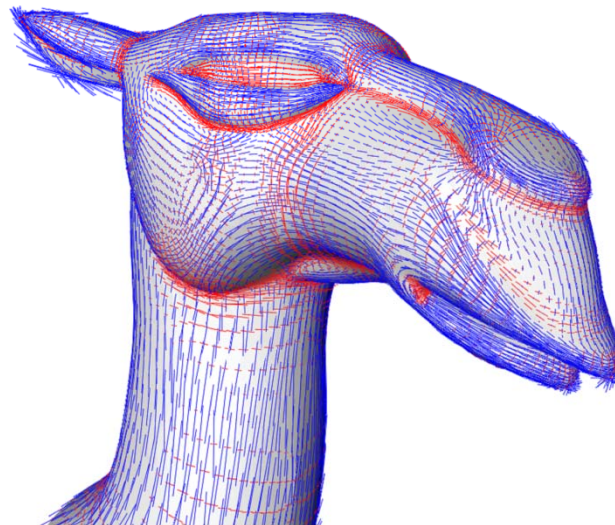
*meshing*



*output mesh*



*after smoothing*



$K_{max}$



$K_{min}$